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Design of the Sliding Mode Controller for Position Control Systems

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Abstract: A sliding mode controller for the DC motor position systems is presented in this paper. The system exists the high nonlinearity caused by the Coulomb friction torque. The controller is designed from the normal variable structure system, then the smoothness of discontinuous control law is obtained by replacing the sigum function by an unit saturation function to reduce the chatter, and then the discrete time control laws are deduced. The algorithm proposed is easily calculated and robust. It is implemented in a DSP-based DC motor position system, and shows the satisfactory results.

Key words: sliding mode control; variable-structure system; error state-space

1 Introduction

Sliding mode control is a special control strategy in which the forcing variable has an ON-OFF shape. In the way an equivalent high gain is applied on the system. Sliding mode control has been used in many different fields. The general application of the sliding model control has been presented by Utkin[1], Slotine[2] and Sarpturk[3]. It has also been successfully applied to motion control problem due to the robustness to the relevant parameter variation and the relevant nonlinaer phenomena in the mechanical load during the varying fast motion[4]. By the design of a suitable control strategy a high quality performance can be attained.

In this paper, a control methodology to achieve accurate position tracking for the DC servomotor caused by the Coulomb friction torque is developed. It is designed by using the normal design method of variable structure system, then the smoothness of the discontinuous control law is obtained by replacing the sigum function by an unit saturation function to reduce the chatter, and then the discrete time control laws are deduced. At the end, the algorithm proposed is implemented in real time and shown completely robustness, simplicty, and easy implementation.

2 Sliding Mode Control

There are several types of friction in a servo drive system. Viscous friction, Coulomb friction and stiction. In practice, the frictions' behaviour appears a nonlinear exponential function (De Carli et al.), and this makes the system the high nonlinearity and it is difficult to be controlled by the conventional controller.

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The description of the differential equation of the electromechanical DC servomotor supplied by the current source is

$$\frac{d\theta}{dt} = \omega(t), \quad \frac{d\omega(t)}{dt} = -\frac{B}{J}\omega(t) + \frac{K_t}{J}i_a - \frac{T_L}{J}, \tag{1}$$

where θ and ω are the angular position and angular velocity respectively; J, B and i_a are the moment of inertia, viscous coefficient, and control current, respectively. All interconnected terms, external disturbances and parameters variations are included in the load torque T_L.

We assume that the disturbance consist of two parts: one is measurable and other is unmeasurable but we know the upper boundary T_l. In our case the Conlomb friction in mechanical system can be estimated by the form

$$T_f(\dot{\theta}) = \begin{cases} T_c & \text{with } \dot{\theta} > 0, \\ -T_c & \text{with } \dot{\theta} < 0. \end{cases} \tag{2}$$

where T_c is a positive constant. So we have $T_L = T_f + T_l$.

The objective of the control is to design a controller to compensate the nonlinear friction and make the output track the desired input. The system should be robust to the parameter variation and disturbances.

For the desired input θ_d, the tracking error is defined as follows:

$$e(t) = \theta_d(t) - \theta(t), \quad \dot{e}(t) = \dot{\theta}_d(t) - \dot{\theta}(t). \tag{3}$$

For sliding mode design to track $\theta(t) \equiv \theta_d(t)$ switch surface $s(\theta,t)$ should be defined. It is generally a linear, stable differential operator acting on the error $e(t)$ in the state space. In our case the sliding mode motion along a straight line is selected $s(\theta,t) = c \cdot e(t) + \dot{e}(t)$, where $e(t)$ is defined by Eq. (3) and c is a positive constant which defines the bandwidth of the error dynamics.

A sufficient sliding condition for such stability is to select the control to satisfy:

$$V = \frac{s^2}{2} > 0, \quad \dot{V} = s \cdot \dot{s} < 0. \tag{4}$$

Under this condition, the system's motion will be confined to the surface $s = 0$ after reaching it. During sliding motion, s remains zero, and the following differential equation

$$s = c \cdot e + \dot{e} = 0 \tag{5}$$

is governing the system dynamics. The error exponentially tends to zero $e(t) = e(0) \cdot \exp(-ct)$. Thus the second order system behaves like an asymptotically stable first order system with time constant $1/c$ during the sliding mode.

3 Control Algorithms

From Eq. (5) we can obtain the dynamics of s as

$$\dot{s} = \frac{ds}{dt} = c \cdot \dot{e} + \ddot{e} = \ddot{\theta}_d - \ddot{\theta} + c \cdot \dot{e} = \frac{B}{J}\omega(t) - \frac{K_t}{J}i_a + \frac{T_L}{J} + \ddot{\theta}_d + c \cdot \dot{e}. \tag{6}$$

when $T_l = 0$, for the system with Conlomb friction T_f only, and let $\dot{s} = 0$. Solve $i_a = i_c$, that is the control i_c that satisfies the condition $ds/dt = 0$, becomes

$$i_c = \frac{J}{K_t}\left(\frac{B}{J}\dot{\theta} + \ddot{\theta}_d + c \cdot \dot{e} + \frac{T_f}{J}\right), \tag{7}$$

The value of i_c is effectively the equivalent control, which maintains the state varibale on the switching surface. When the system has the unmeasurable disturbance with the upper boundary T_l, the discontinuous control that satisfies the stability condition (4) is given as

$$i_a = i_c + K \cdot \mathrm{sgn}(s), \tag{8}$$

the dynamics of s becomes

$$\dot{s} = \frac{T_l}{J} - \frac{K_t}{J} \cdot K \cdot \mathrm{sgn}(s), \quad \text{with} \quad K > \frac{T_l}{K_t}, \tag{9}$$

in which $\mathrm{sgn}(s) = -1$ for $s < 0$, and $\mathrm{sgn}(s) = 1$ for $s > 0$.

By taking into account Eq. (1), the expression (8) can be presented in the form

$$i_a = \frac{J}{K_t}(\frac{B}{J}\dot{\theta} + \ddot{\theta}_d + c \cdot \dot{e} + \frac{T_f}{J}) + K \cdot \mathrm{sgn}(s). \tag{10}$$

4 Smoothness of the Discontinuous Control Laws

Control laws which satisfy the sliding condition (6) are discontinuous across the surface $s(t)$, which can be rewritten as

$$i_a = \begin{cases} i_c + K = i^+, & s > 0, \\ i_c - K = i^-, & s < 0. \end{cases}$$

Under this control law, when the error state variable $e(t)$ crosses the switching line (5) and enters the region $s < 0$, the control value $i_a(t)$ is immediately altered from i^+ to i^-, this causes the state trajectory re-crosses the switching line and enters the region $s > 0$. In this way, the state $e(t)$ is constrained to remain on the switching line $s = 0$ by the control which oscillates between the value i^+ and i^-. This leads to control chattering. In general, chattering is undesirable in practice, to reduce this unwanted control chattering, Slotine and Sastng[2] proposed a "boundary layer" approach which approximates the ideal relay characteristics used by linear saturated amplifier characteristics.

Suppose the objective of control is to make $|e| = |\theta_d - \theta| \leqslant \varepsilon$, a thin boundary layer of s can be selected by $|s| \leqslant \varepsilon c$. In this way, when $|s|$ is within the boundary layer, the control law i_a is chosen by the unit saturation type control function; when $|s| > \varepsilon c$, the control law i_a is the same as before. The control law i_a now becomes

$$i_a = \frac{J}{K_t}(\frac{B}{J}\dot{\theta} + \ddot{\theta}_d + c \cdot \dot{e} + \frac{T_f}{J}) + K \cdot \mathrm{sat}(\frac{s}{\varepsilon c}), \tag{11}$$

where the function sat is defined by

$$\mathrm{sat}(\frac{s}{\varepsilon c}) = \begin{cases} \frac{s}{\varepsilon c}, & |s| \leqslant \varepsilon c, \\ 1, & |s| > \varepsilon c. \end{cases}$$

and the dynamics of equation (6) takes the form of $\dot{s} + \frac{s}{\varepsilon c} = 0$.

5 Discrete-Time Sliding Mode Control Laws

In order to calculate the discrete control law, the discrete time model of the DC motor should be estimated first. Standard linear parameter estimation methods may be applied the Eq. (1) where signals are sampled and filtered. The filter can be optimised based on knowledge of the noise and the known parameters. All estimation algorithms can be characterised by the error model $e(t) = y(t) - \Phi^{T}(t) \cdot \theta$, where the vector $y(t)$ and the regression vector Φ are functions of the data and θ is the vector of the unknown parameters. A recursive least-squares algorithm is then given by the normal estimation method. More details of these techniques can be found in Isermann[6] and Landau[7].

An alternative is to derive a zero-order hold model for the motor representation. In our case the ARMAX model is used. The model for the parameter identification is valid in the linear domain. The transfer function of actuator in z-domain is then obtained by

$$G_p(z) = \frac{bz^{-1}}{(1 - z^{-1})(1 + az^{-1})} = \frac{bz^{-1}}{1 + (a - 1)z^{-1} - az^{-2}}. \tag{12}$$

The discrete-time position control system can be then represented by the following form

$$y(k + 1) = (1 - a) \cdot y(k) + a \cdot y(k - 1) + b \cdot i(k). \tag{13}$$

In a similar manner, the sliding surface is defined for the tracking control problem, in the error space as

$$s(k + 1) = e(k + 1) + \lambda \cdot e(k) \tag{14}$$

and

$$e(k) = r(k) - y(k), \quad \lambda = e^{-T/c}.$$

The sliding condition given by Eq. (4) must be modified. A corresponding version for the discrete system is needed. Let us define first a discrete Lyapunov function for this purpose:

$$V(k) = s^2(k) \tag{15}$$

and then use

$$\Delta V = s^2(k + 1) - s^2(k) \leqslant 0 \tag{16}$$

to represent the discrete sliding condition. According to Sarpturk et al.[3] Eq. (14) can be decomposed into two inequalities as follows:

$$[s(k + 1) - s(k)] \cdot \mathrm{sgn}[s(k)] \leqslant 0, \tag{16a}$$

$$[s(k + 1) + s(k)] \cdot \mathrm{sgn}[s(k)] \geqslant 0. \tag{16b}$$

Eqs. (16a) and (16b) are a sliding condition and a convergence condition, respectively. If discrete dynamics of s is chosen as

$$s(k + 1) = p \cdot s(k), \tag{17}$$

where $p = e^{-T/ec}$ and T is the sampling time, which satisfies Eq. (16). Eq. (17) corresponds to the continuous a dynamics given by (11) for the discrete case. Using Eqs. (13), (14) and (17), we have

$$\begin{aligned} s(k + 1) &= e(k + 1) + \lambda e(k) = r(k + 1) - y(k + 1) + \lambda y(k) \\ &= r(k + 1) - [(1 - a)y(k) + ay(k - 1) + bi(k)] + \lambda r(k) - \lambda y(k). \end{aligned} \tag{18}$$

the discrete control laws, which is to be applied at the instant k, to make $s(k + 1)$ equal to zero, can now be obtained as

$$\begin{aligned} i_a(k) = {} & \frac{1}{b}[(a - 1 - \lambda)y(k) + r(k + 1) + \lambda \cdot r(k) - a \cdot y(k - 1)] + \frac{T_f}{K_t} \\ & + K \cdot \mathrm{sat}(ps), \end{aligned} \tag{19}$$

where $0 < p = e^{-T/ec} < 1$.

From the Eqs. (14) and (17), the dynamics of the sliding surface in discrete time are then governed by the following equation: $e(k + 1) - (\lambda + p)e(k) + \lambda \cdot p \cdot e(k - 1) = 0$. Now the dynamic behaviour depends on the sliding equation only. If we define: $t_1 = -(\lambda + p)$, and $t_2 = \lambda \cdot p$. From the stable condition of the Jury's test, the stable values can be obtained in the ranges of

$$\begin{cases} -1 < t_2 < 1, \\ t_1 > -1 - t_2, \\ t_1 < 1 + t_2. \end{cases} \quad \text{with the selection:} \quad \left.\begin{matrix} t_1 = -1.8 \\ t_2 = 0.8075 \end{matrix}\right\} \rightarrow \begin{cases} \lambda = 0.85, \\ p = 0.75. \end{cases}$$

6 Experimental Results

The block diagram of the system and the experiment system is shown in Fig. 1 and Fig. 2. It is based on a IBM PC 386 equipped with a DSP-32C board. The actuator is a DC servomotor current supplied by PWM power amplifier. The slidig mode control strategy has been programmed in the assembly language of the DSP32C. All the variables used in the control system are D/A converted, scaled, and amplified to produce the current command of te servomotor. The position is measured by a potentiometer and converted by a 16-bit A/D converter. The position control system model is identified in discrete-time by MATLAB as

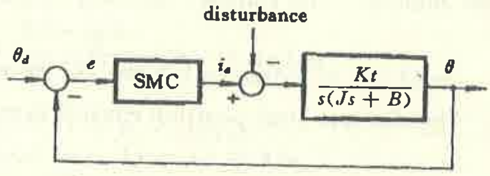


Fig. 1 Block diagram of the control system

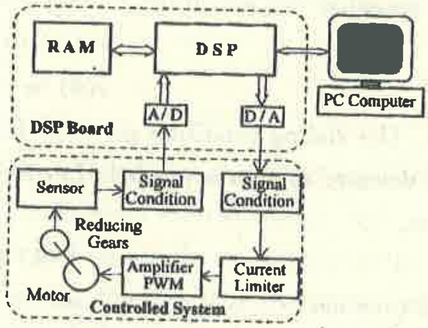


Fig. 2 Block diagram of the experimental system

$$y(k+1) = 1.9932y(k) - 0.9932y(k-1) + 0.01 i_a(k), \tag{20}$$

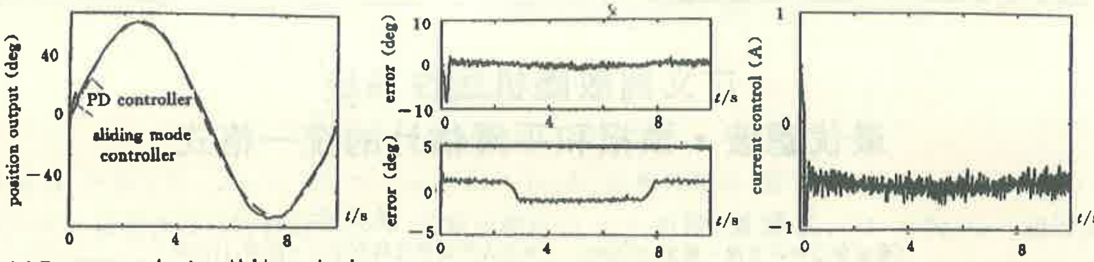
the control law described by (19) is

$$i_a = \frac{1}{0.01}[-1.8432y(k) + r(k+1) + 0.85r(k) + 0.9532y(k-1)] + \frac{T_f}{K_t} + K\mathrm{sat}(0.75s), \tag{21}$$

which is written in assemble language. by selecting value of K the experimental results of the system to the sinusoidal input signal is shown in Fig. 3, For the comparison purpose, we put also the response of the PD controller. It is shown by the dotted line in Fig. 3(a), from which it is seen that when using the conventional PD controller, the position output shows the hysteresis when the motion inverses the direction due to the existence of the nonlinear frictions, while the better response is obtained by sliding mode controller, where the errors caused by the parameter variation and uncertainty disturbance (in our case is the nonlinear frictions) make the system a transient period to achieve the sliding surface and sliding motion occurs for $t > 0.5$s. The errors using sliding mode controller and PD controller are shown in Fig. 3(b), and Fig. 3(c) is the control value.

7 Conclusions

Variable-structure control of the DC servo drive position control system has been described and implemented in discrete time. By ensuring sliding motion on the switching sur-



(a) Responses to the sinusoidal input signal (b) above: error for SM controller below: error for PD controller (c) the control value

Fig. 3 Some response of the sliding mode control system

face, insensitivity to parameter variations and disturbances is achieved. The design technique is straight forward and requires little computational effort. On the other hand, by substituting smooth transitions acrossing a boundary layer to control switching at the sliding surface the chatter is reduced. The experiment has shown the satisfactory ersults.

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位置控制系统滑模控制器的设计

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摘要：介绍了为直流电机控制系统所设计的滑模控制器.原系统存在着由库仑摩擦力矩所导致的非线性.控制器根据常规的变结构系统进行设计,然后通过符号函数的替代来平滑不连续的控制法则,并用一种单位饱和函数来减少其抖动.最后推导出离散时间的控制法则.所提出的算法计算容易且有鲁棒性,已在以数字信号处理器 DSP 为基础的直流电机位置控制系统上得到实现,并显示出令人满意的结果.

关键词：滑模控制；变结构系统；误差状态空间

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