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Stability of Nonlinear Closed-Loop Control System Based on Generalized Frequency Response Functions

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Abstarct: Based on the representation of generalized frequency response functions (GFRF) for a class of nonlinear control systems, the open-loop stability has been investigated in [1], and the closed-loop stability is discussed in this paper, and some input-output stability criterions in the frequency domain are given as well. A simulation example is used for verifying the results.

Key words: nonlinear control system; frequency response function; closed-loop stability

1 Introduction

In recent twenty years, the control theory has been developed so vast and complex that control engineers can not totally understand what to do on earth in the theoretical research. The traditional PID controllers are still used in most of the industrial control systems. It has been a perfect irony and a powerful challenge for the theoretical reserch in control theory. The classical linear frequency analysis (LFA) method is still used to design the industrial controllers not only for its simplicity and utility, but also for its evident physical explanation and experimental verifiability. Control engineers, however, have been puzzled by the nonlinearity in control systems for a long-time. The nonlinear frequency analysis (NFA) method based on GFRF's[2~5] provides a new thought to solve the nonlinear control problems. It is, indeed, an extension of the classical LFA, and easy to be accustomed by engineers. Among all control problems, the stability is the most important one.

An open-loop stability criterion for the polynomial class of nonlinear control systems has been proposed in authors' published paper[1], i. e., based on GFRF's, the sufficient conditions for input-output stability have been given, and the further stability conditions for different special cases have been discussed respectively. In this paper, the relationship between the open-loop GFRF's and the closed-loop GFRF's will be given, and the closed-loop stability conditions will be discussed. In Section 2, a description of the nonlinear closed-loop stability problems will be described, and a general relationship between the open-loop GFRF's and the closed-loop GFRF's will be given in the Section 3. In the Section 4 of this paper, the closed-loop stability conditions will be discussed. Finally, a simulation example is used for verifying the above results.

2 Description of Nonlinear Closed-Loop Stability Problems

Consider the single input-single output nonlinear control system shown in Fig. 1.

Where u ——control; y ——plant output; r ——reference input; w ——disturbance in-

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put; z —— process component of y; v —— disturbance component of y; e —— tracking error; H —— nonlinear operatior of the process, i. e., $z = H(u)$; S —— nonlinear operator of the controller, i. e., $u = S(e)$; P —— nonlinear operator of the disturbance, i. e., $v = P(w)$.

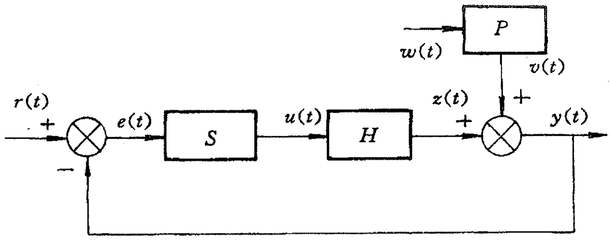

Fig. 1 Diagram of a nonlinear closed-loop control system

Then, the closed-loop control system can be described as

$$y = z + v = H(u) + P(w), \tag{2.1}$$

$$u = S(e), \tag{2.2}$$

$$e = r - y. \tag{2.3}$$

Assumption 2.1 All nonlinear operators in (2.1)~(2.3) have the Volterra series representations, i. e.,

$$z(t) = \sum_{n=1}^{\infty} z_n(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \cdots, \tau_n) \prod_{i=1}^{n} u(t - \tau_i) \mathrm{d}\tau_i, \tag{2.4}$$

$$v(t) = \sum_{n=1}^{\infty} v_n(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p_n(\tau_1, \cdots, \tau_n) \prod_{i=1}^{n} w(t - \tau_i) \mathrm{d}\tau_i, \tag{2.5}$$

$$u(t) = \sum_{n=1}^{\infty} u_n(t) = \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} s_n(\tau_1, \cdots, \tau_n) \prod_{i=1}^{n} e(t - \tau_i) \mathrm{d}\tau_i, \tag{2.6}$$

and corresponding GFRF's $\hat{h}_n(\omega_1, \cdots, \omega_n)$, $\hat{p}_n(\omega_1, \cdots, \omega_n)$ and $\hat{s}_n(\omega_1, \cdots, \omega_n)$ are proper rational fractions of ω_i, $i = 1, 2, \cdots, n$; $\forall\, n \in \mathbb{N}$.

Assumption 2.2 $r, e, y, z, v \in X$, $u \in U$, $w \in W$, where X, U and W are some extended Banach spaces, and

$$H: U \to X,\ P: W \to X,\ S: X \to U. \tag{2.7}$$

Define the compound operator $L = HS: X \to X$, and the sum operator $I + L: X \to X$, where $I: X \to X$ is the identity operator. Then the system (2.1)~(2.3) can be written as

$$y = L(e) + P(w); \tag{2.8}$$

$$e = r - L(e) - P(w). \tag{2.9}$$

Or

$$(I + L)(e) = r - P(w). \tag{2.10}$$

If the operator $I + L$ has its inverse operator $(I + L)^{-1}: X \to X$, then

$$e = (I + L)^{-1}(r) - (I + L)^{-1}P(w), \tag{2.11}$$

$$y = L(I + L)^{-1}(r) + [I - L(I + L)^{-1}]P(w). \tag{2.12}$$

Definition 2.1 The nonlinear closed-loop control system described by (2.1)~(2.3) is L_p-stable, if

$$r, w \in L_p \Rightarrow e, y \in L_p, \quad 1 \leqslant p \leqslant \infty, \tag{2.13}$$

where L_p is the Banach space with norm as

$$\| x \|_p = \left(\int_{-\infty}^{\infty} |x(t)|^p \mathrm{d}t \right)^{1/p}, \quad 1 \leqslant p \leqslant \infty; \quad \| x \|_\infty = \operatorname*{ess\,sup}_{x \in \mathbb{R}} |x(t)|. \tag{2.14}$$

Definition 2.2 For the nonlinear control system (2.1)~(2.3), If $w(t) \equiv 0, \forall\, t \in \mathbb{R}$, it is said to be the tracking stability problem; if $r(t) \equiv 0, \forall\, t \in \mathbb{R}$, it is said to be the disturbance stability problem.

3 Relationship Between Open-Loop GFRF's and Closed-Loop GFRF's

At first the GFRF's of the compound operator are considered.

Lemma 3.1 Let X and U be extended Banach spaces, $H:U\to X$, and $S:X\to U$ be nonlinear operators with GFRF's $\{\hat{h}_n\}$ and $\{\hat{s}_n\}$ respectively, the compound operator $L=HS:X\to X$ has GFRF's $\{\hat{l}_n\}$ and

$$\hat{l}_1(\omega_1)=\hat{h}_1(\omega_1)\hat{s}_1(\omega_1), \tag{3.1}$$

$$\cdots$$

$$\hat{l}_n(\omega_1,\cdots,\omega_n)=\sum_{m=1}^{n}\sum_{k_1+\cdots+k_m=n}\hat{h}_m(\omega_1+\cdots+\omega_{k_1},\omega_{k_1+1}+\cdots+\omega_{k_1+k_2},\cdots,\omega_{n-k_m+1}+\cdots+\omega_n)$$
$$\cdot\hat{s}_{k_1}(\omega_1,\cdots,\omega_{k_1})\hat{s}_{k_2}(\omega_{k_1+1},\cdots,\omega_{k_1+k_2})\cdots\hat{s}_{k_m}(\omega_{n-k_m+1},\cdots,\omega_n),\ \forall\, n\in\mathbb{N},\ n\geqslant 2, \tag{3.2}$$

where $k_i\in\mathbb{N}$.

Proof Because

$$\hat{z}(\omega)=\sum_{n=1}^{\infty}\hat{z}_n(\omega)$$
$$=\sum_{m=1}^{\infty}(2\pi)^{-(m-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{h}_m(\omega-\omega_2-\cdots-\omega_m,\omega_2,\cdots,\omega_m)$$
$$\cdot\hat{u}(\omega-\omega_2-\cdots-\omega_m)\hat{u}(\omega_2)\cdots\hat{u}(\omega_m)\mathrm{d}\omega_2\cdots\mathrm{d}\omega_m$$
$$=\sum_{m=1}^{\infty}(2\pi)^{-(m-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{h}_m(\omega-\omega_2-\cdots-\omega_m,\omega_2,\cdots,\omega_m)$$
$$\cdot\Big[\sum_{k_1=1}^{\infty}(2\pi)^{-(k_1-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{s}_{k_1}(\omega-\omega_2-\cdots-\omega_m-\omega_2^{(1)}-\cdots-\omega_{k_1}^{(1)},\omega_2^{(1)},\cdots,\omega_{k_1}^{(1)})$$
$$\cdot\hat{e}(\omega-\omega_2-\cdots-\omega_m-\omega_2^{(1)}-\cdots-\omega_{k_1}^{(1)})\hat{e}(\omega_2^{(1)})\cdots\hat{e}(\omega_{k_1}^{(1)})\mathrm{d}w_2^{(1)}\cdots\mathrm{d}\omega_{k_1}^{(1)}\Big]$$
$$\cdot\Big[\sum_{k_2=1}^{\infty}(2\pi)^{-(k_2-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{s}_{k_2}(\omega_2-\omega_2^{(2)}-\cdots-\omega_{k_2}^{(2)},\omega_2^{(2)},\cdots,\omega_{k_2}^{(2)})$$
$$\cdot\hat{e}(\omega_2-\omega_2^{(2)}-\cdots-\omega_{k_2}^{(2)})\hat{e}(\omega_2^{(2)})\cdots\hat{e}(\omega_{k_2}^{(2)})\mathrm{d}\omega_2^{(2)}\cdots\mathrm{d}\omega_{k_2}^{(2)}\Big]\cdots$$
$$\cdot\Big[\sum_{k_m=1}^{\infty}(2\pi)^{-(k_m-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{s}_m(\omega_m-\omega_2^{(m)}-\cdots-\omega_{k_m}^{(m)},\omega_2^{(m)},\cdots,\omega_{k_m}^{(m)})$$
$$\cdot\hat{e}(\omega_m-\omega_2^{(m)}-\cdots-\omega_{k_m}^{(m)})\hat{e}(\omega_2^{(m)})\cdots\hat{e}(\omega_{k_m}^{(m)})\mathrm{d}\omega_2^{(m)}\cdots\mathrm{d}\omega_{k_m}^{(m)}\Big]\mathrm{d}\omega_2\cdots\mathrm{d}\omega_m,$$

and $\hat{z}_n$ is a function of n -tuple $\hat{e}$, and by means of comparison of degrees in $\hat{e}$, we have

$$\hat{z}_1(\omega)=\hat{h}_1(\omega)\hat{s}_1(\omega)\hat{e}(\omega)\Rightarrow\hat{l}_1(\omega_1)=\hat{h}_1(\omega_1)\hat{s}_1(\omega_1),$$
$$\hat{z}_2(\omega)=\hat{h}_1(\omega)\cdot(2\pi)^{-1}\int_{-\infty}^{\infty}\hat{s}_2(\omega-\omega_2^{(1)},\omega_2^{(1)})\hat{e}(\omega-\omega_2^{(1)})\hat{e}(\omega_2^{(1)})\mathrm{d}\omega_2^{(1)}$$
$$+(2\pi)^{-1}\int_{-\infty}^{\infty}\hat{h}_2(\omega-\omega_2,\omega_2)\hat{s}(\omega_2)\hat{e}(\omega-\omega_2)\hat{e}(\omega_2)\mathrm{d}\omega_2$$
$$=(2\pi)^{-1}\int_{-\infty}^{\infty}[\hat{h}_1(\omega)\hat{s}_2(\omega-\omega_2,\omega_2)+\hat{h}_2(\omega-\omega_2,\omega_2)\hat{s}_1(\omega-\omega_2)\hat{s}_1(\omega_2)]$$
$$\cdot\hat{e}(\omega-\omega_2)\hat{e}(\omega_2)\mathrm{d}\omega_2$$
$$\Rightarrow\hat{l}_2(\omega_1,\omega_2)=\hat{h}_1(\omega_1+\omega_2)\hat{l}_2(\omega_1,\omega_2)+\hat{h}_2(\omega_1,\omega_2)\hat{l}_1(\omega_1)\hat{l}_1(\omega_2),$$
$$\cdots$$

Similarly, (3.2) can be obtained.

Next, the inverse operators GFRF's are considered.

Lemma 3.2 Let X be an extended Banach space, $L:X\to X$ be a nonlinear operator with GFRF's $\{\hat{l}_n\}$, and $I:X\to X$ be the identity operator, the sum operator $I+L:X\to X$ has its inverse operator $G=(I+L)^{-1}:X\to X$, and its GFRF's are $\{\hat{g}_n\}$, thus

$$\hat{g}_1(\omega_1)=(1+\hat{l}_1)^{-1}, \tag{3.3}$$

$$\cdots$$

$$\begin{aligned}\hat{g}_n(\omega_1,\cdots,\omega_n) = &-(\hat{l}_1(\omega_1+\omega_2+\cdots+\omega_n))^{-1}\\ &\cdot\sum_{m=2}^{n}\sum_{k_1+\cdots+k_m=n}\hat{l}_m(\omega_1+\cdots+\omega_{k_1},\omega_{k_1+1}+\cdots+\omega_{k_1+k_2},\cdots,\omega_{n-k_m+1}+\cdots+\omega_n)\\ &\cdot\hat{g}_{k_1}(\omega_1,\cdots,\omega_{k_1})\hat{g}_{k_2}(\omega_{k_1+1},\cdots,\omega_{k_1+k_2})\cdots\hat{g}_{k_m}(\omega_{n-k_m+1},\cdots,\omega_n),\quad\forall n\geqslant 2,\end{aligned} \tag{3.4}$$

where $k_i\in\mathbb{N},\forall\ j$.

Proof Let $R=I+L:X\to X$ be a nonlinear operator with GFRF's $\{\hat{r}_n\}$, then

$$\hat{r}_1(\omega_1)=1+\hat{l}_1(\omega_1),\cdots,\hat{r}_n(\omega_1,\cdots,\omega_n)=\hat{l}_n(\omega_1,\cdots,\omega_n),\quad n\geqslant 2.$$

Because G is the inverse operator of R, then $GR=RG=I$, or

$$\hat{r}_1(\omega_1)\hat{g}_1(\omega_1)=(1+\hat{l}_1(\omega_1))\hat{g}_1(\omega_1)=1,$$

$$\begin{aligned}&\sum_{m=2}^{n}\sum_{k_1+\cdots+k_m=n}\hat{r}_m(\omega_1+\cdots+\omega_{k_1},\omega_{k_1+1}+\cdots+\omega_{k_1+k_2},\cdots,\omega_{n-k_m+1}+\cdots+\omega_n)\\ &\cdot\hat{g}_{k_1}(\omega_1,\cdots,\omega_{k_1})\hat{g}_{k_2}(\omega_{k_1+1},\cdots,\omega_{k_1+k_2})\cdots\hat{g}_{k_m}(\omega_{n-k_m+1},\cdots,\omega_n)=0,\quad\forall\ n\geqslant 2.\end{aligned}$$

Thus, (3.3) and (3.4) can be proven.

Finally, the relationship between the open-loop GFRF's and closed-loop GFRF's can be obtained.

Theorem 3.1 Suppose that the assumptions (2.1), (2.2) have been made for the nonlinear closed-loop control system (2.1)~(2.3). Let $L=HS:X\to X$ with GFRF's $\{\hat{l}_n\}$, $G=(I+L)^{-1}:X\to X$ with GFRF's $\{\hat{g}_n\}$, then the GFRF's of L and G can be calculated by (3.1) and (3.2) or (3.3) and (3.4) respectively.

Proof This result can be obtained from Lemma 3.1, 3.2 directly.

4 Closed-Loop Stability of Nonlinear Control Systems

Now, the corresponding linear component of the nonlinear closed-loop control system is considered

$$\begin{aligned}\hat{e}_1(\omega)&=\hat{g}_1(\omega)\hat{r}(\omega)-\hat{g}_1(\omega)\hat{p}_1(\omega)\hat{w}(\omega)\\ &=(1+\hat{h}_1(\omega)\hat{s}_1(\omega))^{-1}\hat{r}(\omega)-(1+\hat{h}_1(\omega)\hat{s}_1(\omega))^{-1}\hat{p}_1(\omega)\hat{w}(\omega),\end{aligned} \tag{4.1}$$

$$\begin{aligned}\hat{y}_1(\omega)&=\hat{l}_1\hat{e}_1(\omega)+\hat{p}_1(\omega)\hat{w}(\omega)\\ &=\hat{h}_1(\omega)\hat{s}_1(\omega)[1+\hat{h}_1(\omega)\hat{s}_1(\omega)]^{-1}\hat{r}(\omega)\\ &\quad+[1-\hat{h}_1(\omega)\hat{s}_1(\omega)(1+\hat{h}_1(\omega)\hat{s}_1(\omega))^{-1}]\hat{p}_1(\omega)\hat{w}(\omega).\end{aligned} \tag{4.2}$$

According to [1], if the closed-loop nonlinear control system is L_2-stable, a necessary condition is that the corresponding linear component is L_2-stable, i.e.,

$$\|\hat{g}_1\|_\infty\leqslant\infty,\quad\|\hat{g}_1\hat{p}_1\|_\infty\leqslant\infty,\quad\|\hat{l}_1\hat{g}_1\|_\infty\leqslant\infty,\quad\|(1-\hat{l}_1\hat{g}_1)\hat{p}_1\|_\infty<\infty, \tag{4.3}$$

where $\|\cdot\|_\infty$ is the H$_\infty$-norm. Because the transfer functions $\hat{g}_1,\hat{g}_1\hat{p}_1,\hat{l}_1\hat{g}_1$ and $(1-\hat{l}_1\hat{g}_1)\hat{p}_1$ are all the proper rational fractions of ω, the H$_\infty$-norms of them are dependent on their poles respectively. If there are no poles of them in the right closed half-plane, the corresponding linear component of the nonlinear closed-loop system is L_2 -stable.

Next, the tracking stability problem is considered.

Theorem 4.1 For the nonlinear closed-loop control system (2.1)~(2.3) with $w(t) \equiv 0, \forall t \in \mathbb{R}$, if

1) for any $n \in \mathbb{N}$, $\hat{h}_n(\omega_1, \cdots, \omega_n)$ and $\hat{s}_n(\omega_1, \cdots, \omega_n)$ are proper rational fractions of $\omega_i, i = 1, 2, \cdots, n$;

2) the corresponding linear component of the nonlinear closed-loop system is L_2 -stable;

3) Let $\|\hat{h}_n\|_\infty = \sup_{\omega_1,\cdots,\omega_n} |\hat{h}_n(\omega_1, \cdots, \omega_n)| = \beta_n, \quad \forall n \in \mathbb{N}$,

$$\|\hat{s}_1\|_\infty = \sup_{\omega_1} |\hat{s}_1(\omega_1)| = \lambda_1 > 0,$$

$$\|\hat{s}_n\|_\infty = \sup_{\omega_1,\cdots,\omega_n} |\hat{s}_n(\omega_1, \cdots, \omega_n)| = \lambda_n < \alpha_n \lambda_1^n, \quad n \geqslant 2,$$

$$\varphi_1 = \beta_1, \quad \varphi_n = \sum_{m=1}^{n} \sum_{k_1+\cdots+k_m=n} \beta_m \alpha_{k_1} \cdots \alpha_{k_m}, \quad n \geqslant 2, \quad k_i \in \mathbb{N},$$

$$\|\hat{g}_1\|_\infty = \sup_{\omega_1} |(1 + \hat{h}_1(\omega_1)\hat{s}_1(\omega_1))^{-1}| = \rho_1 > 0,$$

$$\theta_1 = 1, \quad \theta_n = \rho_1 \sum_{m=2}^{n} \sum_{k_1+\cdots+k_m=n} \varphi_m \lambda_1^m \theta_{k_1} \cdots \theta_{k_n}, \quad n \geqslant 2, \quad k_1, k_i \in \mathbb{N},$$

the series $\sum_{m=1}^{\infty} \varphi_m \lambda_1^m$, $\sum_{\beta=1}^{\infty} \theta_n \rho_1^n$ are convergent;

4) $\hat{r} \in H_1(-\infty, \infty)$, i.e., the reference input spectrum is absolutely integrable, then the tracking system is closed-loop internal L_2 -stable, i.e.,

$$r \in L_2(-\infty, \infty) \Rightarrow e \in L_2(-\infty, \infty),$$

or closed-loop input-output L_2 -stable, i.e.,

$$r \in L_2(-\infty, \infty) \Rightarrow y \in L_2(-\infty, \infty).$$

Proof For $w(t) \equiv 0, \forall t \in \mathbb{R}$, the system equation is

$$e = (I + HS)^{-1}(r) = (I + L)^{-1}(r) = G(r),$$

or

$$y = HS(I + HS)^{-1}(r) = L(I + L)^{-1}(r) = LG(r).$$

Because the GFRF's $\{\hat{h}_n\}$ and $\{\hat{s}_n\}$ of H and S are proper fractions of ω_i, therefore, so the GFRF's $\{\hat{e}_n\}$ and $\{\hat{g}_n\}$ of L and G are. Thus, there exists $\omega_0 > 0$ such that

$$\|\hat{h}_n\|_\infty = \sup_{\omega_1,\cdots,\omega_n \leqslant \omega_0} |\hat{h}_n(\omega_1, \cdots, \omega_n)|, \quad \|\hat{s}_n\|_\infty = \sup_{\omega_1,\cdots,\omega_n \leqslant \omega_0} |\hat{s}_n(\omega_1, \cdots, \omega_n)|,$$

$$\|\hat{l}_n\|_\infty = \sup_{\omega_1,\cdots,\omega_n \leqslant \omega_0} |\hat{l}_n(\omega_1, \cdots, \omega_n)|, \quad \|\hat{g}_n\|_\infty = \sup_{\omega_1,\cdots,\omega_n \leqslant \omega_0} |\hat{g}_n(\omega_1, \cdots, \omega_n)|.$$

By 2), $\hat{g}_1(\omega)$ has no poles in the right closed half-plane, so that

$$\|\hat{g}_1\|_\infty = \sup_{\omega_1 \leqslant \omega_0} |(1 + \hat{h}_1(\omega_1)\hat{s}_1(\omega_1))^{-1}| = \rho_1 < \infty.$$

According to Theorem 3.1 and from 3), we have

$$\|\hat{l}_1\|_\infty \leqslant \|\hat{h}_1\|_\infty \|\hat{s}_1\|_\infty = \beta_1 \lambda_1 = \varphi_1 \lambda_1, \quad \cdots,$$

$$\|\hat{l}_n\|_\infty \leqslant \sum_{m=1}^{n} \sum_{k_1+\cdots+k_m=n} \|\hat{h}_m\|_\infty \|\hat{s}_{k_1}\|_\infty \cdots \|\hat{s}_{k_m}\|_\infty \leqslant \sum_{m=1}^{n} \sum_{k_1+\cdots+k_m=n} \beta_m \alpha_{k_1} \cdots \alpha_{k_m} \lambda_1^n = \varphi_n \lambda_1^n, n \geqslant 2.$$

So $\sum_{n=1}^{\infty} \|\hat{l}_n\|_\infty \leqslant \sum_{n=1}^{\infty} \varphi_n \lambda_1^n \leqslant K_0 = e^{k_1} = \sum_{n=1}^{\infty} \frac{K_1^n}{n!}$, where K_0 and K_1 are constants. Therefore, we have

$$\| \hat{h}_n \|_\infty = \beta_n \leqslant \infty, \quad \| \hat{s}_n \|_\infty = \lambda_n < \infty, \quad \forall\, n \in \mathbb{N}.$$

Furthermore, from 3) again, we have

$$\| \hat{g}_n \|_\infty \leqslant \| \hat{g}_1 \|_\infty \sum_{m=2}^{n} \sum_{k_1+\cdots+k_m=n} \| \hat{l}_m \|_\infty \| \hat{g}_{k_1} \|_\infty \cdots \| \hat{g}_{k_m} \|_\infty.$$

Assume that $\| \hat{g}_n \|_\infty \leqslant \theta_n \rho_1^n, \forall\, n \in \mathbb{N}$, the recursive algorithm is obtained by

$$\| \hat{g}_n \|_\infty \leqslant \rho_1 \sum_{m=2}^{n} \sum_{k_1+\cdots+k_m=n} \varphi_m \lambda_1^m \theta_{k_1} \cdots \theta_{k_m} \rho_1^n = \theta_n \rho_1^n.$$

So $\sum_{n=1}^{\infty} \| \hat{g}_n \|_\infty \leqslant \sum_{n=1}^{\infty} \theta_n \rho_1^n \leqslant K_2 = \mathrm{e}^{k_3} = \sum_{n=1}^{\infty} \frac{k_3^n}{n!}$, where K_2 and K_3 are constant too.

If $\| r \|_2, \| \hat{r} \|_1 < L$ (by(4)), then by using Theorem 3. 1 in [1],

$$\| e \|_2 \leqslant \sum_{n=1}^{\infty} \| \hat{e}_n \|_2 \leqslant \sum_{n=1}^{\infty} (K_3 L) n / n! = \mathrm{e}^{K_3 L} = M_1.$$

The closed-loop system is internal L_2 -stable.

Similarly, Let $\{\hat{f}_n\}$ be the GFRF's of the closed-loop nonlinear operator from r to y, then by means of Theorem 3. 1,

$$\| \hat{f}_1 \|_\infty \leqslant \| \hat{l}_1 \|_\infty \| \hat{g}_1 \|_\infty \leqslant \beta_1 \lambda_1 \rho_1, \quad \cdots,$$

$$\begin{aligned} \| \hat{f}_n \|_\infty &\leqslant \sum_{m=1}^{\infty} \sum_{k_1+\cdots+k_m=n} \| \hat{l}_m \|_\infty \| \hat{g}_{k_1} \| \cdots \| \hat{g}_{k_m} \|_\infty \\ &= \| \hat{l}_1 \|_\infty \| \hat{g}_n \|_\infty + \sum_{m=2}^{n} \sum_{k_1+\cdots+k_m=n} \| \hat{l}_m \|_\infty \| \hat{g}_{k_1} \|_\infty \cdots \| \hat{g}_{k_m} \|_\infty \\ &\leqslant \beta_1 \lambda_1 \theta_n \rho_1^n + \rho_1^{-1} \theta_n \rho_1^n = (\beta_1 \lambda_1 + \rho_1^{-1}) \theta_n \rho_1^n = \gamma_1 \theta_n \rho_1^n, \quad \forall\, n \geqslant 2, \end{aligned}$$

where $\gamma_1 = \beta_1 \lambda_1 + \rho_1^{-1}$, and $0 < \gamma_1 < \infty$, so

$$\sum_{n=1}^{\infty} \| \hat{f}_n \|_\infty \leqslant \gamma_1 \sum_{n=1}^{\infty} \frac{K_3^n}{n!}.$$

If $\| r \|_2, \| \hat{r} \|_1 < L$, then by using Theorem 3. 1 in [1],

$$\| y \|_2 \leqslant \sum_{n=1}^{\infty} \| \hat{y}_n \|_2 \leqslant \sum_{n=1}^{\infty} \gamma_1 \frac{(k_3 L)^n}{n!} = r_1 M_1 = M_2.$$

The closed-loop system is also input-output L_2 -stable.

Finally, the disturbance stability problem of the nonlinear closed-loop control system (2. 1)~(2,3) will be discussed.

Theorem 4. 2 For the nonlinear closed-loop control system (2. 1)~(2. 3) with $r(t) \equiv 0, \forall\, t \in \mathbb{R}$; if

1) for any $n \in \mathbb{N}, \hat{h}_n(\omega_1, \omega_2, \cdots, \omega_n), \hat{s}_n(\omega_1, \omega_2, \cdots, \omega_n)$ and $\hat{p}_n(\omega_1, \omega_2, \cdots, \omega_n)$ are all proper rational fractions of $\omega_i, i = 1, 2, \cdots, n$;

2) the corresponding linear component of the closed-loop system is L_2 -stable;

3) let $\| \hat{h}_n \|_\infty = \beta_n, \quad \| \hat{s}_1 \|_\infty = \lambda_1 > 0, \quad \| \hat{s}_n \|_\infty = \lambda_n \leqslant \alpha_n \lambda_1^n, \quad n \geqslant 2;$

$$\| \hat{p}_1 \|_\infty = \mu_1 > 0, \quad \| \hat{p}_n \|_\infty = \mu_n \leqslant \delta_n \mu_1^n, \quad \forall\, n \geqslant 2;$$

$$\varphi_1 = \beta_1, \quad \varphi_n = \sum_{m=1}^{n} \sum_{k_1+\cdots+k_m=n} \beta_m \varphi_{k_1} \cdots \varphi_{k_m}, \quad n \geqslant 2, \quad k_1 \in \mathbb{N},$$

$$\| \hat{g}_1 \|_\infty = \rho_1 > 0,$$

$$\theta_1 = 1, \quad \theta_n = \rho_1 \sum_{m=2}^{n} \sum_{k_1+\cdots+k_m=n} \varphi_m \lambda_1^m \theta_{k_1} \cdots \theta_{k_m}, \quad n \geqslant 2, \quad k_1 \in \mathbb{N},$$

$$\sigma_1 = \rho_1, \quad \sigma_n = \sum_{m=1}^{n} \sum_{k_1+\cdots+k_m=n} \theta_m \rho_1^m \delta_{k_1} \cdots \delta_{k_m}, \quad n \geqslant 2, \quad k_1 \in \mathbb{N}.$$

The series $\sum_{n=1}^{\infty} \delta_n \mu_1^n$, $\sum_{n=1}^{\infty} \varphi_n \lambda_1^n$, $\sum_{n=1}^{\infty} \theta_n \rho_1^n$ and $\sum_{n=1}^{\infty} \sigma_n \mu_1^n$ are convergent;

4) $\hat{w} \in H_1(-\infty,\infty)$,

then system is closed-loop internal L_2 -stable, i.e., $w \in L_2(-\infty,\infty) \Rightarrow e \in L_2(-\infty,\infty)$; or closed-loop input-output L_2 -stable, i.e., $w \in L_2(-\infty,\infty) \Rightarrow y \in L_2(-\infty,\infty)$.

Proof For the system equation

$$e = -(I+HS)^{-1}P(w) = -(I+L)^{-1}P(w) = -GP(w) = Q(w),$$

or $y = [I - HS(I+HS)^{-1}]P(w) = [I - L(I+L)^{-1}]P(w) = [I - LQ]P(w) = F(w)$,

similar to Theorem 4.1, we have

$$\| \hat{g}_1 \|_\infty = \rho_1 < \infty, \quad \| \hat{l}_1 \|_\infty \leqslant \beta_1 \lambda_1 = \varphi_1 \gamma_1,$$

$$\| \hat{l}_n \|_\infty \leqslant \varphi_n \lambda_1^n, \ n \geqslant 2, \quad \sum_{n=1}^{\infty} \| \hat{l}_n \|_\infty \leqslant \sum_{n=1}^{\infty} \varphi_n \lambda_1^n \leqslant K_0 = \mathrm{e}^{k_1},$$

$$\| \hat{g}_n \|_\infty \leqslant \theta_n \rho_1^n, \quad \sum_{n=1}^{\infty} \| \hat{g}_n \|_\infty \leqslant \sum_{n=1}^{\infty} \theta_n \rho_1^n \leqslant K_2 = \mathrm{e}^{k_2},$$

and by 2) and 3), we have

$$\| \hat{q}_1 \|_\infty \leqslant \| \hat{g}_1 \|_\infty \| \hat{p}_1 \|_\infty = \rho_1 \mu_1 = \sigma_1 \mu_1,$$

$$\| \hat{q}_n \|_\infty \leqslant \sum_{m=1}^{n} \sum_{k_1+\cdots+k_m=n} \| \hat{g}_m \|_\infty \| \hat{p}_{k_1} \|_\infty \cdots \| \hat{p}_{k_m} \|_\infty \leqslant \sum_{m=1}^{n} \sum_{k_1+\cdots+k_m=n} \theta_m \rho_1^m \sigma_{k_1} \cdots \sigma_{k_m} \mu_1^n$$

$$= \sigma_n \mu_1^n, \quad n \geqslant 2, \quad k_i \in \mathbb{N},$$

so

$$\sum_{n=1}^{\infty} \| \hat{q}_n \|_\infty \leqslant \sum_{n=1}^{\infty} \sigma_n \mu_1^n \leqslant K_4 = \mathrm{e}^{k_5} = \sum_{n=1}^{\infty} \frac{K_5^n}{n!}.$$

If $\| w \|_2$, $\| \hat{w} \|_1 < L$, then by using Theorem 3.1 in [1],

$$\| e \|_2 \leqslant \sum_{n=1}^{\infty} \| \hat{e}_n \|_2 \leqslant \sum_{n=1}^{\infty} (K_5 L)^n / n! = e^{k_5 L} = M_3,$$

the closed-loop system is internal L_2 -stable.

Similarly, the system is also closed-loop input-output L_2 -stable.

5 Simulation Examples

Example 5.1 The tracking problem is considered as

The plant $(H): \ddot{y} + \mu \dot{y} + y + \delta y^3 = u(t)$; The controller $(S): u(t) = k_p e + k_d \dot{e} + k_i \int e dt$.

Computing $\hat{s}_1(\omega)$, $\hat{s}_2(\omega_1,\omega_2)$, $\hat{s}_3(\omega_1,\omega_2,\omega_3)$, $\hat{h}_1(\omega)$, $\hat{h}_2(\omega_1,\omega_2)$ and $\hat{h}_3(\omega_1,\omega_2,\omega_3)$

$$\hat{s}_1(\omega) = \frac{j\omega}{-k_d\omega^2 + k_p j\omega + k_i}, \quad \hat{s}_2(\omega_1,\omega_2) = 0, \quad \hat{s}_3(\omega_1,\omega_2,\omega_3) = 0,$$

$$\hat{h}_1(\omega) = \frac{1}{-\omega^2 + \mu j\omega + 1}, \quad \hat{h}_2(\omega_1,\omega_2) = 0,$$

$$\hat{h}_3(\omega_1,\omega_2,\omega_3) = -\delta[(-(\omega_1+\omega_2+\omega_3)^2 + \mu j(\omega_1+\omega_2+\omega_3) + 1)(-\omega_1^2 + \mu j\omega_1 + 1) \cdot (-\omega_2^2 + \mu j\omega_2 + 1)(-\omega_3^2 + \mu j\omega_3 + 1)]^{-1}.$$

Assuming that $\mu = 5, \delta = 0.00034, k_p = 1, k_d = 5, k_i = 10$, it is known that they satisfy the condition 1) in Theorem 4.1. By Nyquist stability criteria, it is proven that the corresponding linear component of the closed-loop system is L_2-stable. Furthermore, $\lambda_1 = 1, \alpha_1 = 1, \rho_1 = 1.0802$, and Tab. 1 etc., by using the recursive computing, the series $\sum_{m=1}^{\infty} \varphi_m \lambda_1^m$ and $\sum_{n=1}^{\infty} \theta_n \rho_1^n$ are convergent. In terms of Theorem 4.1, the system is closed-loop L_2-stable. The response diagram of the nonlinear system is shown in Fig. 2, and the phase-plane diagram is shown in Fig. 3. From the diagrams we can see that the system is stable, which is identical with the theoretical result.

Table 1 Parameters

n	β_n	λ_n	α_n	φ_n	θ_n
1	1	1	1	1	1
2	0	0	0	0	0
3	3.4e-4	0	0	3.4e-4	3.68e-4
4	0	0	0	0	0

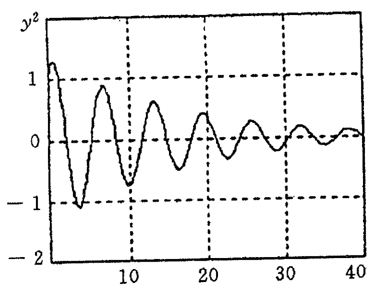

Fig. 2 Response diagram

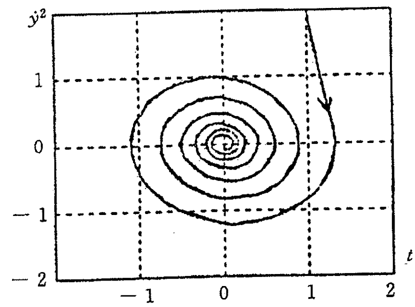

Fig. 3 Phase-plane diagram

Assuming that $\mu = 0.1, \delta = -0.00034, k_p = 1, k_d = 5, k_i = 25$, it is known that they satisfy condition 1) in Theorem 4.1. By Nyquist stability criteria, it is proven that the corresponding linear component of the closed-loop system is L_2-stable. Furthermore, $\lambda_1 = 1$, $\alpha_1 = 1$, $\rho_1 = 1.3318$, and Tab. 2, by using the recursive computing, the series $\sum_{m=1}^{\infty} \varphi_m \lambda_1^m$ is convergent and $\sum_{n=1}^{\infty} \theta_n \rho_1^n$ is divergent. In terms of Theorem 4.1, the system may be not closed-loop L_2-stable. The response diagram of the nonlinear system is shown in

Table 2 Parameters

n	β_n	λ_n	α_n	φ_n	θ_n
1	10.0125	1	1	10.0125	1
2	0	0	0	0	0
3	3.4171	0	0	3.4171	4.5509
4	0	0	0	0	0
5	0.0035	0	0	0.0035	62.137

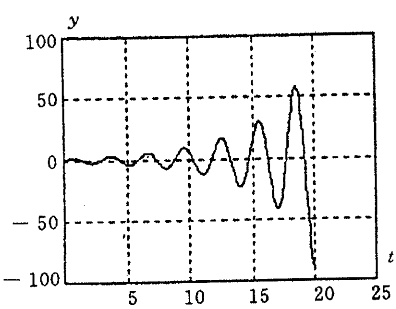

Fig. 4 Response diagram

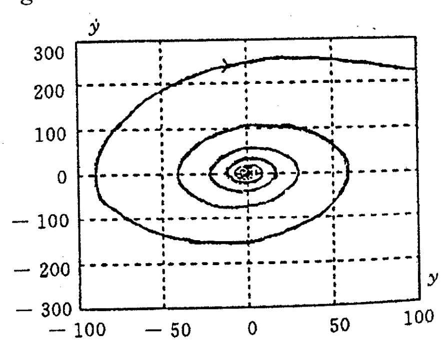

Fig. 5 Phase-plane diagram

Fig. 4, and the phase-plane diagram is shown in Fig. 5. From the diagrams we can see that the system is divergent with oscillating.

6 Conclusion

The closed-loop stability criteria for the polynomial class of nonlinear control systems based on GFRF's are also similar to the ones of linear closed-loop system, the zeros and poles of the linear transfer function $\hat{g}_1(s) = (1 + \hat{h}_1(s)\hat{s}_1(s))^{-1}$ plays main role. But the convergence of GFRF's norm is also a special desire for the nonlinear closed-loop control system.

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基于广义频率响应函数的非线性控制系统的闭环稳定性研究

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摘要:本文基于广义频率响应函数,讨论了一类非线性控制系统的闭环稳定性问题,给出了输入输出频域稳定性判据条件,最后,利用仿真例子对结论进行了验证.

关键词:非线性系统;广义频率响应函数;稳定性

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