Stable Adaptive Controller Design for Manipulators Using Neural Networks

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Abstract: A stable neural network-based adaptive controller design for integrating a neural network (NN) approach with an adaptive implementation of the variable structure control with the sector is presented in this paper for the trajectory tracking control of a robot manipulator with unknown nonlinear dynamics. The variable structure control with the sector serves two purposes, one is to provide the global stability of the closed loop system when the system goes out of the NN control, the other is to improve the tracking performance within the NN approximation region. The system stability and tracking error convergence are proved using Lyapunov stability theory, and the effectiveness of the proposed control approach is illustrated through simulation studies.

Key words: robot adaptive tracking control; neural networks; stability; discrete-time variable structure

1 Introduction

Existing stable adaptive control approaches using neural networks have been almost developed in continuous time systems for the robot trajectory tracking[1,2] because of nice mathematical properties. This paper investigates the discrete time case. A novel scheme for integrating a NN approach with an adaptive implementation of the variable structure control with the sector[3] is developed. The sector, defined by system parameters and NN basis functions, is a reachable region of attraction around the switching hyperplane[3]. The control objective is to force the system tracking error metric[1] to reach and remain inside the sector. While the system tracking error metric is controlled inside the sector, the NN-based adaptive control is stable, and thus a good tracking performance is guaranteed. A complete stability and tracking error convergence proof is given, and the properties of the proposed control approach are demonstrated by the simulations of a two-link manipulator.

2 System Description

The discrete dynamic model of an n -link rigid robot manipulator can be directly obtained by the minimization of the action functional[4] as

$$D(q(k+1))\dot{q}(k+1) - D(q(k))\dot{q}(k) - f(q(k),\dot{q}(k))T = Tu(k), \qquad (2.1)$$

where T is the sampling interval, $D(q(k)) = D^{\mathrm{T}}(q(k))(>0) \in \mathbb{R}^n$ is the inertia matrix, $f(q(k),\dot{q}(k))$ represents centrifugal, Coriolis and gravitational torgues, and $u(k)$ is the piecewise constant generalized force input:

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$$u(t) = u(k) \quad \text{for} \quad kT \leqslant t < (k+1)T. \tag{2.2}$$

The discrete dynamic model (2.1) shows a more accurate performance compared with models obtained through discretization of Euler-Lagrange dynamic equation[4].

According to reference [4], (2.1) can be written in an explicit form by setting

$$\begin{aligned} D(q(k+1)) &\cong D[q(k) + a(k)T\dot{q}(k)] = D(\bar{q}(k)), \\ q(k+1) &\cong \bar{q}(k) = q(k) + a(k)T\dot{q}(k), \end{aligned} \tag{2.3}$$

where $a(k)$ represents the change of the slope of the robot joint trajectories at any discrete time instant. With (2.3), (2.1) can be written through some mathematical operations as

$$\dot{q}(k+1) - \dot{q}(k) + D^{-1}(\bar{q}(k))((D(\bar{q}(k)) - D(q(k)))\dot{q}(k) - Tf(q(k),\dot{q}(k))) = D^{-1}(\bar{q}(k))Tu(k). \tag{2.4}$$

To develop a stable NN-based adaptive control law, the tracking error metric[1] is defined as

$$S(k) = C(X(k) - X_d(k)) = \Lambda q(k) + \dot{q}(k) - CX_d(k). \tag{2.5}$$

where $S(k) = [s_1(k), s_2(k), \cdots, s_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$, $X(k) = [q^{\mathrm{T}}(k), \dot{q}^{\mathrm{T}}(k)]^{\mathrm{T}}, X_d(k) = [q_d^{\mathrm{T}}(k), \dot{q}_d^{\mathrm{T}}(k)]^{\mathrm{T}}$ is the desired state trajectory, and $C = [\Lambda, I] \in \mathbb{R}^{n\times 2n}$, $\Lambda = \Lambda^{\mathrm{T}} > 0$.

Combining (2.5), (2.3) and (2.4) yields

$$S(k+1) = S(k) + \Lambda a(k)T\dot{q}(k) + C(X_d(k) - X_d(k+1)) + F(k) + G(k)u(k), \tag{2.6}$$

where $F(k) = -D^{-1}(\bar{q}(k))((D(\bar{q}(k)) - D(q(k)))\dot{q}(k) - Tf(q(k),\dot{q}(k)))$, $G(k) = D^{-1}(\bar{q}(k))T = D^{-1}(q(k+1))T$. Multiplying $G^{-1}(k)$ to both sides of (2.6) gives

$$\begin{aligned} G^{-1}(k)S(k+1) = {} & G^{-1}(k)rS(k) + G^{-1}(k)(\Lambda a(k)T\dot{q}(k) + \bar{r}S(k) + C(X_d(k) \\ & - X_d(k+1))) + W(k) + u(k), \end{aligned} \tag{2.7}$$

where $r = I - \bar{r} \in \mathbb{R}^{n\times n}$ with $\| r \| = \sqrt{(\mathrm{eig}(r^{\mathrm{T}}r))_{\max}} < 1$, $\mathrm{eig}(\cdot)$ stands for the eigenvalue of a matrix, and

$$W(k) = G^{-1}(k)F(k). \tag{2.8}$$

Under the assumption that the system dynamics can be approximated by neural networks, and with control

$$u(k) = -G^{*-1}(k)(\Lambda a(k)T\dot{q}(k) + \bar{r}S(k) + C(X_d(k) - X_d(k+1))) - W^*(k). \tag{2.9}$$

(2.7) yields

$$S(k+1) = rS(k) + \bar{\varepsilon}(k), \quad \| S(\infty) \| \leqslant \max(\| \bar{r}^{-1}\bar{\varepsilon}(\infty) \|). \tag{2.10}$$

Where $G^*(k), W^*(k)$ are the optimal NN estimations of $G(k)$ and $W(k)$, and $\bar{\varepsilon}(k)$, represents the NN approximation errors.

If $\lambda_i (i = 1, \cdots, n)$ is maximum admission bandwidith of the ith robot joint for not exciting the unmodelled system dynamics, a convenient selection for r is $r = \mathrm{diag}(r_1, \cdots, r_n)$[5], and r and Λ is determined by

$$\Lambda = \mathrm{diag}(\lambda_1, \cdots, \lambda_n), \quad r_i = 2 - \cos(\lambda_i T) - \sqrt{(2 - \cos(\lambda_i T))^2 - 1}, \quad \lambda_i T < \pi, \quad i = 1, \cdots, n, \tag{2.11}$$

thus the stable adaptive control satisfies the matching condition of the controlled system[5], and the asymptotic error of the system is dependent only on inherent NN approximation errors and the frequency range of high-frequency unmodelled dynamics.

3 A Stable NN-Based Adaptive Controller

Consider the following control law

$$u(k) = u_e(k) + \bar{u}(k), \tag{3.1}$$

and $u_e(k) = -(\hat{G}(k))^{-1}(\Lambda a(k)T\dot{q}(k) + \bar{r}S(k) + C(X_d(k) - X_d(k+1))) - \hat{W}(k)$, (3.2)

where $\hat{G}(k)$, $\hat{W}(k)$ are the estimations of $G(k)$ and $W(k)$.

From (3.1) and (3.2), $u_e(k)$ consists of a feedforword equivalent control in the same form as that in the discrete type variable structure control[5] and a PD control ($\bar{r}S(k)$). $\bar{u}(k)$ is the nonlinear control component, used to realize the variable structure control with the sector and compensate for inherent network approximation errors.

Substituting (3.1) and (3.2) into (2.7) yields

$$G^{-1}(k)S(k+1) = G^{-1}(k)rS(k) + [G^{-1}(k) - \hat{G}^{-1}(k)]H(k) + W(k) - \hat{W}(k) + \bar{u}(k), \tag{3.3}$$

where

$$\begin{cases} (G(k))^{-1} = (\bar{G}_{ij}(k)) \in \mathbb{R}^{n\times n}, \quad (\hat{G}(k))^{-1} = (\hat{G}_{ij}(k)) \in \mathbb{R}^{n\times n}, \quad i,j = 1,\cdots,n, \\ W(k) = (w_1(k),\cdots,w_n(k))^{\mathrm{T}} \in \mathbb{R}^n, \quad \hat{W}(k) = (\hat{w}_1(k),\cdots,\hat{w}_n(k))^{\mathrm{T}} \in \mathbb{R}^n, \\ H(k) = (H_1(k),H_2(k),\cdots,H_n(k))^{\mathrm{T}} = \Lambda a(k)T\dot{q}(k) + \bar{r}S(k) + C(X_d(k) - X_d(k+1)) \in \mathbb{R}^n, \\ \bar{u}(k) = (\bar{u}_1(k),\bar{u}_2(k),\cdots,\bar{u}_n(k))^{\mathrm{T}}. \end{cases} \tag{3.4}$$

Assume that $\bar{G}_{ij}(k)$, $w_i(k)$ can be approximated by GRBF networks as

$$\begin{cases} \bar{G}_{ij}(k) = (\Psi_{ij}^*)^{\mathrm{T}} Y_\Psi(k) + v_{ij}(k), \quad w_i(k) = (\varphi_i^*)^{\mathrm{T}} Y_\varphi(k) + \gamma_i(k), \\ \hat{G}_{ij}(k) = (\hat{\Psi}_{ij}(k))^{\mathrm{T}} Y_\Psi(k), \quad \hat{w}_i(k) = (\hat{\varphi}_i(k))^{\mathrm{T}} Y_\varphi(k), \quad i,j = 1,\cdots,n, \end{cases} \tag{3.5}$$

where $Y_\Psi(k) = (Y_\Psi^1(k),\cdots,Y_\Psi^{n_\Psi}(k))^{\mathrm{T}} \in \mathbb{R}^{n_\Psi}$, $Y_\varphi(k) = (Y_\varphi^1(k),\cdots,Y_\varphi^{n_\varphi}(k))^{\mathrm{T}} \in \mathbb{R}^{n_\varphi}$ represent GRBF vectors, $\Psi_{ij}^* \in \mathbb{R}^{n_\Psi}$, $\varphi_i^* \in \mathbb{R}^{n_\varphi}$ are optimal NN weights, $\hat{\Psi}_{ij}(k) \in \mathbb{R}^{n_\Psi}$, and $\hat{\varphi}_i(k) \in \mathbb{R}^{n_\varphi}$ are the NN weight estimations, $v_{ij}(k)$, $\gamma_i(k)$ $(i,j = 1,\cdots,n)$ are the NN approximation errors.

Substituting (3.4) and (3.5) into (3.3) gives

$$G^{-1}(k)(S(k+1) - rS(k)) = \begin{bmatrix} (\Psi_{11}^* - \hat{\Psi}_{11}(k))^{\mathrm{T}} & \cdots & (\Psi_{1n}^* - \hat{\Psi}_{1n}(k))^{\mathrm{T}} \\ \vdots & \cdots & \vdots \\ (\Psi_{n1}^* - \hat{\Psi}_{n1}(k))^{\mathrm{T}} & \cdots & (\Psi_{nn}^* - \hat{\Psi}_{nn}(k))^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} H_1(k)Y_\Psi(k) \\ \vdots \\ H_n(k)Y_\Psi(k) \end{bmatrix}$$
$$+ \begin{bmatrix} (\varphi_1^* - \hat{\varphi}_1(k))^{\mathrm{T}} \\ \vdots \\ (\varphi_n^* - \hat{\varphi}_n(k))^{\mathrm{T}} \end{bmatrix} Y_\varphi(k) + \varepsilon(k) + \bar{u}(k). \tag{3.6}$$

So

$$G^{-1}(k)\tilde{S}(k+1) = G^{-1}(k)(S(k+1) - rS(k)) = \tilde{\theta}^{\mathrm{T}}(k)Y(k) + \varepsilon(k) + \bar{u}(k), \tag{3.7}$$

where

$$\theta^* = \begin{bmatrix} \Psi_{11}^* & \cdots & \Psi_{n1}^* \\ \vdots & \cdots & \vdots \\ \Psi_{1n}^* & \cdots & \Psi_{nn}^* \\ \varphi_1^* & \cdots & \varphi_n^* \end{bmatrix} \in \mathbb{R}^{ne\times n}, \quad \hat{\theta}(k) = \begin{bmatrix} \hat{\Psi}_{11}(k) & \cdots & \hat{\Psi}_{n1}(k) \\ \vdots & \cdots & \vdots \\ \hat{\Psi}_{1n}(k) & \cdots & \hat{\Psi}_{nn}(k) \\ \hat{\varphi}_1(k) & \cdots & \hat{\varphi}_n(k) \end{bmatrix} \in \mathbb{R}^{ne\times n},$$

$$\tilde{\theta}(k) = \theta^* - \hat{\theta}(k) = (\tilde{\theta}_{ij}(k)) \in \mathbb{R}^{ne\times n}, \quad ne = n \times n_\Psi + n_\varphi, \tag{3.8}$$

$$Y(k) = [(H_1(k)Y_\Psi(k))^T, \cdots, (H_n(k)Y_\Psi(k))^T, (Y_\varphi(k))^T]^T \in \mathbb{R}^{ne},$$

$$\varepsilon(k) = (\varepsilon_1(k), \cdots, \varepsilon_n(k))^T, \quad \varepsilon_i(k) = \sum_{j=1}^{n} v_{ij}(k)H_j(k) + \gamma_i(k),$$

$$\| \varepsilon(k) \|_\infty \leqslant \varepsilon_m.$$

After having defined the NN basis function, the nonlinear control $\overline{u}(k)$ is defined as

$$\overline{u}(k) = \sum_{i=1}^{n}\sum_{j=1}^{n_\Psi} a_i^j Y_\Psi^j(k)H_i(k) + \sum_{j=1}^{n_\varphi} b^j Y_\varphi^j(k) - \varepsilon_m \operatorname{sgn}(S(k)). \tag{3.9}$$

Where sgn(•) is the sign function, $a_i^j = (a_{i1}^j, \cdots, a_{in}^j)^T$, $b^j = (b_1^j, \cdots, b_n^j)^T$ are switching-type coefficient vectors, and $\operatorname{sgn}(S(k)) = (\operatorname{sgn}(s_1(k)), \cdots, \operatorname{sgn}(s_n(k)))^T$.

Substituting (3.9) into (3.7) yields

$$G^{-1}(k)\widetilde{S}(k+1) = \widetilde{\theta}^T(k)Y(k) + \varepsilon(k) + \sum_{i=1}^{n}\sum_{j=1}^{n_\Psi} a_i^j Y_\Psi^j(k)H_i(k) + \sum_{j=1}^{n_\varphi} b^j Y_\varphi^j(k) - \varepsilon_m \operatorname{sgn}(S(k)). \tag{3.10}$$

The following theorem yields for stabilizing the closed loop system.

Theorem If the system (2.1) is digitally-controlled by the control law (3.1), and the neural network has the following adaptive learning algorithm:

$$\Delta\hat{\theta}(k) = Y(k)S^T(k)r\eta. \tag{3.11}$$

then the tracking error metric of the system will enter the sector defined as below

$$\Omega(k) = \bigcup_{l=1}^{n} \Gamma_A^l(k) \cup \Gamma_B^l(k). \tag{3.12}$$

where

$$\Gamma_A^l(k) = \bigcup_{j=1}^{n_\Psi}\bigcup_{i=1}^{n} (s_l(k) \,|\, |s_l(k)Y_\Psi^j(k)H_i(k)| \leqslant \rho_{il}^j), \quad \Gamma_B^l(k) = \bigcup_{j=1}^{n_\varphi} (s_l(k) \,|\, |s_l(k)Y_\varphi^j(k)| \leqslant v_l^j),$$

$$\rho_{il}^j = \frac{\alpha(h+p)^2}{2hr_l} |Y_\Psi^j(k)H_i(k)| \Big(\sum_{i=1}^{n}\sum_{j=1}^{n_\Psi} |Y_\Psi^j(k)H_i(k)| + \sum_{j=1}^{n_\varphi} Y_\varphi^j(k) + \kappa \Big),$$

$$v_l^j = \frac{\alpha(h+p)^2}{2hr_l} Y_\varphi^j(k) \Big(\sum_{i=1}^{n}\sum_{j=1}^{n_\Psi} |Y_\Psi^j(k)H_i(k)| + \sum_{j=1}^{n_\varphi} Y_\varphi^j(k) + \kappa \Big),$$

$$(i, l = 1, \cdots, n; \quad j = 1, \cdots, n_\Psi(n_\varphi)).$$

The switching-type coefficient vectors are determined by the following relations:

$$a_{il}^j = \begin{cases} h, & s_l(k)Y_\Psi^j(k)H_i(k) < -\rho_{il}^j, \\ 0, & |s_l(k)Y_\Psi^j(k)H_i(k)| \leqslant \rho_{il}^j, \\ -h, & s_l(k)Y_\Psi^j(k)H_i(k) > \rho_{il}^j, \end{cases} \quad b_l^j = \begin{cases} h, & s_l(k)Y_\varphi^j(k) < -v_l^j, \\ 0, & |s_l(k)Y_\varphi^j(k)| \leqslant v_l^j, \\ -h, & s_l(k)Y_\varphi^j(k) > v_l^j, \end{cases} \tag{3.13}$$

where $\eta = \operatorname{diag}(\eta_1, \cdots, \eta_n)$ is the learning rate matrix, h and $p = \max(\widetilde{\theta}_{ij}(k))$ are constant, $\kappa(p+h) = 8\varepsilon_m, \alpha_m \leqslant \| G(k) \| \leqslant \alpha$. It is assumed that

$$\| \Delta G^{-1}(k) \| \leqslant \min_i (1 - r_i^2)/\alpha + \min_i (r_i^2 \eta_i) \| Y(k) \|^2 \tag{3.14}$$

holds. Where $\Delta G^{-1}(k) = G^{-1}(k) - G^{-1}(k-1)$.

When the system tracking error metric is drived inside the sector, $\|\widetilde{S}(k+1)\|$ is usually in a small magnitude. Let $\|\widetilde{S}(k+1)\| \leqslant \xi \|S(k)\|$, if the learning rate η_l satisfies

$$\eta_l > 2\frac{\max_i(r_i)\xi}{\min_i(r_i^2)\|Y(k)\|^2\alpha_m}, \quad (l = 1,\cdots,n), \tag{3.15}$$

then the NN-based adaptive control inside the sector is stable.

Proof Let the Lyapunov function is defined by

$$V(k) = \frac{1}{2}S^{\mathrm{T}}(k)G^{-1}(k-1)S(k) + \frac{1}{2}\mathrm{tr}(\tilde{\theta}(k-1)\eta^{-1}\tilde{\theta}^{\mathrm{T}}(k-1)),$$

then its first order forward difference is

$$\Delta V(k+1) = V(k+1) - V(k) = S^{\mathrm{T}}(k)rG^{-1}(k)\widetilde{S}(k+1) - \frac{1}{2}S^{\mathrm{T}}(k)(I+r)G^{-1}(k)(I-r)S(k)$$
$$+ \frac{1}{2}\widetilde{S}^{\mathrm{T}}(k+1)G^{-1}(k)\widetilde{S}(k+1) + \frac{1}{2}S^{\mathrm{T}}(k)\Delta G^{-1}(k)S(k)$$
$$- \mathrm{tr}(\Delta\hat{\theta}(k)\eta^{-1}\tilde{\theta}^{\mathrm{T}}(k)) - \frac{1}{2}\mathrm{tr}(\Delta\hat{\theta}(k)\eta^{-1}\Delta\hat{\theta}^{\mathrm{T}}(k)). \tag{3.16}$$

First, the outside of the sector is considered. Because

$$S^{\mathrm{T}}(k)rG^{-1}(k)\widetilde{S}(k+1) - \mathrm{tr}(\Delta\hat{\theta}(k)\eta^{-1}\tilde{\theta}^{\mathrm{T}}(k)) - \frac{1}{2}\mathrm{tr}(\Delta\hat{\theta}(k)\eta^{-1}\Delta\hat{\theta}^{\mathrm{T}}(k))$$
$$= S^{\mathrm{T}}(k)r(\varepsilon(k) + \sum_{i=1}^{n}\sum_{j=1}^{n_\Psi}a_i^jY_\Psi^j(k)H_i(k) + \sum_{j=1}^{n_\varphi}b^jY_\varphi^j(k) - \varepsilon_m\mathrm{sgn}(S(k)))$$
$$- \frac{1}{2}S^{\mathrm{T}}(k)r\eta rS(k)\|Y(k)\|^2$$
$$< \sum_{l=1}^{n}r_l(s_l(k)\varepsilon_l(k) - \varepsilon_m|s_l(k)|)$$
$$- n\frac{\alpha(p+h)^2}{2}(\sum_{i=1}^{n}\sum_{j=1}^{n_\Psi}|Y_\Psi^j(k)H_i(k)| + \sum_{j=1}^{n_\varphi}Y_\varphi^j(k))(\sum_{i=1}^{n}\sum_{j=1}^{n_\Psi}|Y_\Psi^j(k)H_i(k)|$$
$$+ \sum_{j=1}^{n_\varphi}Y_\varphi^j(k) + \kappa) - \frac{1}{2}S^{\mathrm{T}}(k)r\eta rS(k)\|Y(k)\|^2. \tag{3.17}$$

(3.10) gives

$$\|G^{-\frac{1}{2}}(k)\widetilde{S}(k+1)\| \leqslant \sqrt{n}\|G^{\frac{1}{2}}(k)\|((p+h)$$
$$\cdot(\sum_{i=1}^{n}\sum_{j=1}^{n_\Psi}|H_i(k)Y_\Psi^j(k)| + \sum_{j=1}^{n_\varphi}Y_\varphi^j(k)) + 2\varepsilon_m). \tag{3.18}$$

(3.18) results in

$$\widetilde{S}^{\mathrm{T}}(k+1)G^{-1}(k)\widetilde{S}(k+1)$$
$$\leqslant n\|G(k)\|\{(p+h)^2(\sum_{i=1}^{n}\sum_{j=1}^{n_\Psi}|H_i(k)Y_\Psi^j(k)| + \sum_{j}^{n_\varphi}Y_\varphi^j(k))^2$$
$$+ 4\varepsilon_m(p+h)(\sum_{i=1}^{n}\sum_{j=1}^{n_\Psi}|Y_\Psi^j(k)H_i(k)| + \sum_{j=1}^{n_\varphi}Y_\varphi^j(k)) + 4\varepsilon_m^2\}. \tag{3.19}$$

With $\alpha \geqslant \|G(k)\|$ and (3.14), it follows from (3.16), (3.17), (3.19) that if

$$\|\widetilde{S}(k+1)\| \geqslant 3\sqrt{n}\alpha\varepsilon_m = \hat{\varepsilon}_m, \tag{3.20}$$

$\Delta V(k+1) < 0$. If $\|\widetilde{S}(k+1)\| \leqslant \hat{\varepsilon}_m$, an adaptive control relation described by (2.10) will be satisfied whether the system tracking error metric is inside the sector or outside the sector.

The preceding discussion implies either that $\|\tilde{S}(k+1)\|$ and $\|\tilde{\theta}(k)\|$ decrease i. e. $\|\tilde{S}(k+1)\| \to \hat{\varepsilon}_m$ and $\|\tilde{\theta}\| \to o(\hat{\varepsilon}_m)$ as $k \to \infty$ outside the sector or it will be inside the sector.

Inside the sector, the following Lyapunov function candidate is considered

$$V_1(k) = \frac{1}{2}\mathrm{tr}(\tilde{\theta}(k-1)\eta^{-1}\tilde{\theta}^{\mathrm{T}}(k-1)). \tag{3.21}$$

then its first order forward difference is

$$\Delta V_1(k+1) = -\,\mathrm{tr}(\Delta\hat{\theta}(k)\eta^{-1}\tilde{\theta}^{\mathrm{T}}(k)) - \frac{1}{2}\mathrm{tr}(\Delta\hat{\theta}(k)\eta^{-1}\Delta\hat{\theta}^{\mathrm{T}}(k)). \tag{3.22}$$

With $\|\tilde{S}(k+1)\| \leqslant \xi\|S(k)\|$, substituting (3. 11) and (3. 10) into (3. 22) yields

$$\Delta V_1(k+1) \leqslant \sum_{l=1}^{n} r_l(\varepsilon_l s_l(k) - \varepsilon_m|s_l(k)|) + \|S(k)\|^2(\max_i(r_i)\xi\|/\alpha_m$$
$$-\frac{1}{2}\min_i(r_i^2\eta_i)\|Y(k)\|^2), \tag{3.23}$$

with (3. 15), $\Delta V_1(k+1) < 0$. It follows that the NN control inside the sector is stable.

Q. E. D.

4 Selection of the Sector Parameters

From (3. 13), the size of the sector can be estimated by

$$\|S(k)\| \leqslant \frac{\alpha(p+h)^2}{2h}\sqrt{\sum_{l=1}^{n}\frac{1}{r_l^2}(\sum_{l=1}^{n}\sum_{j=1}^{n_\Psi}|Y_\Psi^j(k)H_l(k)| + \sum_{j=1}^{n_\varphi}Y_\varphi^j(k) + \kappa)}, \tag{4.1}$$

which defines a reachable region of attraction around the sliding hyperplane. In order to obtain a better tracking performance, the size of the sector is expected to decrease gradually along with the decrement of the uncertainty bound of parameters $\|\tilde{\theta}(k)\|$ during the NN adaptive learning. If $p \in [p_{\min}, p_{\max}]$ and $h \in [h_{\min}, h_{\max}]$, the parameters p and h can be determined according to the following formula

$$p(k) = (p_{\max} - p_{\min})\alpha_0^k + p_{\min}, \quad h(k) = (h_{\max} - h_{\min})\alpha_0^k + h_{\min}, \tag{4.2}$$

where $\alpha_0 \in [0.9, 0.99]$ denotes the decay rate, $p_{\min}$ and $p_{\max}$ represent the minimal and maximal uncertainty bound of parameters respectively. Referring to [3], a similar sufficient condition for the existence of the sector when the tracking error metric of the system approaches the sector, is given from (4. 1), (3. 10) and (3. 13) as

$$h_{\min} \geqslant \frac{\beta}{2\sqrt{n} - \beta}p_{\min}, \quad \beta = \alpha'\xi\sqrt{\sum_{l=1}^{n}1/r_l^2}. \tag{4.3}$$

where $\alpha' = \alpha/\mathrm{eig}(G(k))_{\max} \geqslant 1$, parameters α and ξ have defined in last section, and besides, $h_{\max}$ is usually chosen as

$$h_{\max} = (2 \sim 3)h_{\min}. \tag{4.4}$$

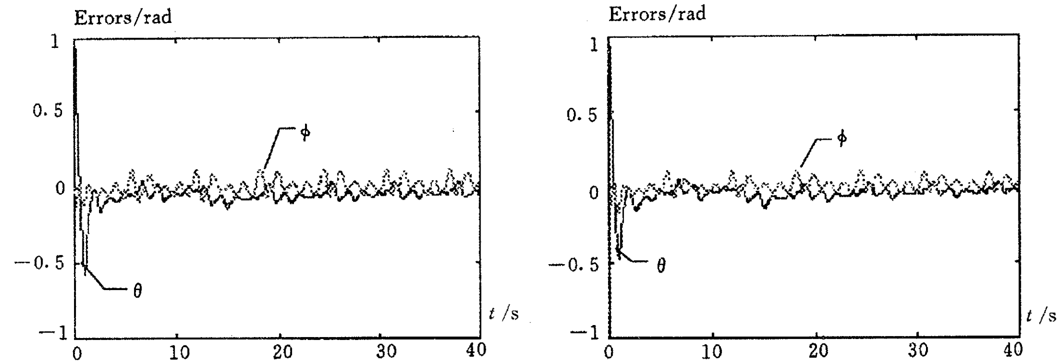
5 Application Example

In this section, the above developed control approach is applied to the position control of a two-link manipulator. The robot dynamics is given in [6]. The desired joint angle trejectories to be tracked are

$$\theta_d(t) = 0.5(\sin t + \sin 2t), \quad \phi_d(t) = 0.5(\cos 3t + \cos 4t). \tag{5.1}$$

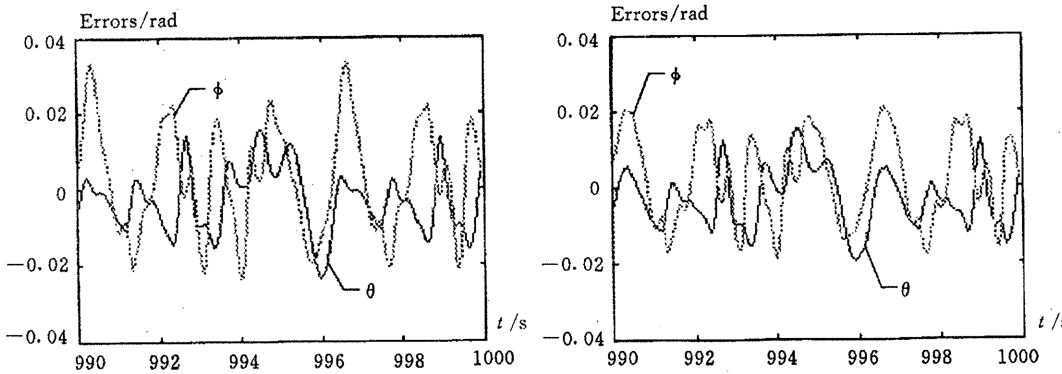
In five dimensional input space, $\theta(k), \phi(k), \bar{\phi}(k) \in [-2, 2]$rad, $\dot{\theta}(k), \dot{\phi}(k) \in [-4, 4]$(rad/s), with 11 even partitions for each input variable, there are 121 divisions for initially positioning the centers of the GRBF vector $Y_\varphi(k)$ according to orthogonal design. Thus 121 GRBF units are required to approximate the nonlinear function $W(k)$. Similarly, 21 GRBF units are required to approximate nonlinear function $G^{-1}(k)$ in input space $\bar{\phi}(k)(=\phi(k) + a(k)T\dot{\phi}(k)) \in [-2, 2]$rad with even partitions. After having initially positioned the GRBF centers, centers can be further modified on line by the recursive K-means clustering algorithm[7] along with the robot simulations for trajectory tracking, and the widths are determined by the P-nearest neighbour heuristics ($P = 14$)[8].

Simulations are done using a four-order Runge-Kutta algorithm with an integral step of 0.001 s., controller sampling interval of 0.05 s. The design parameters are chosen as $\lambda = 8$, $p_{\max} = 12$, $p_{\min} = 0.01$, $h_{\max} = 2.5$, $h_{\min} = 1.2$, $\eta_1 = 0.82$, $\eta_2 = 0.86$, $\alpha = 0.025$, $\alpha_0 = 0.98$, $\alpha(k) = 0.99$, $\varepsilon_m = 0.0045$, the initial learning rates for further updating the GRBF centers are chosen as $\alpha_\Psi(0) = \alpha_\varphi(0) = 0.98$[7]. The initial simulation condition is $[\theta(0), \dot{\theta}(0), \phi(0), \dot{\phi}(0)] = [10, -0.5, 1.0, -2.0]$.



(a) Without the sector (b) With the sector

Fig. 1 Angle following errors of NN control



(a) Without the sector (b) With the sector

Fig. 2 Angle following errors of NN control

Fig. 1 and Fig. 2 present the angle tracking errors during the first 40 seconds and the last 10 seconds of 1000 second's operation. Fig. 1(a) and Fig. 2(a) are obtained by the NN-based adaptive control without the sector, Fig. 1(b) and Fig. 2(b) with the sector. It can be seen that the introduction of the variable structure control with sector improves the tracking performance of the system.

6 Conclusion

This paper presents a stable adaptive controller design approach using neural networks for robot trajectory tracking. The major contribution of the approach is the integration of a NN approach with the adaptive implementation of the variable structure control with the sector. The system stability and tracking error convergence are proved using Lyapunov techniques, and the justification of the theoretical results is illustrated through simulations.

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机械手的神经网络稳定自适应控制器设计

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摘要:本文针对机械手轨迹跟随控制问题,提出了一种稳定的神经网络自适应控制器设计方法,这里机械手的非线性动力学假设是未知的.提出方法是神经网络方法和扇区自适应变结构控制方法的集成.扇区变结构控制的作用有两个,其一是在系统神经网络控制失灵的情形下提供闭环系统的全局稳定性;其二是在神经网络的近似域内改进系统的跟随性能.本文采用李雅普诺夫稳定理论给出了系统的稳定性和跟随误差收敛性的证明,并且通过数字仿真验证了提出方法的有效性.

关键词:机器人自适应跟随控制;神经网络;稳定性;离散变结构

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