On the Tracking Problem of Known Nonminimum Phase Plants *

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Abstract: In this paper, the tracking problem of known discrete time linear nonminimum phase plants is investigated. It is proved that a large class of linear controllers (which will be specified later in our paper) for a known nonminimum phase plant can not track arbitrary bounded set point sequences asymptotically. This indicates that it is impossible to design a Clarke-Gawthrop type self-tuning controller or an adaptive pole placement controller for unknown nonminimum phase plants to track arbitrary bounded set point sequences asymptotically.

Key words: nonminimum phase plants; global stability; asymptotic tracking

1 Introduction

It is well known that one can design adaptive controllers for discrete time linear minimum phase plants which would ensure both global stability and asymptotic tracking[1]. For nonminimum phase plants, many adaptive controllers (including both Clarke-Gawthrop type self-tuning controllers and adaptive pole placement controllers) had been proposed, and global stability of these adaptive controllers had been well estabished ([2～8]). However, the tracking ability of these adaptive controllers is not so well studied such that the following questions are hardly answered:

1) Can these adaptive controllers track arbitrary bounded set point sequences asymptotically?

2) If they can not, what kind of bounded set point sequences can they track asymptotically at most? that is ,what is the largest tracking ability of them?

To the best knowledge of the authors, no satisfactory answers are yet available.

Motivated by the above two questions, the tracking problem of known nonminimum phase plants is investigated in this paper which proves a large class of linear controllers can not track arbitrary bounded set point sequences asymptotically, but they can asymptotically track a special class of bounded set point sequences which satisfy trackable condition. The result of our investigation indicates that it is impossible to design Clarke-Gawthrop type self-tuning controllers or adaptive pole placement controllers for unknown nonminimum phase plants which can track arbitrary bounded set point sequences asymptotically, they may track a special class of bounded set point sequences satisfying trackable condition asymptotically at most.

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2 Structure of the Linear Controller

Consider the control of a discrete time linear SISO plant of the following form

$$A(q^{-1})y(t) = B(q^{-1})u(t) \tag{2.1}$$

where $y(t)$ and $u(t)$ denote the scalar system output and input. $A(q^{-1})$ and $B(q^{-1})$ are scalar polynomials in unit delay operator q^{-1} as follows:

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n}, \quad B(q^{-1}) = b_1q^{-1} + \cdots + b_mq^{-m}.$$

The following assumption will be made.

Assumption A1 $A(q^{-1})$ and $B(q^{-1})$ are relatively prime.

For known nonminimum phase plant (2.1), we investigate the tracking ability of linear controllers which have the following form

$$E(q^{-1})u(t) = -F(q^{-1})y(t) + T(q^{-1})y^*(t) \tag{2.2}$$

where $\{y^*(t)\}$ is an arbitrary bounded set point sequence, $E(q^{-1})$ is a monic polynomial, with $E(q^{-1})$ and $F(q^{-1})$ to be chosen such that $A(q^{-1})E(q^{-1}) + B(q^{-1})F(q^{-1})$ is strictly stable, i.e.

$$A(q^{-1})E(q^{-1}) + B(q^{-1})F(q^{-1}) \neq 0, \quad \text{for } |q| \geqslant 1$$

and with $T(q^{-1})$ to be chosen arbitrarily.

The control system is shown in the following figure.

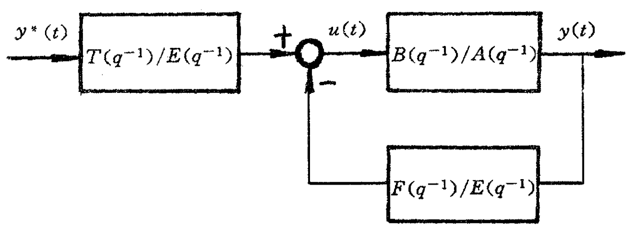

Fig. 1 The control system

Remark 1 Assumption A1 ensures the existence of $E(q^{-1})$ and $F(q^{-1})$. Different choices of $E(q^{-1})$, $F(q^{-1})$ and $T(q^{-1})$ may result in different controllers. It is easy to show that both Clarke-Gawthrop type self-tuning controllers and adaptive pole placement controllers (including the controllers in [2~8]) can be written in the form of (2.2) if they are designed using true parameters of the known plant (2.1). For example, if one takes $E(q^{-1}) = S(q^{-1})$, $F(q^{-1}) = R(q^{-1})$ and $T(q^{-1}) = 1$, (2.2) becomes the controller given in [4]; if one takes $E(q^{-1}) = R(q^{-1})$, $F(q^{-1}) = -S(q^{-1})$ and $T(q^{-1}) = C^*(q^{-1})$, (2.2) becomes the controller given in [7].

In the next section, it will be shown that the linear controllers given by (2.2) can not track arbitrarily bounded set point sequences when they are applied to known nonminimum phase plant (2.1) for any given $E(q^{-1})$, $F(q^{-1})$ and $T(q^{-1})$.

3 A Theorem Regarding Nonminimum Phase Plant

Definition A bounded set point sequence $\{y^*(t)\}$ is said to satisfy Trackable Condition if there exists a monic nonzero polynomial $C(q^{-1})$ such that

i) $\lim\limits_{t\to\infty} C(q^{-1})y^*(t) = 0$. ii) The order of this polynomial is minimal among those polynomials satisfying i). iii) $C(q^{-1})$ and $B(q^{-1})$ are relatively prime.

A theorem is now given as follows.

Theorem 3.1 Under Assumption A1, applying the controller given by (2.2) to the given nonminimum phase plant (2.1), then $\lim_{t\to\infty}[y(t)-y^*(t)]=0$ if and only if the bounded set point sequence $\{y^*(t)\}$ satisfies trackable condition.

To prove Theorem 3.1, we will give two lemmas directly without proof.

Lemma 3.2 For a bounded set point sequence $\{y^*(t)\}$, if there exists a nonzero polynomial $C(q^{-1})$ satisfying i) and ii) in the definition of trackable condition, and if there exists another nonzero polynomial $D(q^{-1})$ such that $\lim_{t\to\infty} D(q^{-1})y^*(t)=0$, then there exists a polynomial $L(q^{-1})$ such that $D(q^{-1})=L(q^{-1})C(q^{-1})$.

Lemma 3.3 For a bounded set point sequence $\{y^*(t)\}$, if there exists a nonzero polynomial $C(q^{-1})$ satisfying i) and ii) in the definition of trackable condition, then

$$C(q^{-1})\neq 0 \quad \text{for } |q|<1.$$

Using Lemma 3.2 and Lemma 3.3, Theorem 3.1 can be proved as follows.

The proof of theorem 3.1 It follows from (2.1) and (2.2) that

$$S(q^{-1})y(t)=B(q^{-1})T(q^{-1})y^*(t), \tag{3.1}$$

$$S(q^{-1})u(t)=A(q^{-1})T(q^{-1})y^*(t). \tag{3.2}$$

where $S(q^{-1})=A(q^{-1})E(q^{-1})+B(q^{-1})F(q^{-1})$.

Since $S(q^{-1})$ is strictly stable and the set point sequence $\{y^*(t)\}$ is bounded, it follows from (3.1) and (3.2) that $\{y(t)\}$ and $\{u(t)\}$ are both bounded sequences. This result shows the controller given by (2.2) can ensure global stability when they are applied to nonminimum phase plants.

From (3.1), one gets

$$S(q^{-1})[y(t)-y^*(t)]=[B(q^{-1})T(q^{-1})-S(q^{-1})]y^*(t). \tag{3.3}$$

It follows from (3.3) that $\lim_{t\to\infty}[y(t)-y^*(t)]=0$ if and only if

$$\lim_{t\to\infty}[B(q^{-1})T(q^{-1})-S(q^{-1})]y^*(t)=0. \tag{3.4}$$

Since the plant (2.1) is nonminimum phase and $S(q^{-1})$ is strictly stable, it is easy to show that

$$B(q^{-1})T(q^{-1})-S(q^{-1})\neq 0. \tag{3.5}$$

If (3.4) is equivalent to $\{y^*(t)\}$ satisfying trackable condition, then Theorem 3.1 is proved.

First, it will be shown that (3.4) implies that $\{y^*(t)\}$ satisfies trackable condition. It follows from (3.4) and (3.5) that there exists a nonzero polynomial $C(q^{-1})$ which satisfies i) and ii) in the definition of trackable condition. Hence it follows again from (3.4), (3.5) and Lemma 3.2 that there exists a polynomial $L(q^{-1})$ such that

$$B(q^{-1})T(q^{-1})-S(q^{-1})=L(q^{-1})C(q^{-1})$$

that is

$$S(q^{-1})=B(q^{-1})T(q^{-1})-L(q^{-1})C(q^{-1}). \tag{3.6}$$

It follows from Lemma 3.3 that all zeros of $C(q^{-1})$ are not within the unit circle in the complex plane. Using this fact and noticing that $S(q^{-1})$ is strictly stable, it follows from (3.6) that $C(q^{-1})$ and $B(q^{-1})$ are relatively prime. Now, it has been shown that $C(q^{-1})$ satisfies i) ~iii) of the definition of trackable condition, i.e. $\{y^*(t)\}$ satisfies trackable condition.

Secondly, if $\{y^*(t)\}$ satisfies trackable condition, we will prove (3.4) can be ensured if

$T(q^{-1})$ is chosen properly. Since $C(q^{-1})$ and $B(q^{-1})$ are relatively prime, it is easy to prove that we can choose two polynomials $T(q^{-1})$ and $L(q^{-1})$ such that

$$S(q^{-1}) = B(q^{-1})T(q^{-1}) - L(q^{-1})C(q^{-1}).$$

Noticing that $\lim_{t\to\infty} C(q^{-1})y^*(t) = 0$, (3.4) follows from the above equation immediately.

By now, (3.4) is proved to be equivalent to the set point sequence $\{y^*(t)\}$ satisfying trackable condition, and this completes the proof of Theorem 3.1.

Remark 2 For nonminimum phase plants, theorem 3.1 shows that the linear controllers given by (2.2) can track asymptotically a special class of bounded set point sequences satisfying trackable condition at most. From Remark 1, this implies that both Clarke-Gawthrop type self-tuning controllers and adaptive pole placement controllers can asymptotically track a special class of bounded set point sequences satisfying trackable condition at most even if they are designed using the true plant parameters.

4 An Example

In order to make the conclusion of Theorem 3.1 clear, the following example is given for illustration.

Example Consider the following nonminimum phase plant

$$y(t) - 2y(t-1) = u(t-1) - u(t-2).$$

If the controller given by (2.2) is applied to the above plant, it is easy to show the following conclusions.

1) The controller given by (2.2) can not asymptotically track nonzero constant sequences $\{y^*(t)\}$ i.e. $y^*(t) \equiv c \neq 0$.

For $y^*(t) \equiv c \neq 0$, one can choose $C(q^{-1}) = 1 - q^{-1}$, noticing that $B(q^{-1}) = q^{-1}(1 - q^{-1})$ and $S(q^{-1})$ is strictly stable, it follows from (3.1) that $\lim_{t\to\infty} y(t) = 0$, therefore 1) is followed.

From the above conclusion, one can see that, though $y^*(t) \equiv c \neq 0$ satisfies i) and ii) in the definition of trackable condition, $\lim_{t\to\infty}[y(t) - y^*(t)] \neq 0$ since $B(q^{-1})$ and $C(q^{-1})$ are not relatively prime. This shows that iii) in the definition of trackable condition is necessary.

2) If $E(q^{-1})$ and $F(q^{-1})$ are chosen such that $S(q^{-1}) = 1$ and $T(q^{-1}) = 1$, and if $y^*(t) = \sin 0.01t$, then the controller given by (2.2) can not track this sequence asymptotically.

Since $T(q^{-1}) = 1$, one gets $S(q^{-1}) - B(q^{-1})T(q^{-1}) = 1 - q^{-1} + q^{-2}$ and therefore $\lim_{t\to\infty}(1 - q^{-1} + q^{-2})y^*(t) = \lim_{t\to\infty}(1 - q^{-1} + q^{-2})\sin 0.01t \neq 0$, then it follows from (3.1) that $\lim_{t\to\infty}[y(t) - y^*(t)] \neq 0$.

From the above conclusion, one can see that $\lim_{t\to\infty}[y(t) - y^*(t)] \neq 0$ although $y^*(t)$ satisfies trackable condition with $C(q^{-1}) = 1 - 2\cos(0.01)q^{-1} + q^{-2}$ since $E(q^{-1})$, $F(q^{-1})$ and $T(q^{-1})$ are not chosen properly.

5 Conclusion

In this paper, the tracking problem of known nonminimum phase plants is studied. It is proved that linear controllers given by (2.2) can and only can track asymptotically a special class of bounded set point sequences satisfying trackable condition. That is, there is no linear controller in the form of (2. 2) which can asymptotically track arbitrarily bounded set point sequences. Since there is little possibility for adaptive controllers to perform any better than their corresponding nonadaptive ones designed using the true parameters of the plants, therefore from Remark 1, one may conclude that both Clarke-Gawthrop type self-tuning controllers and adaptive pole placement controllers may at most track a special class bounded set point sequences satisfying trackable condition asymptotically. Till now, we have given a relatively complete answer to the questions raised in the introduction.

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已知非最小相位系统的跟踪问题研究

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摘要:本文探讨了已知离散时间非最小相位系统的跟踪问题.证明了一大类针对非最小相位系统的线性控制器(其具体形式将在文中给出)不能实现对任意有界设定序列的渐近跟踪.这表明对未知离散非最小相位系统,不可能设计 Clarke-Gawthrop 型自校正控制器或自适应极点配置控制器去渐近跟踪任意有界设定序列.

关键词:非最小相位系统;全局稳定性;渐近跟踪

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