A Random Sampling Recursive Least-Squares Approach to the Design of Linear-Phase FIR Digital Filters *

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Abstract: This paper considers the design problem of linear-phase FIR digital filters with weighted least-squares error criterion and a random sampling recursive least-squares (RS-RLS) algorithm. The design problem is regarded as parameter estimation for a supposed linear system whose parameters are those of the filter to be designed. These parameters are estimated with recursive least-squares (RLS) algorithm. Input signals of the supposed system are generated with a random sampling (RS) method. The proposed RS-RLS algorithm is very simple and easy to use. A simulation example demonstrating the effectiveness of the proposed algorithm is provided.

Key words: digital filter; system identification; recursive least-squares algorithm

1 Intoduction

The frequency response of a FIR digital filter can be expressed as $H(\omega) = \sum_{n=0}^{N-1} h(n)\exp(-jn\omega)$. The design problem of a FIR digital filter is to find a set of parameters $\{h(0), h(1), \cdots h(N-1)\}$ in order that resulting frequency response $H(\omega)$ approximates a desired repsonse $\hat{H}(\omega)$ in some optimal criterion. Minimax design can be formulated as $\min\{\max_{\omega} W(\omega)|H(\omega) - \hat{H}(\omega)|\}$ where $W(\omega)$ is a preset weighting function. Weighted least-squares design is minimizing the cost function given by

$$J = \int_0^{\pi} W(\omega)|H(\omega) - \hat{H}(\omega)|^2 \mathrm{d}\omega. \tag{1}$$

The implementation of minimax design requires sophisticated optimization algorithms such as the Remez exchange algorithm and linear programming. Weighted least-squares methods are easy to implement and the solution can be obtained analytically[1~3]. With suitable weighting function, the weighted least-squares solution is equi-ripple[1,4], but the weighting function must be computed iteratively. Moreover, these methods need computing the inverse of a $m \times m$ matrix where m is the number of independent parameters of the filter. When the order of filter is high, some trouble will be encountered.

In this paper, we look upon the design of linear-phase FIR digital filters as parameter estimation of a linear system. A supposed linear system whose parameters are the parameters of the filter to be designed is established according to the specific structure of the filter. Thus

* Supported by the National Natural Science Foundation of China under Grant (69774002).
Manuscript received Aug. 12, 1996, revised May 13, 1997.

we can use system parameter estimation algorithms to design the filter's parameters. We also take the weighted least-squares error criterion but use the random sampling recursive least-squares (RS-RLS) algorithm rather than the usual weighted least-squares algorithm to design the filter. Input signals of the supposed linear system are generated with a random sampling (RS) method. The parameters of the supposed system are estimated with the recursive least-squares (RLS) algorithm[5,6]. One of the advantages of using recursive algorithm is that matrix inversion is not needed.

2 Formulation of the Design Problem

The frequency response $H(\omega)$ of a linear-phase FIR digital filter with symmetric parameters $h(n)$ can be expressed as

$$H(\omega) = \exp(-j\frac{N-1}{2}\omega)H_g(\omega), \tag{2}$$

where $\exp(-j\frac{N-1}{2}\omega)$ is the linear-phase term and $H_g(\omega)$ is a real frequency response. In the case of odd N:

$$H_g(\omega) = a(0) + \sum_{n=1}^{(N-1)/2} a(n)2\cos n\omega, \tag{3}$$

$$a(n) = h((N-1)/2-n) = h((N-1)/2+n), \quad n = 0,1,\cdots(n-1)/2 \tag{4}$$

and in the case of even N

$$H_g(\omega) = \sum_{n=1}^{(N-1)/2} b(n)2\cos[(n-1/2)\omega], \tag{5}$$

$$b(n) = h(N/2-n) = h(N/2-1+n), \quad n = 1,2,\cdots,N/2. \tag{6}$$

$H_g(\omega)$ in (3) or (5) can be written as following compact form:

$$H_g(\omega) = \theta^\tau\phi(\omega), \tag{7}$$

where for (3) with odd N

$$\theta = [a(0) \quad a(1) \quad a(2) \quad \cdots \quad a((N-1)/2)]^\tau, \tag{8}$$

$$\phi(\omega) = [1 \quad 2\cos\omega \quad 2\cos 2\omega \quad \cdots \quad 2\cos(N-1)\omega/2]^\tau \tag{9}$$

and for (5) with even N

$$\theta = [b(1) \quad b(2) \quad \cdots \quad b(N/2)]^\tau, \tag{10}$$

$$\phi(\omega) = [2\cos(\omega/2) \quad 2\cos(3\omega/2) \quad \cdots \quad 2\cos((N-1)\omega/2)]^\tau. \tag{11}$$

The weighted least-squares design problem of linear-phase FIR digital filters now becomes into finding parameter vector θ such that the cost function

$$J = \int_0^\pi W(\omega)\,|\theta^\tau\phi(\omega) - H_d(\omega)|^2 d\omega \tag{12}$$

is minimized, where $H_d(\omega)$ is the desired magnitude frequency response. If we regard the frequency ω ranging from 0 to π as a random variable whose probability density function is just the error weighting function $W(\omega)$, the cost function J in (12) will be the expected value of function $|\theta^\tau\phi(\omega) - H_d(\omega)|^2$ of random variable ω, i.e.,

$$J = E[\,|\theta^\tau\phi(\omega) - H_d(\omega)|^2] \tag{13}$$

Let $\{\omega_k, k = 1,2,\cdots,L\}$ be a set of samples of random variable ω. If L is large enough the design problem can be approximately formulated as

$$\min_{\theta}\sum_{k=1}^{L}|\theta^{\tau}\phi(\omega_k) - H_d(\omega_k)|^2. \tag{14}$$

3 Derivation of System Parameter Estimation Problem

Suppose a linear system with input $U(t)$, output $y(t)$ and noise $w(t)$. The model of the system is given by

$$y(t) = \theta^{\tau}U(t) + w(t). \tag{15}$$

The least-squares estimation problem for system (15) is estimating the parameter vector θ based on input data $\{U(t), t = 1,2,\cdots,L\}$ and output data $\{y(t), t = 1,2,\cdots,L\}$ in order that the predicted output, $\hat{y}(t) = \theta^{\tau}U(t)$, of model(15)with parameter vector θ approximates the actual output $y(t)$ in the least-squares error sense, i. e. ,

$$\min_{\theta}\sum_{t=1}^{L}[\theta^{\tau}U(t) - y(t)]^2. \tag{16}$$

Comparing equation (16) with equation (14) it can be seen that the design problem of a linearphase FIR filter is equivalent to the parameter estimation problem of linear system (15) under the same parameter vector θ if we set

$$U(t) = \phi(\omega_t), \tag{17}$$

$$y(t) = H_d(\omega_t). \tag{18}$$

Now we focus on the least-squares estimation problem (16), the solution of which denoted by $\hat{\theta}(L)$ can be either non-recursive or recursive. In the recursive case, we can calculate the parameter vector $\hat{\theta}(L)$ by simply updating $\hat{\theta}(L-1)$ without matrix inversion and the covariance matrix in the algorithm can be updated with $U - D$ factorization which has good numerical stability[6]. We use RLS algorithm to estimate the parameter vector θ of system (15). The input and output data of the system can be generated with the Random Sampling (RS) algorithm described below under specified $H_d(\omega)$ and $W(\omega)$ whose maximum value is denoted by $W_{\max}$.

1) Generate a uniform distributed random number ω_t ranging from 0 to π.

2) Generate another uniform random number P ranging from 0 to 1.

3) If $W(\omega_t) \leqslant P \times W_{\max}$, continue. Otherwise, go back to 1).

4) Generate $U(t)$ by equation (9) (or (11)) and equation (17).

5) Generate $y(t)$ by equation (18).

The input data generated with above random sampling algorithm have strong excitation property which ensures the estimated parameter vector converging rapidly. The probability density function $W(\omega)$ is the error weighting function in cost function (12) which determines the weight of errors between $H_g(\omega)$ and $H_d(\omega)$ in different frequency. It should be selected by the designer in accordance with specific design and in general, it should be large for important frequency bands, small for secondary bands and must be zero for transition bands.

4 The Random Sampling Recursive Least-Squares Algorithm

The random sampling recursive least-squares (RS-RLS) algorithm for the design of linearphase FIR digital filters is given below. This algorithm is very simple and easy to implement.

1) Specify $H_d(\omega)$, N and $W(\omega)$.

2) Select recursion number L and initiate parameter vector and covariance matrix as

$$\hat{\theta}(0) = 0, \quad P(0) = \rho I,$$

where ρ is a large number, 100000 for example, I is a unit square matrix.

3) Set $t = 1$.

4) Generate $U(t)$ and $y(t)$ with RS algorithm.

5) Update $\theta(t)$ and $P(t)$ with recursive least-squares algorithm, i.e.,

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-1)U(t)}{1 + U^\tau(t)P(t-1)U(t)}[y(t) - U^\tau(t)\hat{\theta}(t-1)].$$

$$P(t) = P(t-1) - \frac{P(t-1)U(t)U^\tau(t)P(t-1)}{1 + U^\tau(t)P(t-1)U(t)}.$$

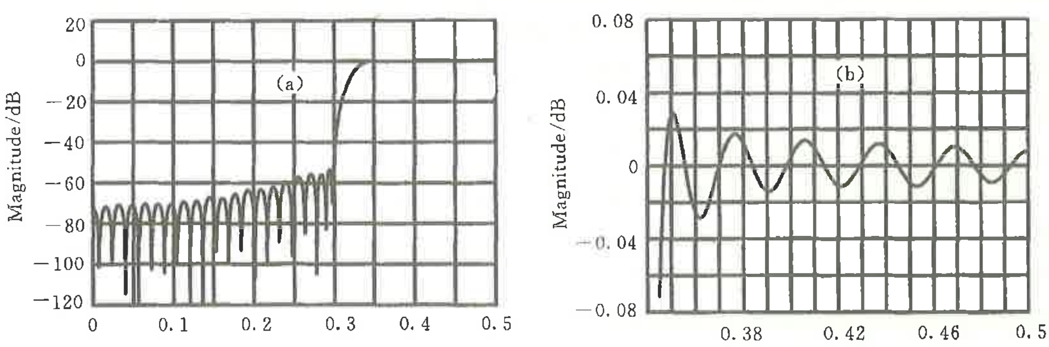
6) if $t < L$, then set $t = t + 1$ and return to 3). Otherwise, continue.

7) Compute $h(0), h(1), \cdots, h(N-1)$ by equation (4) and (8) (or equation (6) and (10)).

8) Set $H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + \cdots + h(N-1)z^{-(N-1)}$ and terminate.

5 A Simulation Example

We consider the design of a 63th-order high-pass filter with stopband [0, 0.3] and passband [0.345, 0.5] in normalized frequency. The weighting function $W(\omega)$ is taken as 0.3, 1.0, 0.0, 1.0, 0.3 for $0 \leqslant \omega < 0.57\pi$, $0.57\pi \leqslant \omega \leqslant 0.60\pi$, $0.60\pi < \omega < 0.69\pi$, $0.69\pi \leqslant \omega \leqslant 0.70\pi$, $0.70\pi \leqslant \omega < 1.00\pi$ respectively. With recursion number $L = 300$, the magnitude frequency response for this example is shown in Fig. 1 and the mean weighted squares error is 1.35×10^{-6}.



(a) Normalized Frequency (b) Normalized Frequency

Fig. 1 Magnitude frequency response of the filter for (a) fullband and (b) passband

6 Conclusion

The design problem of linear-phase FIR digital filters is studied in this paper. Based on the weighted least-squares criterion, a random sampling recursive least-squares algorithm has been proposed. The frequency points are samples of a random variable whose probability density function is the error weighting function and the parameters are estimated with recursive least-squars algorithm. The proposed algorithm is very simple to implement and easy to use, and it was shown to yield excellent solution.

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线性相位 FIR 数字滤波器设计的随机抽样递推最小二乘算法

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摘要：本文考虑了基于加权平方误差准则线性相位 FIR 数字滤波器的设计问题，提出一种随机抽样递推最小二乘(RS-RLS)设计方法．将设计问题看成一个线性系统的辨识问题，辨识参数所需的数据由一随机抽样法产生，辨识算法采用递推最小二乘法．RS-RLS 设计法简单易用，设计范例说明这一方法具有很高的设计精度．

关键词：数字滤波器；系统辨识；递推最小二乘法

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