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Robust Stabilization of Uncertain Nonlinear Systems with Time Delays in both State and Control Input*

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Abstract: For a class of uncertain nonlinear systems with time-delays in both state and control input, when in the linear part of the plant, the uncertainties are unknown but norm-bounded, and the uncertainties in the nonlinear part satisfy the so-called matching condition, this paper deals with the robust stabilization problem. The sufficient conditions for the systems under consideration to be stabilizable via state feedback or output feedback are given in terms of Riccati inequality, in the sense of global exponential stability. And we show that the controller can be designed by a simple procedure so long as obtaining the solutions of the corresponding inequalities.

Key words: robust stabilization; state feedback; output feedback; exponential stability; time delay

1 Introduction

There are lots of literature considering the problem of the robust stabilization for linear uncertain systems with time delays in recent years; see, e. g. Cheng[1], Chio[2] and references therein. But only a few works have been done on the robust stabilization for nonliner uncertain systems with time delays; see, e. g. Thowsen[3], Dawson et al.[4] and Nguang[5]. As it was stated, by using a Riccati equation approach, Thowsen[3] proposed a technique to design a state feedback controller for a class of time-delay nonlinear systems, in the framework of uniform ultimate bounded stability. Nguang[5] studied a calss of time-delay nonlinear systems which consist of a linear time-delay nominal model, and both matched and mismatched nonlinear uncertainties. The time-delay system under consideration does not include delayed control input. And to the best of our knowledge, the problem of robust stabilization for uncertain systems with delayed control inputs still remains an open area. However, control delays exist commonly in practice, e. g. signal transformation and signal processing etc.. Therefore, the problem of robust stabilization for a class of uncertain systmes with control delays is a subject of great practical and theoretical importance.

This paper deals with the problem of robust stabilization for a class of uncertain systems with time-varying delays in both state and control input. The constraints on the considered systems are similar to those of the systems in Nguang[5].

2 Systems and Preliminaries

Consider uncertain time-delay nonlinear systems described by the following state

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equations:

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A(x,t))x(t) + (A_d + \Delta A_d(x,t))x(t-\tau_1(t)) + (B + \Delta B(x,t))u(t) \\ &\quad + (B_d + \Delta B_d(x,t))u(t-\tau_2(t)) + Bf(x,t) + B_d f_d(x(t-\tau_2), t-\tau_2), \\ y(t) &= Cx(t), \\ x_{t_0}(\eta) &= \Psi(\eta) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^p$ stands for the control output, A, B, C, A_d and B_d are known constant matrices with appropriate dimensions, both $f(x,t)$ and $f_d(x(t-\tau_2(t)), t-\tau_2(t))$ are $m \times 1$ vectors representing the nonlinear uncertainties in the plant, $\Delta A(x,t), \Delta A_d(x,t), \Delta B(x,t)$ and $\Delta B_d(x,t)$ represent the uncertainties in linear part of the plant, $\tau_1(t)$ and $\tau_2(t)$ are any continuous bounded functions satisfying

$$0 \leqslant \tau_1(t), \quad \tau_2(t) \leqslant \tau, \quad \dot{\tau}_1(t) \leqslant \alpha < 1, \quad \dot{\tau}_2(t) \leqslant \beta < 1. \tag{2}$$

$\Psi(\eta)$ is a smooth vector-valued function defined in the Banach space $C[-\tau, 0]$ of smooth functions

$$\Psi : [-\tau, 0] \mapsto \mathbb{R}^n \quad \text{with} \quad \|\Psi\|_\infty := \sup_{-\tau \leqslant \eta \leqslant 0} \|\Psi(\eta)\|.$$

If $x : [-\tau, T] \mapsto \mathbb{R}^n$ is smooth and $T > 0$, then for a fixed $t \in [0, T]$, x_t denotes the restriction of x to the internal $[t-\tau, t]$ translated to $[-\tau, 0]$, $x_t \in C[-\tau, 0]$ and $x_t(\eta) = x(t+\eta)$, $-\tau \leqslant \eta \leqslant 0$.

In the following, we make several assumptions.

Assumption 1

$$\Delta A(x,t) = DF(x,t)E_1, \quad \Delta A_d(x,t) = D_d F_d(x,t) E_{d1}, \tag{3}$$

$$\Delta B(x,t) = BJ(x,t)E_2, \quad \Delta B_d(x,t) = B_d J_d(x,t) E_{d2} \tag{4}$$

where $F(x,t), F_d(x,t) \in \mathbb{R}^{k \times j}$ and $J(x,t), J_d(x,t) \in \mathbb{R}^{m \times g}$ are Caratheodory matrix functions (see, e.g. Slotine and Li[6] and references therein) bounded by

$$\|F(x,t)\| \leqslant \xi \quad \text{for some} \quad \xi > 0, \tag{5}$$

$$\|F_d(x,t)\| \leqslant \xi_d \quad \text{for some} \quad \xi_d > 0 \tag{6}$$

and
$$\max_{(x,t) \in \mathbb{R}^n \times \mathbb{R}} \|J(x,t)E_2\| \leqslant \gamma_1 \leqslant \gamma < 1, \quad \max_{(x,t) \in \mathbb{R}^n \times \mathbb{R}} \|J_d(x,t)E_{d2}\| \leqslant \gamma_2 \leqslant \gamma < 1, \tag{7}$$

D, E_1, E_2, D_d, E_{d1} and E_{d2} are known real matrices which characterize the uncertainties.

The nonlinear functions $f(x,t)$ and $f_d(x,t)$ are also assumed to be Caratheodory functions and to satisfy the following assumption:

Assumption 2 There exist a positive scalar function $\rho(x,t)$ such that

$$\|f(x,t)\| \leqslant \rho(x,t), \quad \|f_d(x,t)\| \leqslant \rho(x,t)$$

where $\|\cdot\|$ denotes the Euclidean norm.

In this paper we also use the notion of global exponential stability which was introduced in [6].

Definition Given an n-dimensional continuous nonlinear system. Suppose there exists a Lyapunov function $V(x,t)$ with the following properties:

$$\lambda_1 \|x(t)\| \leqslant V(x,t) \leqslant \lambda_2 \|x(t)\|, \ \dot{V}(x,t) \leqslant -\lambda_3 \|x(t)\| + \varepsilon e^{-\lambda t}, \ \forall\ (x,t) \in \mathbb{R}^n \times \mathbb{R}$$

where $\lambda_1, \lambda_2, \lambda_3, \varepsilon$ and λ are some positive scalar constants. Then the system (1) is said to be global exponential stable.

Obviously, we have the following Lemma.

Lemma Given system (1). If there exists a Lyapunov function $V(x,t)$ satisfying

$$\lambda_1 \| x(t) \| \leqslant V(x,t) \leqslant \lambda_2 \| x(t) \| ,$$

$$\dot{V}(x,t) \leqslant - x^{\mathrm{T}} M(P) x + \varepsilon_1 \mathrm{e}^{-\lambda t} + \varepsilon_2 \mathrm{e}^{-\lambda(t-\tau)}, \quad \forall \ (x,t) \in \mathbb{R}^n \times \mathbb{R}$$

where $\lambda_1, \lambda_2, \lambda_3, \varepsilon_1, \varepsilon_2$ and λ are some positive scalar constants, and inequality $M(P) < 0$ has a positive-definite symmetric solution P. Then the system (1) is global exponential stable.

3 Main Results

In this section, first present a sufficient condition for the nonlinear system to be global exponentially stabilizable via state feedback under assumptions 1, 2. And then extend this result to the case using static output feedback. In the following we are searching for a state feedback control law of the form

$$u(t) = \phi_c(x,t) \tag{8}$$

where $\phi_c(x,t)$ is a Caratheodory function.

Now we state our main result.

Theorem 1 The system (1) satisfying Assumption 1, 2 is global exponential stabilizable via a nonlinear state feedback controller of the form (8) if there exists a positive constant ε such that the following Riccati inequality

$$\begin{aligned} & PA + A^{\mathrm{T}}P - P\left(\left(2 - \frac{1}{\varepsilon(1-\beta)}\right)BB^{\mathrm{T}} - \varepsilon(A_d A_d^{\mathrm{T}} + B_d B_d^{\mathrm{T}} + \xi DD^{\mathrm{T}} + \xi_d D_d D_d^{\mathrm{T}})\right)P \\ & + \frac{1}{\varepsilon} E_1^{\mathrm{T}} E_1 + \frac{1}{\varepsilon(1-\alpha)}(I + E_{d1} E_{d1}^{\mathrm{T}}) < 0 \end{aligned} \tag{9}$$

has a positive definite symmetric solution matrix P which should satisfy following inequality

$$\| B_d^{\mathrm{T}} P x(t) \| \leqslant \| B^{\mathrm{T}} P x(t-\tau_2) \| . \tag{10}$$

And if this is the case, a suitable stabilizing control law is given by

$$u(t) = - Kx(t) - \frac{1}{1-\gamma}\phi_c(x,t) \tag{11}$$

where $\quad K = B^{\mathrm{T}}P, \quad \phi_c(x,t) = \dfrac{B^{\mathrm{T}}Px(\rho(x,t) + \gamma \| B^{\mathrm{T}}Px \|)^2}{\| B^{\mathrm{T}}Px \| (\rho(x,t) + \gamma \| B^{\mathrm{T}}Px \|) + \varepsilon^* \mathrm{e}^{-\lambda t}},$

λ and ε^* are any positive scalars.

Proof Suppose there exists a positive-definite symmetric matrix P such that Riccati inequality (10) holds. Then a suitable Lyapunov candidate is

$$V(x_t,t) = x^{\mathrm{T}}Px + \int_{t-\tau_1}^{t} x^{\mathrm{T}}Sx\mathrm{d}t + \int_{t-\tau_2}^{t} x^{\mathrm{T}}Wx\mathrm{d}t$$

where S and W are positive-definite symmetric matrices yet to be chosen. It follows that the derivative of $V(x_t,t)$ along the trajectory of (1) is given by

$$\begin{aligned} \dot{V}(x_t,t) = & 2x^{\mathrm{T}}P(A + \Delta A(x,t)) + x^{\mathrm{T}}(S + W)x + 2x^{\mathrm{T}}P(A_d + \Delta A_d(x,t))x(t-\tau_1) \\ & - 2x^{\mathrm{T}}P(B + \Delta B(x,t))\left(Kx(t) + \frac{1}{1-\gamma}\phi_c(x,t)\right) \\ & - 2x^{\mathrm{T}}P(B_d + \Delta B_d(x,t))\left(Kx(t-\tau_2) + \frac{1}{1-\gamma}\phi_c(x(t-\tau_2),t-\tau_2)\right) \\ & + 2x^{\mathrm{T}}P(Bf(x,t) + B_d f_d(x(t-\tau_2),t-\tau_2)) \end{aligned}$$

$$- (1 - \dot{\tau}_1)x^{\mathrm{T}}(t - \tau_1)Sx(t - \tau_1) - (1 - \dot{\tau}_2)x^{\mathrm{T}}(t - \tau_2)Wx(t - \tau_2).$$

Note that

$$2x^{\mathrm{T}}PBf(x,t) - 2x^{\mathrm{T}}PBJE_2Kx - 2\frac{1}{1-\gamma}x^{\mathrm{T}}PB(I + JE_2)\phi(x,t)$$

$$\leqslant \frac{2\,\|B^{\mathrm{T}}Px\|\,(\rho(x,t) + \gamma\,\|Kx\|)\varepsilon^* \mathrm{e}^{-\lambda t}}{\|B^{\mathrm{T}}Px\|\,(\rho(x,t) + \gamma\,\|B^{\mathrm{T}}Px\|) + \varepsilon^* \mathrm{e}^{-\lambda t}}, \tag{12}$$

$$2x^{\mathrm{T}}PB_d(f_d(x(t-\tau_2),t-\tau_2) - J_dE_{d2}Kx(t-\tau_2))$$

$$- \frac{2}{1-\gamma}x^{\mathrm{T}}PB_d(I + J_dE_{d2})\phi_c(x(t-\tau_2),t-\tau_2)$$

$$\leqslant \frac{2\,\|B^{\mathrm{T}}Px(t-\tau_2)\|\,(\rho(x(t-\tau_2),t-\tau_2) + \gamma\,\|Kx(t-\tau_2)\|)\varepsilon^* \mathrm{e}^{-\lambda(t-\tau_2)}}{\|B^{\mathrm{T}}Px(t-\tau_2)\|\,(\rho(x(t-\tau_2),t-\tau_2) + \gamma\,\|B^{\mathrm{T}}Px(t-\tau_2)\|) + \varepsilon^* \mathrm{e}^{-\lambda(t-\tau_2)}}, \tag{13}$$

and Chose
$$S = \frac{1}{\varepsilon(1-\alpha)}(I + E_{d1}E_{d1}^{\mathrm{T}}), \quad W = \frac{1}{\varepsilon(1-\beta)}PBB^{\mathrm{T}}P, \tag{14}$$

it is easy to obtain that

$$\dot{V}(x_t,t)$$

$$\leqslant x^{\mathrm{T}}(PA + A^{\mathrm{T}}P - P((2 - \frac{1}{\varepsilon(1-\beta)})BB^{\mathrm{T}} - \varepsilon(A_dA_d^{\mathrm{T}} + B_dB_d^{\mathrm{T}} + \xi DD^{\mathrm{T}} + \xi_dD_dD_d^{\mathrm{T}}))P$$

$$+ \frac{1}{\varepsilon}E_1^{\mathrm{T}}E_1 + \frac{1}{\varepsilon(1-\alpha)}(I + E_{d1}E_{d1}^{\mathrm{T}}))x + 2\varepsilon^* \mathrm{e}^{-\lambda t} + 2\varepsilon^* \mathrm{e}^{-\lambda(t-\tau_2)}. \tag{15}$$

Considering the Lemma in Section 2, we conclude Theorem 1.

Remark 1 Several previous results[5,7,8] are special cases of our result.

Remark 2 For an asymptotically stable system, inequality (10) is easy to satisfy.

In the next, we make some additional conditions and extend the state feedback stabilization result in Theorem 1 to the static output feedback.

Assumption 3 There exists a positive scalar Caratheodory function

$$\rho(y,t) \geqslant f(x,t), f_d(x,t), \quad \forall\ (x,t) \in \mathbb{R}^n \times \mathbb{R},$$

where $y = Cx$ are in Eqn. 1.

Now, the static output feedback stabilization result can be presented.

Theorem 2 Given system (1) satisfying Assumptions 1~3, it is globally exponentially stabilizable via a nonlinear and time-invariant static output feedback controller, if there exists a constant $\varepsilon > 0$ and a constant matrix $H \in \mathbb{R}^{m\times p}$ such that the following Riccati inequality

$$PA + A^{\mathrm{T}}P - P((2 - \frac{1}{\varepsilon(1-\beta)})BB^{\mathrm{T}} - \varepsilon(A_dA_d^{\mathrm{T}} + B_dB_d^{\mathrm{T}} + \xi DD^{\mathrm{T}} + \xi_dD_dD_d^{\mathrm{T}}))P$$

$$+ \frac{1}{\varepsilon}E_1^{\mathrm{T}}E_1 + \frac{1}{\varepsilon(1-\alpha)}(I + E_{d1}E_{d1}^{\mathrm{T}}) < 0$$

has a positive definite symmetric solution P which satisfies inequality (10) and following constraint:

$$B^{\mathrm{T}}P = HC.$$

And if this is the case, a suitable stabilizing control law is given by

$$u(t) = -Hy(t) - \frac{1}{1-\gamma}\phi_c(y,t)$$

where
$$\phi_c(y,t) = \frac{Hy(\rho(y,t) + \gamma \| Hy \|)^2}{\| Hy \| (\rho(y,t) + \gamma \| Hy \|) + \varepsilon^* e^{-\lambda t}}.$$

4 Conclusions

This paper has solved the problem of the robust stabilization for a class of uncertain nonlinear systems with time-varying delays in state and control input via a memoryless state feedback controller in the sense of global exponential stability. A sufficient condition for the system to be stabilizable is presented and the construction of the controller is given. Several previous results are special cases of our results. Finally, this result is extended to the case using static-output feedback.

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具有状态时滞和控制时滞的不确定性非线性系统的鲁棒控制

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摘要：对于具有状态时滞和控制时滞的不确定性非线性系统，当其线性部分的不确定性满足范数有界条件而非线性部分的不确定性满足匹配条件时，本文研究其鲁棒镇定问题. 在整体指数稳定的意义下，我们得到了上述这类系统可由线性状态反馈和输出反馈镇定的充分条件——某个代数黎卡提不等式有解. 并且只要得到了黎卡提不等式的解，就可很方便地设计出对应的控制器.

关键词：鲁棒镇定；状态反馈；输出反馈；指数稳定；时滞

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