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Robust H_∞ Output Feedback Controller Design for Linear Time-Varying Uncertain Systems with Delayed State *

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Abstract: This paper focuses on analysis and synthesis of robust H_∞ control for linear time-varying uncertain dynamic systems with delayed state. A dynamic output feedback controller is presented to quadratically stabilize the plant and reduce the effect of the disturbance input on the controlled output to a prescribed level for all admissible uncertainties. Two equivalent linear time-invariant structural descriptions for the time-varying uncertain systems with delayed state are obtained to get the controller gain matrix and the observer gain matrix.

Key words: robust H_∞ control; time-delay; uncertainty; output feedback

状态时滞时变不确定系统的鲁棒 H_∞ 输出反馈控制器设计

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摘要: 主要研究了存在状态滞后的线性时变不确定时滞系统的鲁棒 H_∞ 控制分析和综合问题. 给出了对所有容许不确定性, 被控对象可二次镇定和满足从干扰输入到控制输出的 H_∞ 范数界约束的动态输出反馈鲁棒 H_∞ 控制分析结果. 将不确定时滞系统的鲁棒 H_∞ 输出反馈控制器设计问题等价为两个线性时不变系统的状态反馈标准 H_∞ 控制问题, 并由此得到反馈阵和观测阵, 最终得到鲁棒 H_∞ 控制器综合设计方法.

关键词: 鲁棒 H_∞ 控制; 时滞; 不确定性; 输出反馈

1 Introduction

Robust H_∞ control problem for systems with parameter uncertainties has received a considerable amount of attention in recent years. Several related results for delay-free uncertain linear systems have been reported[1]. All these research results require the condition that all states of uncertain systems must be obtained, a natural question is what additional requirements (if any) are needed to ensure stabilizability of the systems for admissible uncertainties, when none of the states can be measured. The study of robust stabilization for uncertain linear dynamic systems with output feedback controller has been reported in several literature[2]. The robust stabilization of uncertain linear systems with dynamic output feedback based on the notion of quadratic stability also has been studied by some researchers[3].

Recently, H_∞ control problem for systems with time-delay has also been studied[4]. However, few studies of robust H_∞ state feedback control problem for time-delay systems with parameter uncertainties have been reported, let alone studies on robust H_∞ output feedback control problem for time-delay systems with time-varying parameter uncertainties. In this paper, attention is focused on the robust H_∞ output feedback control analysis and synthesis of linear time-varying uncertain systems with delayed state. The analysis and synthesis of robust H_∞ control problem addressed here are to design a linear time-invariant dynamic output feedback control law such

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that the closed-loop system is quadratically stable with an H_∞ norm bound constraint.

2 System description and definitions

Consider a linear time-varying uncertain system with delayed state,

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + \\ \qquad (A_1 + \Delta A_1(t))x(t-\tau) + \\ \qquad (B + \Delta B(t))u(t) + D_1 w(t), \\ y(t) = Cx(t), \quad z(t) = D_2 x(t), \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^r$ is the output vector, $w(t) \in \mathbb{R}^p$ is the disturbance input vector which belongs to $L_2[0,\infty)$, and $z(t) \in \mathbb{R}^q$ is the controlled output vector, $\Delta A(t)$, $\Delta A_1(t)$ and $\Delta B(t)$ are real-valued matrices whose elements are continuous functions with respect to time t, representing time-varying parameter uncertainties in the system, $\varphi(t) \in \mathbb{C}^n[-\tau, 0]$ is a real-valued continuous vector initial function.

Assume the nominal systems of (1) are stabilizable and detectable. Suppose the time-varying uncertain structures are given by,

$$\begin{aligned} [\Delta A(t)\ \Delta B(t)] &= H_0 F(t)[E_0 \quad E_1], \\ \Delta A_1(t) &= H_1 F(t) E_2 \end{aligned} \tag{2}$$

where $F(t) \in \mathbb{R}^{i \times j}$ is an unknown matrix function satisfying the following inequality,

$$F^{\mathrm{T}}(t)F(t) \leqslant I. \tag{3}$$

To facilitate further description, we propose some necessary definitions. The following Definition 1 can be regarded as an extension of existing definition in [5] to output feedback case.

Definition 1 The system of (1) (with $u(t) = 0$, $w(t) = 0$) is said to be quadratically stable if there exist a positive definite symmetric matrix P and a positive constant α such that for any admissible uncertainty the derivative of a Lyapunov function candidate

$$V(x,t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-\tau}^{t} x^{\mathrm{T}}(\theta)Rx(\theta)\mathrm{d}\theta$$

with respect to time t satisfies

$$\dot{V} \leqslant -\alpha \| x \|_2 \tag{4}$$

for all pairs $(x,t) \in \mathbb{R}^n \times \mathbb{R}$. The system of (1) (with $w(t) = 0$) is said to be quadratically stabilizable via linear dynamic observer $u = -K\hat{x}$ (where $\hat{x}$ is the observer state vector which will be defined in the following part, introduce the observer error $e(t) = x(t) - \hat{x}(t)$) if there exist positve definite symmetric matrices P_c and P_o and a positive constant α such that for any admissible uncertainty the derivative of a Lyapunov function candidate

$$V(x,e,t) = [x^{\mathrm{T}}(t) \quad e^{\mathrm{T}}(t)]\begin{bmatrix} P_c & 0 \\ 0 & P_o \end{bmatrix}\begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \int_{t-\tau}^{t} [x^{\mathrm{T}}(\theta) \quad e^{\mathrm{T}}(\theta)]\begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}\begin{bmatrix} x(\theta) \\ e(\theta) \end{bmatrix}\mathrm{d}\theta$$

with respect to time t satisfies

$$\mathrm{d}V/\mathrm{d}t \leqslant -\alpha \left\| \begin{bmatrix} x \\ e \end{bmatrix} \right\|_2 \tag{5}$$

for all pairs $(x,e,t) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}$.

Definition 2[1] For a given constant $\gamma > 0$, the uncertain system of (1) (with $u(t) = 0$) is said to be quadratically stable with an H_∞ norm bound γ if there exists a linear time-invariant dynamic output feedback control law, such that for any admissible time-varying parameter uncertainty the following two conditions are satisfied: a) The system is quadratically stable; b) Subject to the assumption of the zero initial condition, constraint $\| z(t) \|_2 \leqslant \gamma \| w(t) \|_2$ is satisfied. The system of (1) is said to be quadratically stabilizable with an H_∞ norm bound γ via output feedback if there exists a linear time-invariant dynamic output feedback control law such that the closed-loop system satisfies both condition a) and condition b).

In this paper, we consider the following linear robust H_∞ output feedback control law

$$\begin{aligned} u(t) &= -K\hat{x}(t), \\ \dot{\hat{x}}(t) &= A\hat{x}(t) + A_1\hat{x}(t-\tau) + \\ &\qquad Bu(t) + L(y(t) - \hat{y}(t)), \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{6}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the observer state vector, $\hat{y}(t) \in \mathbb{R}^r$ is the observer output vector, K is controller gain matrix, and L is observer gain matrix such that the closed-loop system is quadratically stable with a given H_∞ norm bound constraint $\| z(t) \|_2 \leqslant \gamma \| w(t) \|_2$.

3 Robust H_∞ control analysis

In this section, we present sufficient conditions for the systems (1) ~ (3) to be quadratically stabilizable with an H_∞ norm bound γ via linear dynamic output feedback control law (6).

We first introduce some useful lemmas.

Lemma 1 Suppose that x and y are vectors with appropriate dimensions, then,

$$2x^{\mathrm{T}}y \leqslant \varepsilon x^{\mathrm{T}}Qx + \varepsilon^{-1}y^{\mathrm{T}}Q^{-1}y$$

where ε is a positive constant and Q is a positive definite matrix with appropriate dimension.

Lemma 2[6] Let A, D, E be matrices of compatible dimensions. Then the following statements are equivalent:

a) A is a stability matrix and $\| E(sI - A)^{-1}D \|_\infty < 1$.

b) There exists a positive definite symmetric matrix $X > 0$ such that

$$A^{\mathrm{T}}X + XA + XDD^{\mathrm{T}}X + E^{\mathrm{T}}E < 0.$$

The quadratically stabilizable condition of the system (1) is derived as follows.

Theorem 1 For the system (1) ~ (3), let $R_1, R_2 \in \mathbb{R}^{n\times n}$ be given positive definite symmetric matrices, suppose that the disturbance input is zero for all time and there exist two positive definite symmetric matrices P_{c} and P_{o} satisfying the following matrix inequalities respectively,

$$\begin{aligned} S_1 = & (A - BK)^{\mathrm{T}}P_{\mathrm{c}} + P_{\mathrm{c}}(A - BK) + \\ & P_{\mathrm{c}}(2H_0H_0^{\mathrm{T}} + H_1H_1^{\mathrm{T}} + A_1R_1^{-1}A_1^{\mathrm{T}} + BB^{\mathrm{T}})P_{\mathrm{c}} + \\ & (R_1 + 2E_2^{\mathrm{T}}E_2 + 2(E_0 - E_1K)^{\mathrm{T}}(E_0 - E_1K)) < 0, \end{aligned} \tag{7}$$

$$\begin{aligned} S_2 = & (A - LC)^{\mathrm{T}}P_{\mathrm{o}} + P_{\mathrm{o}}(A - LC) + \\ & P_{\mathrm{o}}(2H_0H_0^{\mathrm{T}} + H_1H_1^{\mathrm{T}} + A_1R_2^{-1}A_1^{\mathrm{T}})P_{\mathrm{o}} + \\ & (R_2 + K^{\mathrm{T}}K + 2K^{\mathrm{T}}E_1^{\mathrm{T}}E_1K) < 0 \end{aligned} \tag{8}$$

then the closed-loop system of (1) and (6) is quadratically stabilizable.

Proof Denote $x_\tau = x(t - \tau)$ and $e_\tau = e(t - \tau)$, omit independent variable t, we get,

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \left\{ \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} + \begin{bmatrix} \Delta A - \Delta BK & \Delta BK \\ \Delta A - \Delta BK & \Delta BK \end{bmatrix} \right\} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} A_1 + \Delta A_1 & 0 \\ \Delta A_1 & A_1 \end{bmatrix} \begin{bmatrix} x_\tau \\ e_\tau \end{bmatrix}.$$

The Lyapunov function candidate for this system is chosen as follows,

$$V(e, x, t) = \begin{bmatrix} x^{\mathrm{T}} & e^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} P_{\mathrm{c}} & 0 \\ 0 & P_{\mathrm{o}} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \int_{t-\tau}^{t} \begin{bmatrix} x(\theta) \\ e(\theta) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_1 + 2E_2^{\mathrm{T}}E_2 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} x(\theta) \\ e(\theta) \end{bmatrix} \mathrm{d}\theta. \tag{9}$$

By using Lemma 1 and rearranging, the derivative of (9) with respect to time t is obtained,

$$\dot{V} \leqslant \xi^{\mathrm{T}}(t)S\xi(t) \leqslant \lambda_{\max}(S)\xi^{\mathrm{T}}(t)\xi(t)$$

where $\xi(t) = [x^{\mathrm{T}}(t) \quad e^{\mathrm{T}}(t)]^{\mathrm{T}}, S = \mathrm{diag}(S_1, S_2)$ and $\lambda_{\max}(S)$ denotes the maximum eigenvalue of matrix S. Therefore inequality (5) is satisfied with $\alpha = -\lambda_{\max}(S) > 0$. Thus the quadratic stabilization of closed-loop system of (1) and (6) follows easily from Definition 1. Q.E.D.

The main result of this section is the following theorem.

Theorem 2 For the system (1) ~ (3), for given positive constants $\gamma^2 > \lambda > 0$, let $R_1, R_2 \in \mathbb{R}^{n\times n}$ be given positive definite symmetric matrices, suppose that there exist two positive definite symmetric matrices P_{c} and P_{o} satisfying the following matrix inequalities respectively,

$$\begin{aligned} T_1 \doteq & (A - BK)^{\mathrm{T}}P_{\mathrm{c}} + P_{\mathrm{c}}(A - BK) + (R_1 + 2E_2^{\mathrm{T}}E_2 + \\ & 2(E_0 - E_1K)^{\mathrm{T}}(E_0 - E_1K) + D_2^{\mathrm{T}}D_2) + \\ & P_{\mathrm{c}}(2H_0H_0^{\mathrm{T}} + H_1H_1^{\mathrm{T}} + A_1R_1^{-1}A_1^{\mathrm{T}} + \\ & BB^{\mathrm{T}} + (\gamma^2 - \lambda)^{-1}D_1D_1^{\mathrm{T}})P_{\mathrm{c}} < 0, \end{aligned} \tag{10}$$

$$\begin{aligned} T_2 = & (A - LC)^{\mathrm{T}}P_{\mathrm{o}} + P_{\mathrm{o}}(A - LC) + P_{\mathrm{o}}(2H_0H_0^{\mathrm{T}} + \\ & H_1H_1^{\mathrm{T}} + A_1R_2^{-1}A_1^{\mathrm{T}} + \lambda^{-1}D_1D_1^{\mathrm{T}})P_{\mathrm{o}} + \\ & (R_2 + K^{\mathrm{T}}K + 2K^{\mathrm{T}}E_1^{\mathrm{T}}E_1K) < 0 \end{aligned} \tag{11}$$

then the closed-loop system of (1) and (6) is quadratically stable with an H_∞ norm bound γ.

Proof The proof suffices to prove the inequality $\| z(t) \|_2 \leqslant \gamma \| w(t) \|_2$ from Therorem 1.

From Definition 2, assume that $x(t) = 0, t \in [-\tau, 0]$, consider the following index,

$$J = \int_0^\infty (z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t))\mathrm{d}t.$$

From Theorem 1, the closed-loop system of (1) and (6) is quadratically stable, so we can conclude that for any nonzero $w(t) \in L_2[0, \infty)$ the following equality can be obtained.

$$J = \int_0^\infty (z^{\mathrm{T}}(t)z(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) +$$

$$\frac{\mathrm{d}}{\mathrm{d}t}V(x,e,t))\mathrm{d}t - x^{\mathrm{T}}(\infty)P_c x(\infty) - e^{\mathrm{T}}(\infty)P_o e(\infty) - U_\infty - V_\infty \tag{12}$$

where $V(x,e,t)$ is defined in (9) and U_∞, V_∞ are defined as follows.

Obviously, the following four inequalities are true.

$$0 \leqslant x^{\mathrm{T}}(\infty)P_c x(\infty) < \infty,$$

$$0 \leqslant e^{\mathrm{T}}(\infty)P_o e(\infty) < \infty,$$

$$U_\infty = \lim_{t\to\infty}\int_{t-\tau}^{t} x^{\mathrm{T}}(\theta)(R_1 + 2E_2^{\mathrm{T}}E_2)x(\theta)\mathrm{d}\theta \geqslant 0,$$

$$V_\infty = \lim_{t\to\infty}\int_{t-\tau}^{t} e^{\mathrm{T}}(\theta)R_2 e(\theta)\mathrm{d}\theta \geqslant 0.$$

From the proof of Theorem 1, (12) becomes,

$$J \leqslant \int_0^\infty (z^{\mathrm{T}}z - \gamma^2 w^{\mathrm{T}}w + x^{\mathrm{T}}S_1 x + e^{\mathrm{T}}S_2 e + 2x^{\mathrm{T}}P_c D_1 w + 2e^{\mathrm{T}}P_o D_1 w)\mathrm{d}t.$$

Using the following inequalities

$$2x^{\mathrm{T}}P_c D_1 w \leqslant (\gamma^2 - \lambda)^{-1}x^{\mathrm{T}}P_c D_1 D_1^{\mathrm{T}}P_c x + (\gamma^2 - \lambda)w^{\mathrm{T}}w$$

$$2e^{\mathrm{T}}P_o D_1 w \leqslant \lambda^{-1}e^{\mathrm{T}}P_o D_1 D_1^{\mathrm{T}}P_o e + \lambda w^{\mathrm{T}}w$$

and from (1), (10) and (11), we can easily get $J \leqslant \int_0^\infty (x^{\mathrm{T}}T_1 x + e^{\mathrm{T}}T_2 e)\mathrm{d}t < 0$. Therefore $\| z(t) \|_2 \leqslant \gamma \| w(t) \|_2$, and thus we completes the proof.

Q.E.D.

Based on the above results, we present the following two conditions:

Condition 1a The closed-loop system of (1) and (6) is said to satisfy Condition la if there exist an controller gain matrix K and a positive definite matrix P_c for a constant γ such that (10) holds.

Condition 1b The closed-loop system of (1) and (6) is said to satisfy Condition 1b if there exist an observer gain matrix L and a positive definite matrix P_o such that (11) holds.

Remark 1 Using Theorem 2, it follows immediately that any system satisfying both Condition 1a and Condition 1b will be quadratically stable with an H_∞ norm bound γ. Therefore both Condition 1a and Condition 1b are sufficient conditions for the closed-loop system of (1) and (6) to be quadratically stable with an H_∞ norm bound γ .

4 Robust H_∞ control synthesis

In this section, we will present a design procedure for controller (6) such that the closed-loop system of (1) and (6) will be quadratically stable with an H_∞ norm bound γ .

From Lemma 2, Condition 1a is equivalent to the following statements:

Statement a For a new linear time-invariant system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \bar{D}_c w(t), \\ z(t) &= \bar{E}_{c1}x(t) + \bar{E}_{c2}u(t) \end{aligned} \tag{13}$$

where

$$\bar{D}_c = [\sqrt{2}H_0, H_1, A_1 R_1^{-1/2}, B, (\gamma^2 - \lambda)^{-1/2}D_1],$$

$$[\bar{E}_{c1}\ \bar{E}_{c2}] = \begin{bmatrix} R_1^{1/2} & 0 \\ D_2 & 0 \\ \sqrt{2}E_2 & 0 \\ \sqrt{2}E_0 & \sqrt{2}E_1 \end{bmatrix}$$

with a memoryless state feedback control law

$$u(t) = -Kx(t),$$

the closed-loop system is asymptotically stable and satisfies the H_∞ norm bound constraint

$$\| (\bar{E}_{c1} - \bar{E}_{c2}K)(sI - A + BK)^{-1}\bar{D}_c \|_\infty < 1.$$

Thus Condition 1a can be converted into a linear time-invariant H_∞ control problem.

Let $s = \mathrm{rank}(\bar{E}_{c2})$ and let $U_c \in \mathbb{R}^{(n+q+2j)\times s}$, $V_c \in \mathbb{R}^{s\times m}$ be any matrices such that

$$\bar{E}_{c2} = U_c V_c, \quad \mathrm{rank}(U_c) = \mathrm{rank}(V_c) = s.$$

Next let $\Phi_c \in \mathbb{R}^{(m-s)\times m}$ be chosen such that

$$\Phi_c V_c^{\mathrm{T}} = 0, (\Phi_c = 0 \text{ if } s = m). \tag{14}$$

Define

$$\Xi_c = V_c^{\mathrm{T}}(V_c V_c^{\mathrm{T}})^{-1}(U_c^{\mathrm{T}}U_c)^{-1}(V_c V_c^{\mathrm{T}})^{-1}V_c.$$

Now we are ready to state one main result in this section to get the controller gain matrix K.

Theorem 3 For a given constant $\gamma^2 > \lambda > 0$, let $\Phi_c \in \mathbb{R}^{(m-s)\times m}$ be chosen such that (14) is satisfied and $\hat{Q}_c \in \mathbb{R}^{n\times n}$ be a given positive definite symmetric matrix, then the uncertain time-delay system (1) with controller (6) satisfies Condition 1a if and only if there exists a positive scalar ε_c such that the algebraic Riccati equation

$$(A - 2B\Xi_c E_1^{\mathrm{T}}E_0)^{\mathrm{T}}X + X(A - 2B\Xi_c E_1^{\mathrm{T}}E_0) + X\widetilde{M}_c X + \widetilde{Q}_c + \varepsilon_c\hat{Q}_c = 0$$

where

$$\widetilde{M}_c = \overline{D}_c\overline{D}_c^{\mathrm{T}} - B\Xi_c B^{\mathrm{T}} - \frac{1}{\varepsilon_c}B\Phi_c^{\mathrm{T}}\Phi_c B^{\mathrm{T}},$$

$$\overline{D}_c\overline{D}_c^{\mathrm{T}} = 2H_0H_0^{\mathrm{T}} + H_1H_1^{\mathrm{T}} + A_1R_1^{-1}A_1^{\mathrm{T}} + BB^{\mathrm{T}}(\gamma^2 - \lambda)^{-1}D_1D_1^{\mathrm{T}},$$

$$\widetilde{Q}_c = R_1 + D_2^{\mathrm{T}}D_2 + 2E_2^{\mathrm{T}}E_2 + 2E_0^{\mathrm{T}}(I - 2E_1\Xi_cE_1^{\mathrm{T}})E_0$$

has a positive definite symmetric solution X. Furthermore, if such a solution exists, a suitable robust H_∞ control law for the equivalent linear time-invariant system (13) is given by

$$u(t) = -Kx(t)$$

where

$$K = \left(\frac{1}{2\varepsilon_c}\Phi_c^{\mathrm{T}}\Phi_c + \Xi_c\right)B^{\mathrm{T}}X + 2\Xi_cE_1^{\mathrm{T}}E_0.$$

Proof Similar to the proof in [6] and omitted due to length limitation.

Obviously, inequality (11) is equivalent to the following inequality

$$P_o^{-1}(A^{\mathrm{T}} - C^{\mathrm{T}}L^{\mathrm{T}}) + (A^{\mathrm{T}} - C^{\mathrm{T}}L^{\mathrm{T}})^{\mathrm{T}}P_o^{-1} + (2H_0H_0^{\mathrm{T}} + H_1H_1^{\mathrm{T}} + A_1R_2^{-1}A_1^{\mathrm{T}} + \lambda^{-1}D_1D_1^{\mathrm{T}}) + P_o^{-1}(R_2 + K^{\mathrm{T}}K + 2K^{\mathrm{T}}E_1^{\mathrm{T}}E_1K)P_o^{-1} < 0.$$

From Lemma 2 and (15), Condition 1b is equivalent to the following statements:

Statement b For a new linear time-invariant system

$$\dot{x}(t) = A^{\mathrm{T}}x(t) + C^{\mathrm{T}}u(t) + \overline{D}_ow(t), z(t) = \overline{E}_{o1}x(t) \tag{16}$$

where

$$\overline{D}_o = [R_2^{1/2} \quad K^{\mathrm{T}} \quad \sqrt{2}K^{\mathrm{T}}E_1^{\mathrm{T}}],$$

$$\overline{E}_{o1}^{\mathrm{T}} = [\sqrt{2}H_0 \quad H_1 \quad R_2^{-1/2}A_1 \quad \lambda^{-1/2}D_1]$$

with a memoryless state feedback control law

$$u(t) = -L^{\mathrm{T}}x(t),$$

the closed-loop system is stable and satisfies the H_∞ norm bound constraint

$$\| \overline{E}_{o1}(sI - A^{\mathrm{T}} + C^{\mathrm{T}}L^{\mathrm{T}})^{-1}\overline{D}_o \|_\infty < 1.$$

Thus Condition 1b can also be converted into a linear time-invariant H_∞ control problem.

Now we are ready to state another main result to get the observer gain matrix L.

Theorem 4 For a given constant $\lambda > 0$, let $\Phi_o \in \mathbb{R}^{r\times r}$ be chosen to be any nonsingular matrix, $\hat{Q}_o \in \mathbb{R}^{n\times n}$ be a given positive definite symmetric matrix and $K \in \mathbb{R}^{m\times n}$ is assumed to be obtained from Theorem 3 then the uncertain time-delay system (1) with controller (6) satisfies Condition 1b if and only if there exists a positive scalar ε_o such that the algebraic Riccati equation

$$AY + YA^{\mathrm{T}} + Y\widetilde{M}_oY + \widetilde{Q}_o + \varepsilon_o\widetilde{Q}_o = 0$$

where

$$\widetilde{M}_o = R_2 + K^{\mathrm{T}}K + 2K^{\mathrm{T}}E_1^{\mathrm{T}}E_1K - \frac{1}{\varepsilon_o}C^{\mathrm{T}}\Phi_o^{\mathrm{T}}\Phi_oC,$$

$$\widetilde{Q}_o = 2H_0H_0^{\mathrm{T}} + H_1H_1^{\mathrm{T}} + A_1R_2^{-1}A_1^{\mathrm{T}} + \lambda^{-1}D_1D_1^{\mathrm{T}}$$

has a positive definite symmetric solution Y. Furthermore, if such a solution exists, a suitable robust H_∞ control law for the equivalent linear time-invariant system (16) is given by

$$u(t) = -L^{\mathrm{T}}x(t)$$

where
$$L^{\mathrm{T}} = \frac{1}{2\varepsilon_o}\Phi_o^{\mathrm{T}}\Phi_oCY.$$

Proof Similar to the proof in [6] and omitted due to length limitation.

5 Conclusion

The robust H_∞ output feedback control analysis and synthesis are obtained for the linear time-delay systems including time-varying uncertainties in system matrices which do not need to satisfy the so-called matching conditions. Based on the notion of quadratic stabilization with H_∞ norm bound and Riccati equation approach, sufficient conditions for the solvability of the robust H_∞ control problem are obtained to ensure not only the quadratic stabilization but also the H_∞ norm bound constraint of the closed-loop system. Two equivalent linear time-invariant structural descriptions for the linear time-varying uncertain systems with delayed state are used to construct linear time-invariant dynamic output feedback controller.

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$$S = \left[\begin{array}{c|cc} A - B_1 D_{21}^{-1} C_2 & B_2 - B_1 D_{21}^{-1} D_{22} & B_2 \\ \hline C_1 - D_{11} D_{21}^{-1} C_2 & D_{12} - D_{11} D_{21}^{-1} D_{22} & D_{11} D_{21}^{-1} \\ -D_{21}^{-1} C_2 & -D_{21}^{-1} D_{22} & D_{21}^{-1} \end{array}\right] = \left[\begin{array}{cc|cc} -2 & 0 & -1 & 1 \\ 0 & -1 & -1 & 2 \\ \hline 2 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right].$$

Step 2 Find (J, J')-loseless factorization:

In this case, $m = 2, r = p = q = 1$. First, we solve (2) to get a $D_\pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then by solving two Riccati equations we get $X = \mathrm{Ric}(Hx) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0, Y = \mathrm{Ric}(H_y) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \geqslant 0$. Thus we get (J, J')-loseless factorization:

$$\Theta(s) = \left[\begin{array}{c|cc} -2 & -1 & 1 \\ \hline 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{array}\right], \Pi(s) = \left[\begin{array}{c|cc} -5 & 1 & -2 \\ \hline 2 & 1 & 0 \\ 3 & 0 & 1 \end{array}\right].$$

Step 3 Compute H_∞ controller.

a) Compute stable H_∞ controller.

From Theorem 2, take $Q(s) = D_q$, where D_q is a constant number $(p = q = 1)$. We have: $K(s) = \left[\begin{array}{c|c} 1 - 3D_q & -D_q + 2 \\ \hline -2 + 3D_q & D_q \end{array}\right]$, $A_k = 1 - 3D_q$. If $Q(s) = D_q = 0$, $A_k = 1$, we get a unstable controller. Clearly we can get lower order (order less than 2) stable controller by taking any $1/3 < D_q < 1$. For example if we take $Q(s) = D_q = 5/6$, we have $K(s) = (5s + 11)/(6s + 9)$, which is a 1-st order stable controller.

b) Compute reduced order H_∞ controller.

$\Pi_{11}(s) = (s + 7)/(s + 5), \Pi_{12}(s) = -4/(s + 5), \Pi_{21}(s) = 3/(s + 5), \Pi_{22}(s) = (s - 1)/(s + 5)$. Solve equation (15) we get common zero: $s = -1$. From equation (16) we have $Q(-1) = 2/3$. Taking $Q(s) = 2/3$ we get reduced order controller $K(s) = 2/3$, which is a zero-th order proportional controller.

Remark For the above plant, the controller designed by DGKF method in Robust Toolbox in Matlab is $(Q(s) = 0)$: $K(s) = \left[\begin{array}{cc|c} -2 & 3 & 1 \\ 0 & 1 & 2 \\ \hline 0 & 2 & 0 \end{array}\right]$. Cleary $K(s)$ is unstable and its order is higher than that of our controller.

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