X-Q Adaptive PID Controller and Its Application to Multiobjective Control System with Satisfied Performance

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Abstract: This paper gives a new adaptive PID controller named X-Q. It has novel frequency characteristics of adaptability: For the control action it is an adaptive PID controller without any phase lag, but there is an amplifier with the gain k in the high frequency and an integrator with integral time constant τ in the low frequency; For the harmonic input it is still an adaptive PID controller, but the transfer function in the high frequency is $W = 0.81ke^{-j51.85°}$, the transfer function in the medium-low frequency is $W = \frac{0.41}{\tau\omega}e^{-j57.52°}$. Finally, employ the new adaptive PID controller to realize satisfied control of the multiobjective control system (MOCS). The results show that its performances are much better than the ones of ITAE optimum control system.

Key words: adaptive PID; satisfied control of MOCS; ITAE optimum control; frequency domain

X-Q 自适应 PID 及其在多目标满意控制系统中的应用

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摘要: 给出一种新型 X-Q 自适应控制器，它具有新颖的自适应频率特性：对于控制信号而言，它是一个没有任何相位落后的 PID，且高频段为一放大倍数为 k 的放大器，低频段为一积分时间常数为 τ 的积分器；而对谐波输入信号而言，它仍然是自适应 PID，但高频段的传递函数 $W = 0.81ke^{-j51.85°}$，低频段的传递函数 $W = \frac{0.41}{\tau\omega}e^{-j57.52°}$. 最后，应用这种新型的 X-Q 自适应控制器去实现多目标满意控制，结果表明，它的性能指标比 ITAE 最优控制律好得多.

关键词: 自适应 PID; 多目标满意控制; ITAE 最优控制; 频率法

1 Presentation of the problem

The reference signal of control system is generally a function of time t with low order. That is to say it contains not only step input, but also ramp and acceleration, etc. A system with optimum performance should be with good responses not only for the step, but also for the ramp and the acceleration, etc. We define such a system that has satisfied more than two optimum performances as multi-objective optimum control system (MOCS).

It is impossible to realize the optimum control of MOCS with linear control theory. That is because linear system is a minimum phase system. An increase of one degree of zero steady-state error will bring 90° phase lag to the system, which would worsen the dynamic performance. Reference [1] points out that multi-value nonlinearity would break the stability of ITAE optimum control system with zero steady-state error of uniform velocity. Whatever kind of nonlinearity may be present in the system, it will always break the stability of ITAE optimum system with zero steady-state error of uniform acceleration. So it is very difficult to apply the two kinds of optimum control system to engineering.

In order to realize satisfied control of MOCS the following problems should be solved:

First, One needs some practical nonlinear integrators in engineering. Its amplitude frequency characteristic can make integral action, but its phase lag must be as small as possible. Reference [2] gives a nonlinear integrator with variable phase lag. Reference [3] gives the one with zero phase lag. But they both have too big higher-harmonics. Reference [4] gives an only 38.1° phase lag nonlinear integrator named Clegg by us. Reference [5]

Manuscript received Jun. 5, 1998, revised Nov. 13, 1998.

gives an intelligent integrator. Its phase lag is only 27.6°. There are not too big higher harmonics in the last two kinds of integrator. So they can be applied to engineering, but they are both of a nonlinear integrator type. Here a new adaptive PID controller is given. It can automatically change its control function according to the transient process developing, starting from amplifier state, passing through PID, then coming into integrator state at the end. This wonderful characteristic just satisfies the requirement presented to the controller by optimum control law. In Sections 2 and 3 its basic principle and adaptability are described.

Secondly, the system is nonlinearized after leading nonlinear controller into it. Designing a nonlinearization system with optimum performance is very difficult. References [6,7] have already discussed how to use equal-amplitude principle and artificial experience to design optimum control of MOCS with Clegg integrator. And those techniques are called twice optimum control. Its performance is much better than the one of linear system with ITAE optimum control law, but it has not yet met the practical requirement. Optimum solution of MOCS is multi-valued. Generally we can only take its sectional optimum solution. It is very difficult to get the optimum solution in the whole field. In this paper, we prefer looking for its satisfied optimum solution with CAD. It must be pointed out that satisfied solution is no worse than the sectional optimum solution sometimes. This problem will be discussed in Section 4. Section 5 is the conclusion.

2 Basic principle

Assume the controller with the model

$$e_0(t) = \begin{cases} \frac{1}{\tau}\int e_i(t)\mathrm{d}t, & e_i(t)\dot{e}_i(t) < 0, \\ ke_i(t), & e_i(t)\dot{e}_i(t) > 0, \\ 0, & e_i(t) = 0, \end{cases} \tag{1}$$

where $e_i(t)$, $e_0(t)$, k and τ are input, output, gain and integral time constant of the controller respectively.

Setting

$$e_i(t) = A_m \sin\omega t, \tag{2}$$

where A_m is amplitude; ω is frequency. $e_0(t)$ can be expanded into Fourier's series, and with only the basic harmonic is taken, then

$$e_0(t) \approx e_{01}(t) = A_1 \sin(\omega t + \theta_1), \tag{3}$$

where

$$A_1 = \frac{A_m}{2\pi\tau\omega}\sqrt{(2k\tau\omega+\pi)^2+[k\tau(\pi+4)\omega+2]^2}; \tag{3.1}$$

$$\theta_1 = -\,\mathrm{tg}^{-1}\frac{2k\tau\omega + \pi}{(\pi + 4)k\tau\omega + 2}. \tag{3.2}$$

Definition 1 Setting

$$W(\omega, k, \tau) = \frac{E_{01}(\omega)}{E_i(\omega)} = H(\omega, k, \tau)\mathrm{e}^{\mathrm{j}\theta_1(\omega,k,\tau)}. \tag{4}$$

It is defined as the transfer function of the controller, where $E_{01}(\omega)$ and $E_i(\omega)$ indicate the Fourier transform of $e_{01}(t)$ and $e_i(t)$ respectively,

$$H(\omega, k, \tau) = \frac{\sqrt{13.9}}{2\pi\tau\omega}\sqrt{\cdot}, \tag{4.1}$$

here,

$$\sqrt{\cdot} = \sqrt{\left(\frac{\omega}{\omega_z}\right)^2 + 2\zeta\left(\frac{\omega}{\omega_z}\right) + 1}, \tag{4.2}$$

$$\omega_z = \sqrt{\frac{13.9}{55}\frac{1}{k\tau}} \approx 0.5\frac{1}{k\tau}, \tag{4.3}$$

$$\zeta = \frac{41.12}{2\sqrt{13.9 \times 55}} = 2.35. \tag{4.4}$$

Theorem 1 If $\omega \to \infty$, then $W(\infty, k) = 1.16k\mathrm{e}^{-\mathrm{j}15.6°}$; If $0 \leqslant \omega \leqslant \omega_z$, then $W(\omega, \tau) = \frac{1.38}{\tau\omega}\mathrm{e}^{\mathrm{j}\theta_1}$, where, $\theta_1 \in [-40°, -57.52°]$. So $W(\omega, k, \tau)$ is the transfer function of adaptive PID controller.

Proof Take the limit for Eq. (4), it becomes:

$$W(\infty, k) = \lim_{\omega\to\infty} W(\omega, k, \tau) = \frac{\sqrt{55}}{2\pi}k\mathrm{e}^{-\mathrm{j}\,\mathrm{tg}^{-1}(2/7.14)} = 1.16k\mathrm{e}^{-\mathrm{j}15.6°}, \tag{5.1}$$

$$W(\omega_z, k, \tau) = \lim_{\omega\to\omega_z} W(\omega, k, \tau) = \frac{1.38}{\tau\omega}\mathrm{e}^{-\mathrm{j}40°}, \tag{5.2}$$

$$W(0, \tau) = \lim_{\omega\to 0} W(\omega, k, \tau) = \frac{\sqrt{13.9}}{2\pi}\frac{1}{\tau\omega}\mathrm{e}^{-\mathrm{j}\,\mathrm{tg}^{-1}(\frac{\pi}{2})} = \frac{0.6}{\tau\omega}\mathrm{e}^{-\mathrm{j}57.52°}. \tag{5.3}$$

Q.E.D.

Theorem 1 shows that the controller possesses a frequency characteristic of adaptability in the transient process: There is a gain with phase lag 15.6° and it only depends on the k in the high frequency; and an integrator with phase lag $\theta_1 \in [-40°, -57.52°]$ in the medium-low frequency as $\omega \in [0, \omega_z]$ and it only depends on the τ. That is to say, its control function automatically transforms as following transient development from amplifier state at the beginning, passing through PID into an integrator state at the end. It means this controller can separately control the quickness, smoothness and steady-state precision of system with k and τ. This kind of adaptability is different from the others. So we name it X-Q adaptive PID.

Adaptive PID is a nonlinear controller itself. The big or small higher harmonic can damage the performance of system in different degree. We can count the ratio of all kinds of higher harmonics to the basic. The results show that the greater the gain k, the smaller the influence of higher harmonic on the performance.

3 Adaptability

In order to bring to light the adaptability of the controller it is necessary to study its asymmetrical frequency characteristic.

where, A_0 is DC component of controller input.

The $e_0(t)$ can be expanded as Fourier's series, and only DC and basic harmonic are taken, then the following equation can be obtained.

$$E_0(\omega t) = H_0(\omega,r,k,\tau)A_0 + W_1(\omega,r,k,\tau)A_m\sin\omega t = E_{0d} + E_{01}, \tag{7}$$

where, E_{0d} and E_{01} reqresent DC component and basic harmonic of the controller output respectively.

$$H_0(\omega,r,k,\tau) = \frac{k_0}{\tau\omega}\left(\frac{\omega}{\omega_{z0}} + 1\right), \tag{8}$$

$$k_0 = \frac{\pi^2 + 4(\arcsin r)^2 + 8}{4\pi}, \tag{8.1}$$

$$\omega_{z0} = \frac{\pi^2 + 4(\arcsin r)^2 + 8}{8(\pi + \frac{1}{r}\arcsin r)k\tau}, \tag{8.2}$$

$$W_1(\omega,r,k,\tau) = H_1(\omega,r,k,\tau)e^{-j\theta_1(\omega,r,k,\tau)}, \tag{9}$$

$$H_1(\omega,k,\tau,r) = \frac{\sqrt{N}}{2\pi\tau\omega}\sqrt{\left(\frac{\omega}{\omega_{z1}}\right)^2 + 2\zeta_1\frac{\omega}{\omega_{z1}} + 1}, \tag{9.1}$$

Setting $$e_i(t) = A_0 + A_m\sin\omega t, \tag{6}$$

$$\theta_1(\omega,k,\tau,r) = -\mathrm{tg}^{-1}\frac{2k\tau(1+r^2)\omega + (2\pi r^2 + \pi)}{(\pi + 4\sqrt{1-r^2})k\tau\omega + 2(1 - 3r^2 + 2r\sqrt{1-r^2}\arcsin r)}, \tag{9.2}$$

here,

$$\omega_{z1} = \sqrt{\frac{N}{L_r}\frac{1}{k\tau}};\ \zeta_1 = \frac{M_r}{2\sqrt{NL_r}};\ r = \frac{A_0}{A_m};$$

$$N = 4[(\pi^2+9)r^4 + (\pi^2-6)r^2 + \frac{\pi}{4} + 1 + 4(1-3r^2)r\sqrt{1-r^2}\arcsin r + 4r^2(1-r^2)\arcsin r];$$

$$L_r = 4r^4 - 8r^2 + 8\pi\sqrt{1-r^2} + \pi^2 + 20;$$

$$M_r = 8\pi(r^4 - 2r\sqrt{1-r^2}\arcsin r + 1) + 4(4-12r^2)\sqrt{1-r^2} + 32r(1-r^2)\arcsin r.$$

Definition 2 Setting

$$H_0(\omega,r,k,\tau) = \frac{k_0}{\tau\omega}\left(\frac{\omega}{\omega_{z0}} + 1\right) = \frac{E_{0d}}{A_0}.$$

Define it as DC component transfer function of the controller.

Theorem 2 If $\omega \to \infty$, $r \to 1$, then $H_0(\infty,1,k) = 3k$; If $\omega = \omega_{z0}, 0 < r < 1$, then $H_0(\omega,r,\tau) = \frac{h_0(r)}{\tau\omega}$, where $h_0(r) = \frac{\pi^2 + 4(\arcsin r)^2 + 8}{2\pi}$; If $\omega \to 0$; $r \to 0$, then $H_0(0,0,\tau) = 1.5/(\tau\omega)$. So H_0 is the transfer function of an adaptive PID controller without any phase lag.

Proof Because $\omega \to \infty \Leftrightarrow r \to 1$; $\omega \to 0 \Leftrightarrow r \to 0$[7], take the limit of Eq.(8), one will get the above results. Q.E.D.

Definition 3 Setting

$$W_1(\omega,r,k,\tau) = H_1(\omega,r,k,\tau)e^{-j\theta_1(\omega,r,k,\tau)} = \frac{E_{01}(\omega)}{E_{i1}(\omega)}. \tag{10}$$

Define it as basic harmonic transfer function of the controller.

All the same, theorem 3 is obtained.

Theorem 3 If $\omega \to \infty$, $r \to 1$, then $W_1(\infty,1,k) = 0.8ke^{-j51.85°}$; If $\omega \to 0$, $r \to 0$, then $W_1(0,0,\tau) = \frac{0.41}{\tau\omega}e^{-j57.52°}$. It is still an adaptive PID controller for basic harmonic signal.

Fig. 1 gives Bode plots of Eq.(8) and Eq.(9). The dotted lines indicates Bode plots for different r, because r and ω are variable in the transient process, so the actual Bode plots are solid lines indicated. It gives full proof of the adaptability for this controller in the transient process. It is pointed out that the influence of r on H_0 and H_1 is small, but the influence on θ_1 is very big. This is because $\omega_{z1}(r)$ will decrease very quickly as r decreases. That is to say, the smaller the r, the bigger the θ_1 and the smoother the transition.

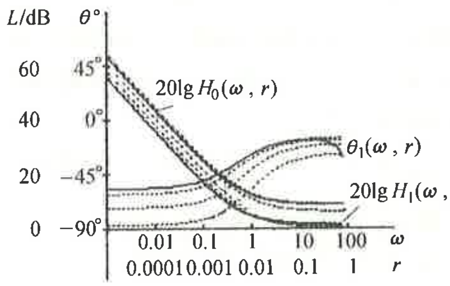


Fig. 1 Bode plots of X-Q adaptive PID

Compare the Theorem 2, 3 with 1, and we see that the basic harmonic is weakened by asymmetrical component, while control action (DC component) is stronger. That means the controller will have a better adaptability.

4 Application in MOCS

Optimum control of MOCS is a deep question in the linear control theory for a long time. This is because if you want to realize the optimal control system with zero steady-state error of uniform velocity or acceleration, then its step response is very bad. But this problem can be easily solved by X-Q adaptive PID controller.

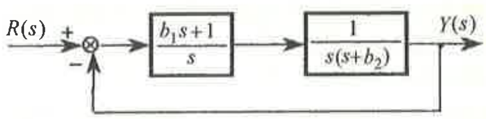


Fig. 2 Three order control system with two integrators

Example 1 Fig.2 gives a system with zero steady-state error of uniform velocity. Where transfer function of controlled object is $\frac{1}{s(s+b_2)}$, $\frac{b_1 s+1}{s}$ is transfer function of linear controller. Here S is standard differential operator. Design task is to select optimum parameter set (b_1, b_2) for making the system double objective optimum performance. According to the ITAE linear optimum control law[8], $b_1 = 3.25$, $b_2 = 1.75$. Its responses are as follows[8]:

1) Step response: $Q_{dl}\% = 38.7\%$, $t_{dl} = 7.2\text{s}$;

2) Ramp response: $t_{vl} = 4.73\text{s}$.

where $Q_{dl}\%$ is overshoot, t_{dl}, t_{vl} is transient time as $e(\infty) \leqslant 2\%$, here subindicator "l" indicates linear.

Now, utilize X-Q adaptive PID controller to make above system double objective satisfied control.

Record

$$s_n = \frac{2\pi\tau\omega}{\sqrt{13.9\sqrt{\cdot}}} e^{-j\theta_1(\omega,k,\tau)} = \tau\bar{s}_n,$$

$$\text{then} \quad \bar{s}_n = \frac{2\pi\omega}{\sqrt{13.9\sqrt{\cdot}}} e^{-j\theta_1(\omega,k,\tau)}. \tag{11}$$

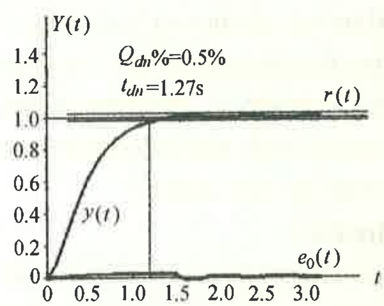
Use $\frac{b_1\bar{s}_n + 1}{\bar{s}_n}$ instead of linear PID controller $\frac{b_1 s + 1}{s}$ in Fig.2. Its open-loop transfer function takes the form of

$$W(s,\bar{s}_n) = \frac{b_1\bar{s}_n + 1}{\bar{s}_n s(s+b_2)}. \tag{12}$$

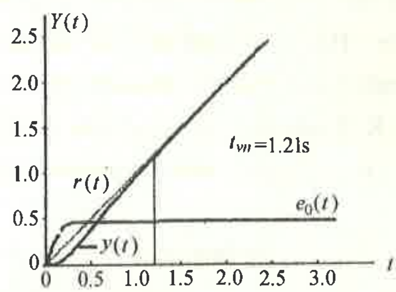
It is a very complicated nonlinear equation. To take its analysis solution is very difficult. We can look for its satisfied solution with CAD. First, give a set of parameters, make out logarithmic frequency characteristic of Eq.(12), then revise them step by step to get the maximum phase margin. Take this parameter set as initial value to look for the optimum parameter set to satisy the following performance function

$$J = Q_{dn}\% t_{dn} = \min. \tag{13}$$

With the system the better performance as $b_1 \in [19,35]$, $b_2 \in [9,70]$ can be achieved. Finally, among the above number field, a satisfied parameter set, that makes the system with response time t_{vn} as quickly



(a) Step response



(b) Ramp response

Fig. 3 Response curves of double objective satisfied control system as possible for the uniform velocity input, can be found as follows: $b_1 = \tau = 19.8$, $b_2 = 9.6$, $k = 3$. Its responses are shown in Fig.3.

where

$$Q_{dn}\% = 0.5\% \ll Q_{dl}\% = 38.7\%,$$
$$t_{dn} = 1.27\mathrm{s} \ll t_{dl} = 7.2\mathrm{s},$$
$$t_{vn} = 1.21\mathrm{s} \ll t_{vl} = 4.73\mathrm{s}.$$

The results show that the performance of double objective control system with X-Q adaptive PID is much better than the one of ITAE linear optimum system.

Example 2 It is known that the ITAE optimum open-loop transfer function of system with three integrators is as follows[8]:

$$W_l(s) = \frac{(b_1 s + 1)(b_2 s + 1)}{s^3} = \frac{(4.235s + 1)(0.705s + 1)}{s^3}. \tag{14}$$

Its responses are as follows: $Q_{dl}\% = 24.8\%$, $t_{dl} = 3.37\mathrm{s}$; $t_{vl} = 2.82\mathrm{s}$; $t_{al} = 3.16\mathrm{s}$.

By the same method, its open-loop transfer function can be obtained as follows

$$W(s, \bar{s}_n) = \frac{(b_1 \bar{s}_n + 1)(b_2 s + 1)}{\bar{s}_n s^2}. \tag{15}$$

We can obtain the satisfied control parameter set: $k = 3$, $b_1 = \tau = 5$, $b_2 = 1$. Its response curves are shown in Fig. 4, where,

$$Q_{dn}\% = 3.5\% \ll Q_{dl}\% = 24.8\%,$$
$$t_{dn} = 0.79\mathrm{s} \ll t_{dl} = 3.37\mathrm{s},$$
$$t_{vn} = 0.72\mathrm{s} \ll t_{vl} = 2.82\mathrm{s},$$
$$t_{an} = 1.57\mathrm{s} \ll t_{al} = 3.16\mathrm{s}.$$

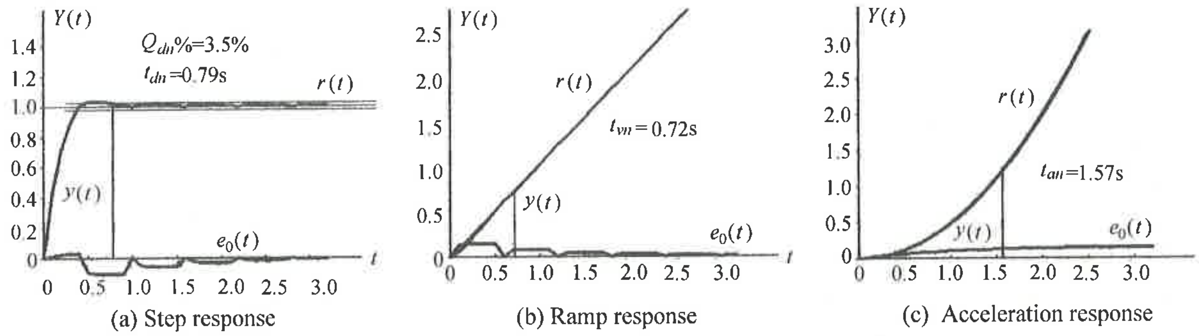


Fig. 4 Response curves of three objective satisfied control system

The Results show that the performance of three objective satisfied control system with X-Q adaptive PID is also much better than the one of ITAE linear optimum system. Indeed, its quality is very good. People must regard it as a system with satisfied control performance.

So does it for the other system.

5 Conclusion

Results investigated show that X-Q adaptive PID controller has a very good adaptability. By means of this a system can multiobjectively satisfy its required control performance. This is the need of many industrial control systems, especially in servomechanism, and it is easy to realize the X-Q adaptive PID algorithm with microprocessors. So it is of great value for practical application.

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(12), (13) requires additional costs for the calculation of the symmetric matrix-function $\Psi(t, u)$. The expected effect of this information is connected with the improvement of the quality of each iteration. This theoretical prediction is justified by the results of the numerical testing, when the quasigradient procedures (12), (13) show their advantages in comparison with alternative methods.

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