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Quasigradient Methods for Solving Optimal Control Problems

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Abstract: In this paper, the nonlinear optimal control problem connected with the ordinary differential system is considered, Two modifications to the standard gradient procedures are constructed. The presented methods are based on the qualitative approximations of the cost functional. For linear-quadratic problems, the modifications have the property of the nonlocal improvement in contrast to the standard gradient procedures. Some results relating to the convergence of the new methods are proved.

Key words: functional increment; quasigradient; approximation

解决最优控制问题的准梯度方法

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摘要: 讨论非线性最优控制问题构造的标准梯度方法的两种改进方法. 文中的方法是以罚函数的有效近似为基础, 与标准梯度方法相比, 该方法对于非线性二次问题具有非局部的特点, 同时给出相关收敛结果.

关键词: 函数增量; 准梯度; 近似化

1 Introduction

Traditionally the gradient methods are the standard tool for solving optimal control problems[1~3]. The technology of their construction and analysis on the function level is developed quite well and completely corresponds to the finite-dimensional situation. However, the specific character of optimal control problems allows to introduce certain corrections in the ultimate structure of the gradient procedures. As a rule, these modifications have nontrivial character and help to discover the additional information to improve the quality of some methods [4,5].

In this paper there are presented two quasigradient methods that use some corrections of the usual gradient. The basis of the modification is the nonstandard formula of the functional increment with improved characteristics of the approximation. This fact increases the quality of corresponding methods which obtain the second order approximation with respect to the phase variables. Consequently, appear to be the property of the nonlocal improvement for the linear-quadratic problem that takes off the subproblem of the parametric search on each iteration.

The methods are developed for the typical problem of the optimal control without terminal constraints

$$\Phi(u) = \varphi(x(t_1)) + \int_T F(x,u,t)\mathrm{d}t \to \min, \quad (1)$$

$u \in V$,

$$\dot{x} = f(x,u,t), \ x(t_0) = x^0, \quad (2)$$

$$V = \{u \in PC(T): u(t) \in U, t \in T\}. \quad (3)$$

Here $t \in T = [t_0, t_1]$ is an independent variable, $u(t) \in \mathbb{R}^r$ is a control, $x(t) \in \mathbb{R}^n$ is a phase state.

Suppose that the terminal function $\varphi(x)$ is twice continuously-differentiable in $\mathbb{R}^n$, the integrand $F(x,u,t)$ and the vector-function $f(x,u,t)$ are continuous in $\mathbb{R}^n \times U \times T$ together with first and second order derivatives with respect to x. Suppose also that there exist the continuous derivatives $F_u(x,u,t), f_u(x,u,t)$ satisfying the Lipschitz condition with respect to x on the set $\mathbb{R}^n \times$

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$U \times T$.

The set of admissible controls V includes the piece-wise-continuous vector-functions $u(t)$ with the restriction $u(t) \in U, t \in T$, where $U \subset \mathbb{R}^r$ is a convex compact set. Suppose that each admissible control $u \in V$ generates the unique absolutely-continuous solution $x(t, u)$, $t \in T$, of the system (2) and the family of the phase trajectories $\{x(t, u)\}$ is bounded: $x(t, u) \in X$, $t \in T, u \in V$, where $X \subset \mathbb{R}^n$ is the compact set.

2 Quasigradient methods of improvement

Define the Pontryagin function (Hamiltonian)

$$H(\psi, x, u, t) \leqslant \langle \psi, f(x, u, t)\rangle F(x, u, t)$$

with the conjugate variable $\psi \in \mathbb{R}^n$.

Let $u(t)$ be an admissible control with the phase trajectory $x(t, u), t \in T$. We denote

$$f[t, u] = f(x(t, u), u(t), t).$$

Introduce the vector-matrix problem for the conjugate objects: $(n \times 1)$ vector-function $\psi(t)$ and $n \times n$ matrix-function $\Psi(t)$

$$\begin{cases} \dot{\psi} = -H_x(\psi, x(t, u), u(t), t), \\ \dot{\Psi} = -f_x[t, u]^{\mathrm{T}}\Psi - \Psi f_x[t, u] - \\ \qquad H_{xx}(\psi, x(t, u), u(t), t), \end{cases} \tag{4}$$

$$\begin{cases} \psi(t_1) = -\varphi_x(x(t_1, u)), \\ \Psi(t_1) = -\varphi_{xx}(x(t_1), u)). \end{cases} \tag{5}$$

Let $\psi(t, u), \Psi(t, u), t \in T$, be the solution of Cauchy problem (4,(5). Note that $\Psi(t)$ is the symmetric matrix.

Let $w(t) = u(t) + \Delta u(t)$ be an admissible control with the phase trajectory $x(t, w) = x(t, u) + \Delta x(t)$, $t \in T$. Consider the known formula of the functional increment[6]

$$\Phi(w) - \Phi(u) =$$
$$-\int_T \Delta_{w(t)} H(\psi(t, u) +$$
$$\Psi(t, u)\Delta x(t), x(t, w), u(t), t)\mathrm{d}t + \eta, \tag{6}$$
$$\eta = o_\varphi(\|\Delta x(t_1)\|^2) + \int_T o_F(\|\Delta x(t)\|^2)\mathrm{d}t -$$
$$\int_T \langle \psi(t, u), o_f(\|\Delta x(t)\|^2)\rangle \mathrm{d}t -$$
$$\int_T \langle o_f(\|\Delta x(t)\|), \Psi(t, u)\Delta x(t)\rangle \mathrm{d}t.$$

Here $\Delta_w H$ is the partial increment of Hamiltonian with respect to the control on the pair (u, w), o_φ, o_F, o_f are the remainder terms in the expansions of the corresponding functions with respect to the increment Δ_x.

An estimate for the phase increment is as follows

$$\|\Delta x(t)\| \leqslant C\int_T \|\Delta u(t)\| \mathrm{d}t. \tag{7}$$

It should be noted that the remainder η in (6) has the order higher than second with respect to $\|\Delta(x)\|$. Thus, the formula (6) defines the quadratic phase approximation of the functional Φ.

Linearize the integrand in (6) with respect to $\Delta u(t) = w(t) - u(t)$. Then we have the expression

$$\Delta_w \Phi(u) = \delta_2\Phi(u, w) + \eta_2,$$
$$\delta_2\Phi(u, w) =$$
$$-\int_T \langle H_u(\psi(t, u) +$$
$$\Psi(t, u)\Delta x(t), x(t, w), u(t), t), \Delta u(t)\rangle \mathrm{d}t, \tag{8}$$
$$\eta_2 = \eta - \int_T o_H(\|\Delta u(t)\|)\mathrm{d}t.$$

The basis of the approximation (8) is quasigradient —— the derivative H_u, calculated along the mixed system of the arguments. As a corollary the quality of the approximation with respect to Δx is growing on the order. The effect is especially obvious for the problem, linear with respect to control, when $o_H(\|\Delta u(t)\|) = 0, \eta_2 \sim o(\|\Delta x\|^2), \eta_0 \sim o(\|\Delta x\|)$.

For given admissible pair $(u(t), x(t, u))$ we find the solutions $\psi(t, u), \Psi(t, u)$ of the vector-matrix Cauchy problem (4),(5). Form the vector-function

$$p(x, t) = \psi(t, u) + \Psi(t, u)(x - x(t, u)),$$
$$x \in \mathbb{R}^n,\ t \in T.$$

Determine the auxiliary control by one of the two variants:

1) Conditional quasigradient method:

$$\bar{u}(x, t) = \arg\max_{v \in U}\langle H_u(p(x, t), x, u(t), t), v\rangle, \tag{9}$$

2) Projection quasigradient method:

$$\bar{u}(x, t) = P_U(u(t) + H_u(p(x, t), x, u(t), t)). \tag{10}$$

Conduct the variation procedure with parameter $\alpha \in [0,1]$

$$u(x,t,\alpha) = u(t) + \alpha(\bar{u}(x,t) - u(t)),$$
$$x \in \mathbb{R}^n,\ t \in T. \tag{11}$$

Find the solution $x_\alpha(t)$ of the phase system

$$\dot{x} = f(x, u(x,t,\alpha), t),\ x(t_0) = x^0. \quad (12)$$

Denote $u_\alpha(t) = u(x_\alpha(t), t, \alpha)$ and define the problem for parameter $\alpha \in [0,1]$ by the improvement conditon

$$\Phi(u_\alpha) \leqslant \Phi(u). \quad (13)$$

Let's justify the described procedure. Suppose that the solution $x_\alpha(t)$ of the phase system (12) exists on T for each $\alpha \in [0,1]$. Note that the trajectory $x_\alpha(t)$ corresponds to the control $u_\alpha(t)$: $x_\alpha(t) = x(t, u_\alpha)$. According to the estimate (7) for $\Delta u(t) = u_\alpha(t) - u(t)$ we obtain

$$\| \Delta x(t) \| = \| x_\alpha(t) - x(t,u) \| \leqslant C_\alpha,\ t \in T.$$

Denote

$$\bar{u}_\alpha(t) = \bar{u}(x_\alpha(t), t),\ p_\alpha(t) = p(x_\alpha(t), t).$$

Then the variation procedure takes the form

$$u_\alpha(t) = u(t) + \alpha(\bar{u}_\alpha(t) - u(t)),\ t \in T.$$

Define the nonnegative value

$$\delta_\alpha(u) = \int_T \langle H_u(p_\alpha(t), x_\alpha(t), u(t), t), \bar{u}_\alpha(t) - u(t) \rangle \mathrm{d}t.$$

Then $\delta_2 \Phi(u, u_\alpha) = -\alpha\delta_\alpha(u)$ and on the basis of the formula(8), we have the expression

$$\Phi(u_\alpha) - \Phi(u) = -\alpha\delta_\alpha(u) + o(\alpha).$$

For comparison we reduce the standard gradient approximation of the functional[1,2]

$$\Delta_w \Phi(u) = \delta_0 \Phi(u, w) + \eta_0, \quad (14)$$

$$\delta_0 \Phi(u,w) = -\int_T \langle H_u(\psi(t,u), x(t,u), u(t), t), \Delta u(t) \rangle \mathrm{d}t,$$

$$\eta_0 = -\int_T o_H(\| \Delta u(t) \|)\mathrm{d}t - \int_T \langle \Delta_w H_x, \Delta x(t) \rangle \mathrm{d}t - \int_T o_H(\| \Delta x(t) \|)\mathrm{d}t + \int_T o_F(\| \Delta x(t) \|)\mathrm{d}t + o_\varphi(\| \Delta x(t_1) \|).$$

Here the quality of the approximation with respect to Δx is explicitly worse; even for the bilinear problem $\eta_0 \neq 0$.

The formula (14) defines the gradient of the functional Φ on the control $u(t)$

$$\nabla \Phi(u) = -\mathrm{H}_u(\psi(t,u), x(t,u), u(t), t)$$

and generates the necessary condition of optimality-the differential maximum principle (DMP)

$$u(t) = \arg\max_{v \in U} \langle \mathrm{H}_u(\psi(t,u), x(t,u), u(t), t), v \rangle,\ t \in T.$$

Using the projection operator P_U on the set U in Euclidean norm DMP, $u(t)$ can be written in the form

$$u(t) = P_U(u(t) + H_u(\psi(t,u), x(t,u), u(t), t),\ t \in T,$$

Lastly we can formulate two gradient methods for our problem. The variation procedure and parametric subproblem are the same

$$u_\alpha(t) = u(t) + \alpha(\bar{u}(t) - u(t)),\ t \in T.$$

$$\alpha \in [0,1]: \Phi(u_\alpha) \leqslant \Phi(u).$$

The auxiliary control $\bar{u} \in V$ is determined in two variants according to the above optimality conditions:

1) Conditional gradient method

$$\bar{u}(t) = \arg\max_{v \in U} \langle \mathrm{H}_u[t,u], v \rangle,\ t \in T. \quad (15)$$

2) Projection gradient method

$$\bar{u}(t) = P_U(u(t) + \mathrm{H}_u[t,u]),\ t \in T. \quad (16)$$

The value $\delta_0(u) = -\delta_0\Phi(u, \bar{u})$ has the meaning of DMP discrepancy on the control $u \in V$:

$$\delta_0(u) \geqslant 0,\ \delta_0(u) = 0 \Leftrightarrow \text{DMP}.$$

The improvement property is ensured by the formula

$$\Phi(u_\alpha) - \Phi(u) = -\alpha\delta_0(u) + o(\alpha),$$

which follows the approximation (14).

In order to prove the improvement property, we determine the connection between values $\delta_\alpha(u), \delta_0(u)$.

Lemma 1 For the conditional quasigradient method (9), (11) ~ (13), there is estimate

$$| \delta_\alpha(u) - \delta_0(u) | \leqslant C\alpha,\ C = \text{const}. \quad (17)$$

Proof By definition, we have

$$\delta_\alpha(u) - \delta_0(u) = \int_T \langle H_u(p_\alpha(t), x_\alpha(t), u(t), t), \bar{u}_\alpha(t) - u(t) \rangle \mathrm{d}t - \int_T \langle H_u(\psi(t,u), x(t,u), u(t), t), \bar{u}(t) - u(t) \rangle \mathrm{d}t.$$

According to the maximum condition for the control $\bar{u}(t)$, there is an inequality

$$\delta_\alpha(u) - \delta_0(u) \leqslant \int_T \langle \mathrm{H}_u[t,\alpha] - \mathrm{H}_u[t,u], \bar{u}_\alpha(t) - u(t) \rangle \mathrm{d}t. \quad (18)$$

The set U is bounded. Therefore $\| \bar{u}_\alpha(t) - u(t) \| \leqslant$

$C_1, t \in T$. Furthermore,

$$\| p_\alpha(t) - \psi(t,u) \| = \| \Psi(t,u)(x_\alpha(t) - x(t,u)) \| \leqslant C_2\alpha. \quad (19)$$

Estimate the increment of the derivative H_u from (18)

$$\| H_u(p_\alpha(t), x_\alpha(t), u(t), t) - H_u(\psi(t,u), x(t;u), u(t), t) \|$$
$$\| f_u(x_\alpha(t), u(t), t)^{\mathrm{T}} p_\alpha(t) - F_u(x_\alpha(t), u(t), t) - f_u(x(t,u), u(t), t)^{\mathrm{T}} \psi(t,u) + F_u(x(t,u), u(t), t) \| \leqslant$$
$$\| F_u(x_\alpha(t), u(t), t) - F_u(x(t,u), u(t), t) \| + \| (f_u(x_\alpha(t), u(t), t) - f_u(x(t,u), u(t), t))^{\mathrm{T}} p_\alpha(t) + f_u(x(t,u), u(t), t)^{\mathrm{T}} (p_\alpha(t) - \psi(t,u)) \|.$$

Using Lipschitz condition for the derivatives F_u, f_u with respect to x and the estimate (19), it can be written

$$\| H_u[t,\alpha] - H_u[t,u] \| \leqslant C_3\alpha.$$

Then from inequality (18) we have the upper bound

$$\delta_\alpha(u) - \delta_0(u) \leqslant C\alpha. \quad (20)$$

Furthermore, we consider the inverse difference $\delta_0(u) - \delta_\alpha(u)$ and use the maximum condition for the control $\bar{u}_\alpha(t)$. It leads to the inequality

$$\delta_0(u) - \delta_\alpha(u) \leqslant \int_T \langle \mathrm{H}_u[t,u] - \mathrm{H}_u[t,\alpha], \bar{u}_\alpha(t) - u(t) \rangle dt.$$

The following deduction is realized as above. As the result we have the estimate $\delta_0(u) - \delta_\alpha(u) \leqslant C\alpha$. With regard to (20), this proves Lemma 1.

Lemma 2 For the projection quasigradient method (13 ~ 16), the estimate (17) is true.

Proof Note that by definition for the projection variant

$$\bar{u}_\alpha(t) = P_U(u(t) + \mathrm{H}_u(p_\alpha(t), x_\alpha(t), u(t), t)), \quad t \in T.$$

Then using Lipschitz condition for the projection operator P_U we obtain

$$\| \bar{u}_\alpha(t) - u(t) \| \leqslant \| H_u[t,\alpha] - H_u[t,u] \|.$$

Represent the difference $\delta_\alpha(u) - \delta_0(u)$ in the following way

$$\delta_\alpha(u) - \delta_0(u) = \int_T \langle H_u[t,\alpha], \bar{u}_\alpha(t) - u(t) \rangle dt + \int_T \langle H_u[t,\alpha] - H_u[t,u], \bar{u}(t) - u(t) \rangle dt.$$

As in Lemma 1

$$\| H_u[t,\alpha] - H_u[t,u] \| \leqslant C_1\alpha,$$
$$\| \bar{u}(t) - u(t) \| \leqslant C_2, \ t \in T.$$

Hence, the required estimate (17) is true. Lemma 2 is proved.

Remark According to the proof scheme of Lemma 1,2, the constant C in (17) can be taken without the dependence of the control $u(t)$ and the corresponding trajectories.

Theorem 1 If the control $u \in V$ does not satisfy DMP in the problem (1) ~ (3), then for both the quasigradient methods

$$\Phi(u_\alpha) < \Phi(u) \text{ for small } \alpha > 0.$$

Proof It is determined by the following relations

$$\Phi(u_\alpha) - \Phi(u) = -\alpha\delta_\alpha(u) + o(\alpha) = -\alpha\delta_0(u) + \alpha(\delta_0(u) - \delta_\alpha(u)) + o(\alpha) = -\alpha\delta_0(u) + o_1(\alpha).$$

3 The convergence for LQ-problem

Consider the above quasigradient procedure for LQ-problem. Let

$$\varphi(x) = \langle c, x \rangle + 1/2 \langle x, Dx \rangle,$$
$$F(x,u,t) = b_0(u,t) + \langle a(u,t), x \rangle + 1/2 \langle x, Q(u,t), x \rangle,$$
$$f(x,u,t) = A(u,t)x + b(u,t),$$

and the function $b_0(u,t)$, the vector-functions $a(u,t)$, $b(u,t)$, the matrix-functions $Q(u,t)$, $A(u,t)$ are linear with respect to u.

We call the corresponding problem the linear-quadratic problem(LQ-problem). If the quadratic terms are absent ($D = 0, Q(u,t) = 0$), then we obtain the bilinear problem. Note that in LQ-problem the set of the phase trajectories $\{x(t,u)\}$ is bounded.

Consider the remainder η_2 for LQ-problem. As the function H is linear with respect to u, then $o_H(\| \Delta u(t) \|) = 0$. According to the quadratic property of $\varphi(x)$ and $F(x,u,t)$ with respect to x, we have $o_\varphi(\| \Delta x(t_1) \|^2) = 0, o_F(\| \Delta x(t) \|^2) = 0$. In view of the linear dependence of the function f on x, it should be $o_f(\| \Delta x(t) \|) = 0$. Thus, we conclude that $\eta_2 = 0$. Therefore,

$$\Phi(u_\alpha) - \Phi(u) = -\alpha\delta_\alpha(u) \leqslant 0, \ \alpha \in [0,1].$$

Thus, the quasigradient methods in LQ-problem guarantee the nonlocal improvement: for each $\alpha \in [0,1]$ the control u_α is not worse by functional than the initial control u.

We describe the methods (12) ~ (16) in the iterative form and consider the convergence property of the successive approximations in LQ-problem.

Let $k = 1,2,\cdots$ be the index of the iteration, $u^k(t)$ —— the corresponding control, $u_\alpha^k(t)$ —— the α- parametric family of the varied controls. The step α_k is determined according to the steepest descent rule

$$\alpha_k = \arg\min_{\alpha \in [0,1]} \Phi(u_\alpha^k). \tag{21}$$

The next approximation has the form

$$u^{k+1}(t) = u_{\alpha_k}^k(t), \quad t \in T.$$

It should be noted that in the LQ-problem DMP is equivalent to the maximum principle (MP). Therefore $\delta_0(u^k)$ is the discrepancy of MP. If $\delta_0(u^k) = 0$, then the control u^k satisfies MP and the procedure is finished. In the case $\delta_0(u^k) > 0$, the improvement property $\Phi(u^{k+1}) < \Phi(u^k)$ is true. The convergence is described by the following statement.

Theorem 2 In LQ-problem the sequence $u^k, k = 1,2,\cdots$, of the quasigradient methods converges with respect to the discrepancy of MP:

$$\delta_0(u^k) \to 0, \quad k \to \infty.$$

Proof is omited.

Consider the convex LQ-problem, which is characterized by the following expressions

$$\varphi(x) = \langle c, x\rangle + 1/2\langle x, Dx\rangle, \ D \geqslant 0,$$
$$F(x,u,t) = \langle b(t), u\rangle + \langle a(t), x\rangle + 1/2\langle x, Q(t)x\rangle, \ Q(t) \geqslant 0,$$
$$f(x,u,t) = A(t)x + B(t)u + c(t).$$

In this problem the functional $\Phi(u)$ is convex on the set V with the estimate

$$\Phi(w) - \Phi(u) \geqslant \delta_0\Phi(u,w), \ u, w \in V.$$

Let $u^* \in V$ be the optimal control in the convex LQ-problem.

Theorem 3 In the convex LQ-problem the quasigradient methods generate the minimizing sequence of controls:

$$\Phi(u^k) \to \Phi(u^*), \quad k \to \infty.$$

Proof First, consider the conditional quasigradient method. We have $\delta_0(u^k) = -\delta_0\Phi(u^k, \bar{u}^k)$ and the control $\bar{u}^k$ is the solution to the problem

$$\delta_0\Phi(u^k, w) \to \min, \ w \in V.$$

So, it follows

$$\Phi(u^*) - \Phi(u^k) \geqslant \delta_0\Phi(u^k, u^*) \geqslant \delta_0\Phi(u^k, \bar{u}^k) = -\delta_0(u^k).$$

Using the statement of Theorem 2, we conclude that $\Phi(u^*) - \Phi(u^k) \to 0, k \to \infty$. For the projection quasigradient method, we have

$$\bar{u}^k(t) = P_U(u^k(t) + \mathrm{H}_u[t, u^k]), \ t \in T.$$

In the equivalent form it can be written

$$\langle \bar{u}^k(t) - u^k(t) - \mathrm{H}_u[t,u^k], \bar{u}^k(t) - w(t)\rangle \leqslant 0, \ w(t) \in U.$$

Setting $w(t) = u^k(t)$, after integration we obtain the estimate

$$\delta_0(u^k) \geqslant \int_T \| \bar{u}^k(t) - u^k(t) \|^2 \mathrm{d}t.$$

Thus, the last integral tends to zero when $k \to \infty$. Taking $w(t) = u^*(t)$, we have the inequality

$$\langle H_u[t,u^k], u^*(t)\rangle \leqslant \langle \bar{u}^k(t) - u^k, u^*(t) - u^k(t)\rangle + \langle H_u[t,u^k], u^k(t)\rangle.$$

Then on the basis of the convex condition

$$\Phi(u^k) - \Phi(u^*) \leqslant -\delta_0\Phi(u^k, u^*) = \int_T \langle H_u[t,u^k], u^*(t) - u^k(t)\rangle \mathrm{d}t \leqslant$$
$$\int_T \langle \bar{u}^k(t) - u^k(t), u^*(t) - u^k(t)\rangle \mathrm{d}t + \delta_0(u^k) \leqslant$$
$$C\int_T \| \bar{u}^k(t) - u^k(t) \| \mathrm{d}t + \delta_0(u^k).$$

Passing to the limit for $k \to \infty$, we obtain the required convergence result. Theorem 3 is proved.

4 Conclusion

On the whole, the efficiency of considered methods is defined by the quality of the corresponding approximations and connected with possibilities of the nonlocal improvement for the certain classes of problems. Gradient methods (10), (11) do not have the property of the nonlocal improvement. The Quasigradient methods (12), (13) possess this property for the bilinear and LQ-problems, respectively. This fact is rather important because the main part of the computer calculations is connected with the α-parametric search.

The computer realization of the quasigradient methods

(12), (13) requires additional costs for the calculation of the symmetric matrix-function $\Psi(t,u)$. The expected effect of this information is connected with the improvement of the quality of each iteration. This theoretical prediction is justified by the results of the numerical testing, when the quasigradient procedures (12), (13) show their advantages in comparison with alternative methods.

References

1 Phedorenko R P. Approximate Solving Optimal Control Problems. Moscow: Nauka, 1978

2 Vasiliev F P. Methods of Solving External Problems. Moscow: Nauka, 1981

3 Jones D I and Finch J W. Comparison of optimization algorithms. Int. J. Control, 1984, 40(4): 747 – 761

4 Antonik V G and Srochko V A. The solution of optimal control problems using linearization methods. Comput. Maths. Math. Phys., 1992, 32(7): 859 – 871

5 Srochko V A. Phase linearization method for optimal control problems without terminal constraints. Heilongjiang University Journal of Natural Science, 1994, 11(3): 20 – 26

6 Srochko V A. The method of quadratic phase approximation for optimal control problems. Izvestiya vuzov. Matematika, 1993, Ⅳ(12): 81 – 87

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References

1 McClamroch N H and Wang D. Feedback stabilization and tracking of constrained robot. IEEE Trans. Automat. Contr., 1988, 33(5): 419 – 426

2 Bloch A M, Reyhanoglu M and McClamroch N H. Control and stabilization of a nonholonomic Caplygin dynamic Systems. IEEE Trans. Automat. Contr., 1992, 37(11): 1746 – 1757

3 You L S and Chen B S. Tracking control designs for both holonomic and nonholonomic constrained mechanical system: a unified viewpoint. Int. J Control, 1993, 58(3): 587 – 612

4 Krishnan H and McClamroch N H. Tracking in nonlinear differential-algebraic control systems with applications to constrained robot systems. Automatica, 1994, 30(12): 1885 – 1897

5 Isidori A. Nonlinear Control Systems. Berlin: Springer-Verlag, 1989

6 Wen X C and Liu Y Q. Neuromorphic position/force control for constrained robot manipulator using sliding mode. Control Theory and Applications, 1995, 12(2): 162 – 169

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