H_∞/μ Robust Control Based Active Vibration Control for Tall Buildings Using ATMD

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Abstract: In this paper, the modeling work has been done for the flexible building structure with n-th order of freedom and the reduced model can be gotten to design the controller for ATMD. According to the analysis, the displacement and velocity of the building structure can be chosen as the output feedback variables for the controller. By using the equivalent two-port model of the building structure with ATMD, a robust controller can be designed for ATMD based on the H_∞/μ frequency shaping synthesis theory. With the results of simulation, we have found that the control algorithm is very effective. The small mass has the reasonable movement space and the controller has the reasonable output energy. The system has good robustness for performance and stability in case of parameter variations and unstructured uncertainty caused by system un-modeling parts and system model reduction.

Key words: active vibration control; robust control; computer simulation

基于 H_∞/μ 鲁棒控制原理和使用 ATMD 的高楼主动振动控制策略研究

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摘要: 对使用主动调频质量块 ATMD(Active Tuned Mass Damper)的具有 n 阶自由度的柔性大楼结构建立了全阶模型,并由此将得到的结构降阶模型用于设计 ATMD 的控制器.通过分析,大楼结构的位置和速度可被选为控制器的输出反馈变量.将等效两端口模型应用于带 ATMD 的大楼结构上,使用 H_∞/μ 频域定型综合理论,设计出 ATMD 的鲁棒控制器.通过模拟分析,可见该控制算法是非常有效的,ATMD 中的小质量块的运动空间合理有限,控制器输出能量(力)合理,对于由于参数变化和非结构化的系统不确定具有很好的控制性能和系统稳定的鲁棒性(Robustness).文中对执行机构的实现也给出了分析和结果.

关键词: 主动振动控制; 鲁棒控制; 计算机模拟

1 Introduction

Design of an active controller for civil structures in seismic and windy zones consists of two distinct parts: development of a theoretical controller to meet specific performance criteria in the presence of parametric uncertainty and nonlinear actuator saturation effects; and application of this controller to a realistic control system[1~10]. This paper will pay attention to the first part.

It is well known that the control performance depends on the characteristics of controller. There are many control system design methods for active vibration control system of structural control. The H_∞/μ robust control synthesis methods are new and powerful tool to design the controller for active vibration control system.

In this paper, the state space model and the reduced order model for flexible building structure have been given out. After the analysis of this model, we will choose the displacement and velocity states of the building structure as the feedback control variables because the effect of this feedback states can directly modify the equivalent coefficients of stiffness and damping of the building structure respectively. We have formulated the control problem as the two-port formulation and design the H_∞ controller for ATMD. The simulation results of the design example show that the H_∞/μ based robust control synthesis methods are very suitable for tall building ac-

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tive vibration control. The design, synthesis and simulation works are supported by the MATLAB with toolbox[11,12]. We will verify all these results in the experiment rig in Tongji University.

2 Modelling and analysis

The model of the n degree-of-freedom flexible buiding structure of a tall buiding is shown in Fig. 1. An Active Tuned Mass Damper (ATMD) is mounted at the top of the primary building structure along the x-axis. The building structure is assumed to be excited by a wind force f (or earthquake acceleration a) input along the x-axis too.

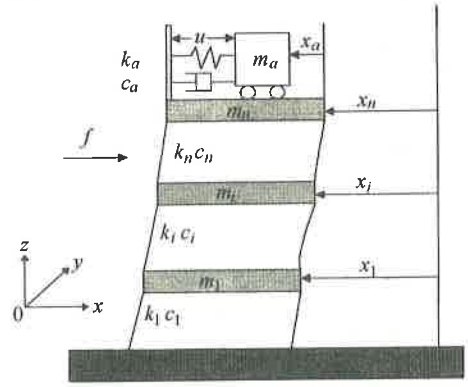


Fig. 1 The flexible building structure model of tall building

The specification of the building structure and the ATMD of the experiment rig are shown below. In this ATMD system, c_a can be used to absorb the disturbance energy and k_a can be used to adjust the dynamic response of the primary building structure and limit the small mass movement.

$m_i = 6.78/\text{n kg},\ i = 1,\cdots,n,\ m_a = 0.135\text{ kg},$

$k_i = 2600\text{N/m},\ i = 1,\cdots,n,\ k_a = 22.0\text{N/m},$

$c_i = 0.08\text{Ns/m},\ i = 1,\cdots,n,\ c_a = 0.37\text{Ns/m}.$

In our research, the main problems are

1) The limitation of small mass moving space and the limitation of control force $U(t)$ caused by the actuator output force limited will cause difficulty in achieving the best control performance.

2) There are structured and unstructured uncertainty in the plant model. For instance, the mass of the building M, damping and stiffness coefficients C, K will vary in a range and the real building contains nonlinear characteristics. It perhaps will cause the unstability and bad performance of the control system.

Therefore, Our research aims are

1) Within the limitation, try to find the best algorithm $K(s)$ for the controller to modify the disturbance dynamics of the building structure with reasonable energy.

2) By using the H_∞/μ robust control theory, try to synthesise the robust controller to deal with the uncertainty and the nonlinear problems.

Our research area is focused on below

1) We just deal with the one freedom ATMD system. In real building practice, the one freedom ATMD system can be installed in each independent freedom and direction as required.

2) We will only check the disturbance response of impulse (or step) input, because any real wind and earthquake input can be divided into several frequency order sine-wave inputs and the impulse (or step) input contains low and high frequency order sine-wave inputs.

According to Fig. 1, the dynamic equation of motion of the i-th mall is described by

$$m_i\ddot{x}_i + c_i\dot{x}_i + k_ix_i = f + c_{i+1}(\dot{x}_{i+1} - \dot{x}_i) + k_{i+1}(x_{i+1} - x_i),\quad i = 1,\cdots,n-1. \tag{1}$$

The dynamic equation of motion of the n-th mass is described by

$$m_n\ddot{x}_n + c_n\dot{x}_n + k_nx_n = f - u + c_a\dot{x}_a + k_ax_a. \tag{2}$$

Using equation (1) and (2), the dynamic equation of motion of the primary building structure is that

$$M\ddot{x}_s + C\dot{x}_s + Kx_s = ef + h(u - c_a\dot{x}_a - k_ax_a). \tag{3}$$

Where,

$$x_s = [x_1 \cdots x_n]^{\mathrm{T}},\ e = [1 \cdots 1]^{\mathrm{T}},$$

$$h = [0 \cdots 0\ -1]^{\mathrm{T}},$$

$M = \mathrm{diag}\,(m_1 \cdots m_n)$ is the mass matrix,

$$C = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & 0 & c_{n-1} + c_n & -c_n \\ 0 & \cdots & \cdots & 0 & c_n \end{bmatrix},$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & 0 \\ \cdots & \cdots & 0 & k_{n-1} + k_n & -k_n \\ 0 & \cdots & \cdots & 0 & k_n \end{bmatrix}$$

are the damping matrix and the stiffness matrix respec tively.

The dynamic equation of motion of ATMD mass is described by

$$m_a(\ddot{x}_a + \ddot{x}_n) = u - c_a\dot{x}_a - k_a x_a. \quad (4)$$

We can assume the transfer function of actuator as a constant $k_a(k_a = 1)$, because the response of the actuator is faster than the resonance frequency of the building stucture system and disturbance exciting inputs are always the low frequency signals. With H_∞/μ synthesis method, we can get the reasonable output control energy. So we can neglect the actuator saturaion and dynamic function in the system model (the model of actuator can be involved into the model of controller).

The measuring sensor noise is $n(t)$ and is always a high frequency signal. The measuring time delay is small enough to ignore it.

According to equation (1) to (4) and using the state vector

$$x_f = [x_a \quad \dot{x}_a \quad x_1 \quad \cdots \quad x_n \quad \dot{x}_1 \quad \cdots \quad \dot{x}_n]^T,$$

we can build the state equation of full order model as below

$$\dot{x}_f = A_f x_f + B_f u + d_f f. \quad (5)$$

Let's assume $n = 1$ and there is no limitation of small mass moving space. Using displacement and velocity states directly feedback to $u(t)$, we find that the coefficients of displacement and velocity states directly feedback can directly modify the equivalent coefficients of stiffness and damping of the building structure respectively. This has been described in Fig. 2. The damping of the building structure will effect the consumption of disturbance energy and the change of the stiffness of building structure will change the dynamic behavior of the tall building. For this reason, the displacement and velocity states of the building structure will be chosen as the feedback states. The controller cannot be realized in this way directly, because the small mass will move to infinitive position under step disturbance input. The controller must be designed by H_∞/μ synthesis method.

According to the above analysis, we can use the structure reducing method to get the reduced order model of building structure with ATMD. The state equation of the reduced order model can be written as below

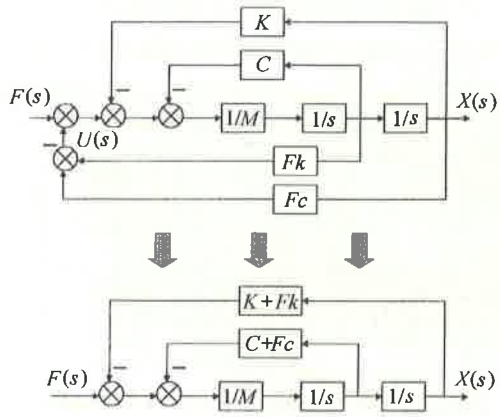


Fig. 2 The affect of displacement state and velocity state feedback

$$\dot{x}_r = A_r x_r + B_r u + D_r f. \quad (6)$$

Where

$$x_r = [x_a \quad \dot{x}_a \quad x_n \quad \dot{x}_n]^T, \ n = 1,$$

$$A_r = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_a(M+m_a)}{Mm_a} & -\dfrac{c_a(M+m_a)}{Mm_a} & \dfrac{K}{M} & \dfrac{C}{M} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_a}{M} & \dfrac{c_a}{M} & -\dfrac{K}{M} & -\dfrac{C}{M} \end{bmatrix},$$

$$B_r = \begin{bmatrix} 0 \\ \dfrac{M+m_a}{Mm_a} \\ 0 \\ -\dfrac{1}{M} \end{bmatrix}, \quad D_r = \begin{bmatrix} 0 \\ -\dfrac{1}{M} \\ 0 \\ \dfrac{1}{M} \end{bmatrix}.$$

Where M, K, C are the equivalent mass, stiffness and damping of the building structure respectively.

We will use this reduced model to design the robust controller for ATMD. There are many other model reduction methods that can be used for this purpose such as the additive model reduction methods and multiplicative model reduction methods. These methods are all supported by MATLAB robust control toolbox. The model reduction caused the unstructured uncertainty.

3 H_∞/μ robust control synthesis

The active vibration control problem mentioned above can be described in Fig. 3(a). According to the H_∞ robust control theory, this control problem can be changed and described as the two-port formation of the active control problem for H_∞ controller shown in Fig. 3(b).

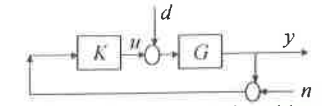


Fig. 3(a) Active control problem

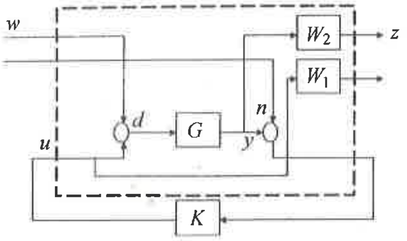


Fig. 3(b) The wot-port formation of the active control problem for H_∞ controller

In Fig. 3(a) and (b), G is the controlled plant (building structure), d is either the external disturbance caused by wind and earthquake or unmodeled dynamics in the system and n is either actual measuring sensor noise or a representation of high frequency unmodeled dynamics, K is the H_∞ controller, W_1, W_2 are the weighting function that can describe the stability and performance robustness and specification.

It is important that the control signal be included in the regulated outputs so that we can bound its magnitude to prevent actuator saturation problems.

If we consider the modeled uncertainty Δ , we can use the μ synthesis method to design the controller. The two-port formulation of the active controll problem for μ controller is shown in Fig. 4.

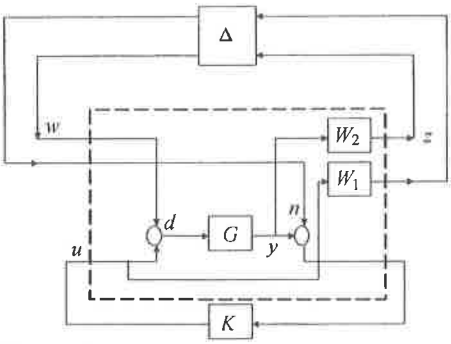


Fig. 4 The two-port formation of the active control problem for μ controller

The μ robust control synthesis method has the same frequency shaped principal with the H_∞ robust control synthesis method but with different algorithm. So in this paper, we just deal with the problems of how can we design the H_∞ controller.

With the two-port formulation, the plant can also be represented in state space form as

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u, \\ z = C_1 x + D_{11} w + D_{12} u, \\ y = C_2 x + D_{21} w + D_{22} u. \end{cases} \tag{7}$$

Using the packed-matrix notation, we get

$$P(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}. \tag{8}$$

For the reduced model of the designing example, we can choose $W_1 = W_2 = 1$ and

$$W = [d \quad n_1 \quad n_2]^T, \; Z = [x_1 \quad \dot{x}_1 \quad u]^T,$$

$$Y = [x_1 + n_1 \quad \dot{x}_1 + n_2]^T.$$

Here, n_1, n_2 are measuring sensor noise.

$$A = A_r,$$

$$B_1 = \begin{bmatrix} 0 & 0 & 0 \\ -1/M & 0 & 0 \\ 0 & 0 & 0 \\ -1/M & 0 & 0 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ \dfrac{M + m_a}{Mm_a} \\ 0 \\ -1/M \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; D_{21} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$D_{12} = [0 \quad 0 \quad 0], \; D_{22} = [0 \quad 0],$$

our objective is to keep the plant output small despite disturbances acting on the system and measurement noise. It is also desired to keep the actuator effort down to conserve control energy.

The main difference between the LQR/LQG/LTR and H_∞ control is the norm that is used. The former uses the 2-norm and the latter uses the ∞-norm. the main properties and advantages of the ∞-norm over the 2-norm are the following:

1) The ∞-norm is a gain (the L2 gain of the system). It can also be interpreted as the energy gain of the system. The 2-norm is not a gain.

2) The ∞-norm minimizes the worst case RMS value of the regulated variables when the disturbance has unknown spectra. The 2-norm minimizes the RMS values of the regulated variables when the disturbances are unit intensity white processes.

3) H_∞ control results in guaranteed stability margins (i. e. it is robust), whereas LQG has no guaranteed margins.

The H_∞ control problem is formulated as follows: consider the two-port diagram in Fig. 3, and find an internally sabilizing controller $K(s)$, for the plant $P(s)$, such that the ∞-norm of the closed loop transfer function T_{zw} is below a given level γ (a positive scalar). This problem is called the standard H_∞ control problem. If the γ is the smallest value, the problem is called the optimal H_∞ control problem. The standard problem is more practical because we must consider the moving space limitation of small mass and reasonable control energy output.

It should be obvious that H_∞ control problems cannot be solved manually. Computer software such as MATLAB with toolbox can be used to solve these problems. A summary of the solving steps is given below.

1) Set up the problem to obtain the state space representation for $P(s)$.

2) Check if the assumptions (the rank conditions) are satisfied. If they are not, reformulate the problem by adding weights or adding (fictitious) inputs or outputs.

3) Select a large positive value for γ.

4) Solve the two Riccati equations. Determine if the solution is positive semi-definit; also verify that the spectral radius condition is met.

5) If all the above conditions are satisfied, lower the value of γ. Otherwise, increase it. Repeat steps 4 and 5 until either an optimal or satisfactory solution is obtained.

4 Simulation results

With the model of the system and the robust controller algorithm described above, we can use MATLAB to get the simulation results about the system responses. The system reponses without active control are shown in Fig. 5. Fig. 6 shows the singular values graph of the open loop controlled plant.

From Fig. 5, we can find that the building structure has the characteristics of high stiffness and light damping. From Fig. 6, we can find the system has a high resonance peak and has not very good low frequency pass characteristic. The gain in low frequency region is low. It will cause system more oscillation when disturbances occurr. If the resonance peak can be relatively reduced and has a signififcantly low frequency pass characteristic, the system will become more stable. This is just the aim of the vibration control.

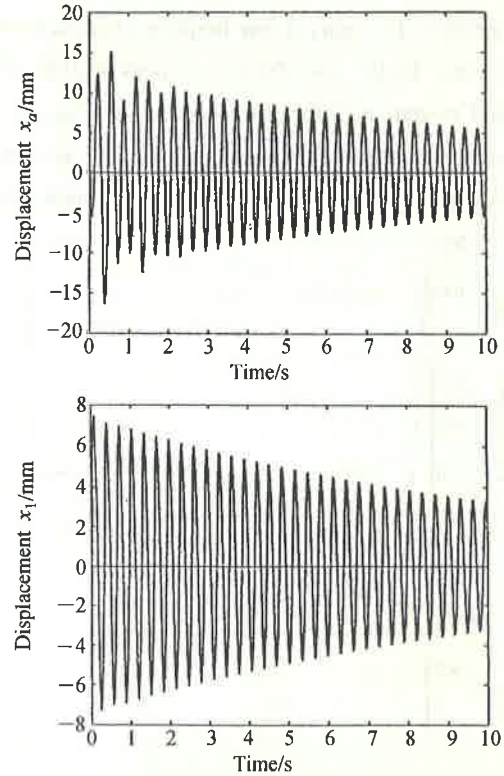


Fig. 5 Building structrue impulse responses without active control

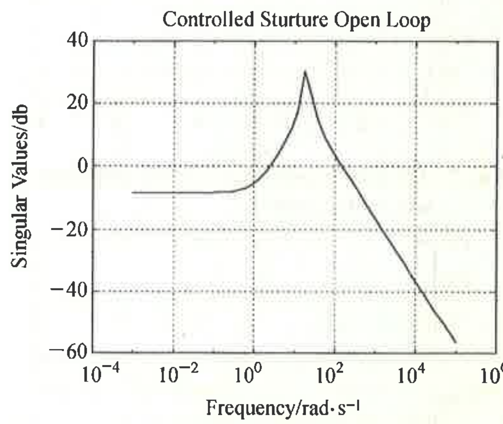


Fig. 6 Singular values of the open loop controlled plant

With the two-port model $(A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22})$, the optimal H_∞ controller can be designed and we can get the closed loop control system. Fig. 7 shows the building structure impulse responses with active control. Fig. 8 shows the impulse response of control force. It seems that the amplitude and the setting time of displacement of building and small mass are reduced significantly and the control force remains in reasonable area. Fig. 9 is the singular-value graph of the closed loop controlled plant. We find that the resonance peak of the closed loop control system has been relatively reduced and has a significantly low frequency pass characteristic. The gain in low frequency region becomes higher. Due to the low frequency pass feature of the controled system and the small energy feature of high frequency part of disturbance, the building can't be influenced by the high frequency part of disturbance greatly.

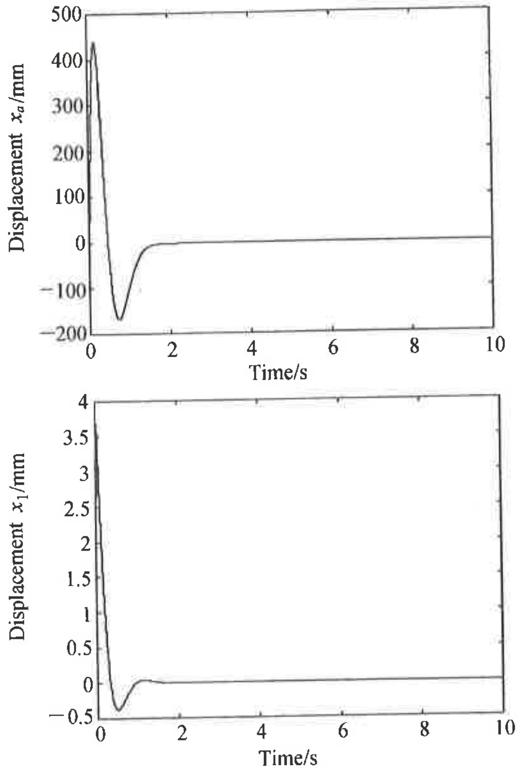


Fig. 7 The building structure impulse responses with active control

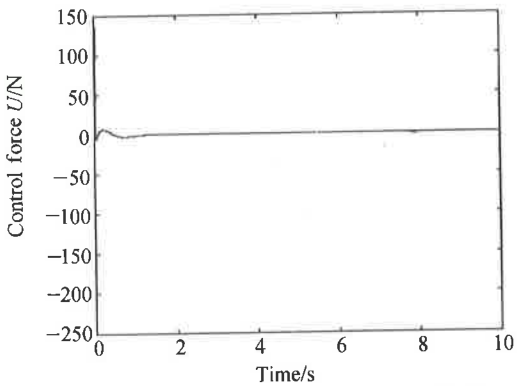


Fig. 8 The impulse response of control force

For the realization purpose, we should consider the actuator's dynamics and limit its output force further. Let's assume that the equivalent transfer function of actuator $K_a(s)$ is an idea 2-order system

$$k_a(s) = K/(T^2 s^2 + 2\xi Ts + 1), \xi = 0.7 \quad (9)$$

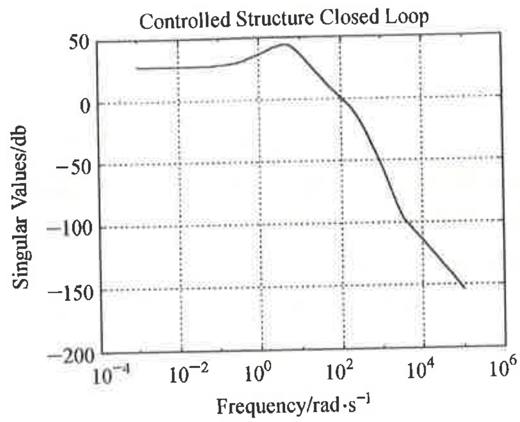
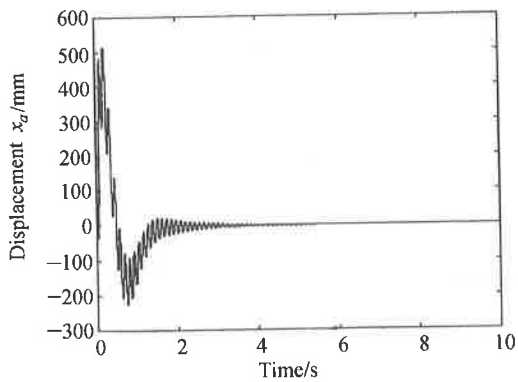


Fig. 9 The singular values graph of the closed loop controlled plant

and series with controller $K(s)$.

From the simulation results with different K and T, we find that the time constant T must be small enough and the gain K can be chosen less than one. From Fig. 10 ~ Fig. 12, we find that the system disturbance response will contain high frequency parts if T is not small enough. The singular values have a second resonance peak. Although the control force is reduced, yet it can not be realized in practice easily. If we reduce the gain K and let the time constant T be small enough, we can get very satisfying control results shown in Fig. 13 ~ 15. The system disturbance responses are satisfied enough and do not contain high frequency parts and the control force is more reduced. It is very easy to realize in practice, because the actuator, which is described by formula (9) can be realized easily by hydraulic and electrical devices. These results verify that the H_∞ active vibration control system has significant robustness.

If we design the μ controller, we can get better vibration control effect. The principal is the same as H_∞ controller.



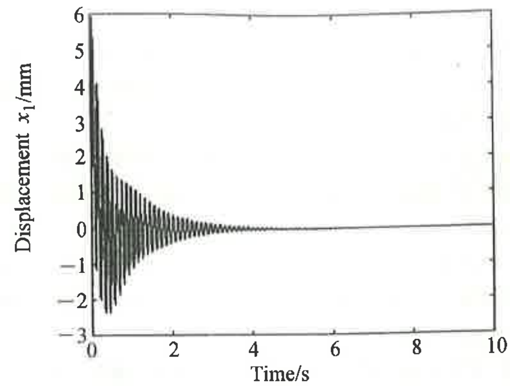


Fig. 10 The building structure impulse responses with active control (with actuator T=1/100s, K=1)

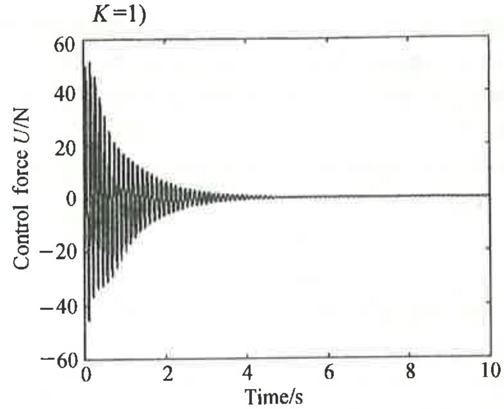


Fig. 11 The impulse response of control force (with actuator T=1/100s, K=1)

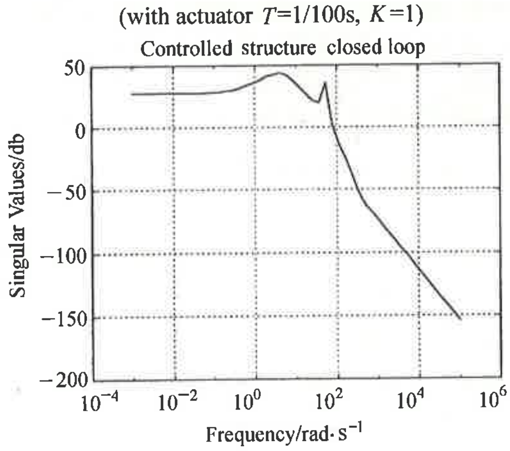
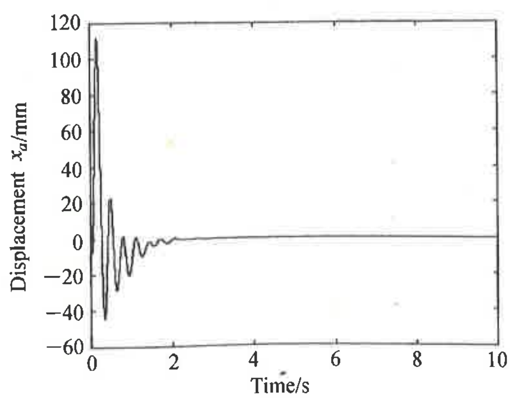


Fig. 12 The singular values graph of the closed loop controlled plant (with actuator T=1/100s, K=1)



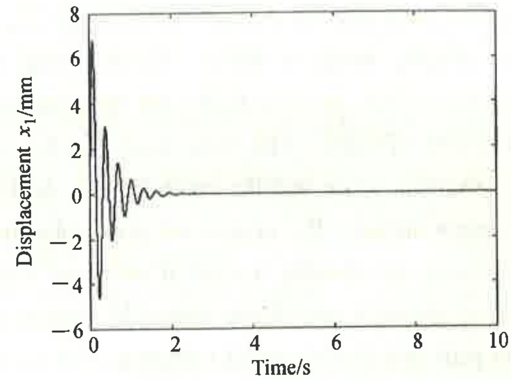


Fig. 13 The building structure impulse responses with active control (with actuator T=1/200s, K=0.125)

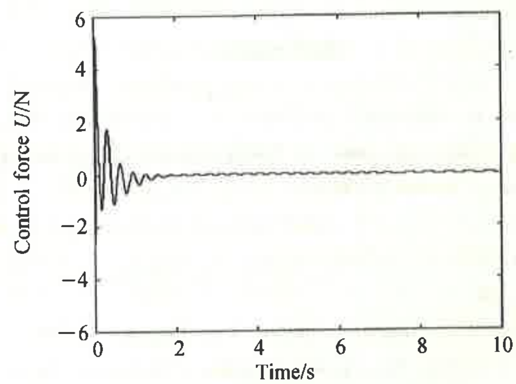


Fig. 14 The impulse response of control force (with actuator T=1/200s, K=0.125)

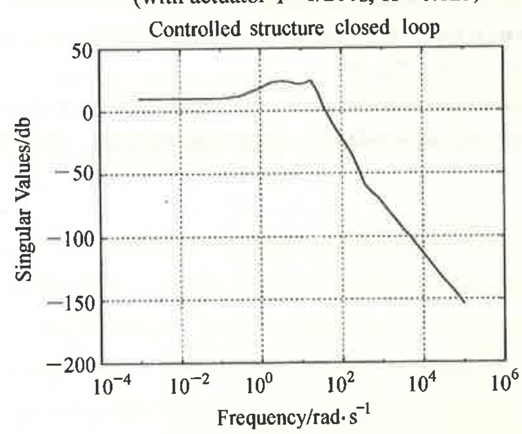


Fig. 15 The singular values graph of the closed loop controlled plant (with actuator T=1/200s, K=0.125)

5 Conclusions

Firstly, we have done the modeling work for the flexible building structure with n-th freedom order. We find that we can get the reduced model to design the controller for ATMD. According to the analysis, we find that we can choose the displacement and velocity of the building structure as the output feedback variables for the controller. Secondly, by using the equivalent two-port model of the building structure with ATMD, we can design a robust controller for ATMD based on the H_∞/μ frequency shaping synthesis theory. Thirdly, with the results of simulation, we have found that the control algorithm is very effective. The small mass has the reasonable movement space and the controller has the reasonable output energy. The system has good robustness for performance and stability in case of parameter variations and unstructured uncertainty caused by system unmodeling parts and system model reduction. All the results can be examined on the experimental rig in Tongji University.

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