# Delay-Dependent Robust Stability and Stabilization of Uncertain Systems with Multiple State Delays

Cao Yongyan　and　Sun Youxian
(National Laboratory of Industrial Control Technology, Zhejiang University·Hangzhou, 310027, P.R.China)

**Abstract**: A new delay-dependent criteria has been proposed and a method of robust stabilization has been presented via linear memoryless state feedback control for a class of uncertain time-delay systems with multiple state delays and norm-bounded parameter uncertainty. The results depend on the size of the delays and are given in terms of several linear matrix inequalities.

**Key words**: uncertain linear systems; time-delay; robust stability; linear matrix inequality(LMI)

# 不确定多状态滞后系统时滞相关鲁棒稳定与镇定

曹永岩　孙优贤
(浙江大学工业控制技术国家重点实验室·杭州, 310027)

**摘要**: 针对具有范数有界参数不确定性的多状态滞后系统,本文提出与系统时滞相关的鲁棒稳定性判据,并给出了一种无记忆的时滞相关的鲁棒镇定控制率的设计方法.本文的结果与系统的时滞大小有关,并采用 LMI 描述.

**关键词**: 不确定线性系统; 时滞; 鲁棒稳定性; 线性矩阵不等式(LMI)

## 1 Introduction

Dynamic systems with time-delay are common in chemical processes and long transmission lines in pneumatic, hydraulic, and rolling mill systems. A major problem in the analysis and design of this class of systems is related to their stability and stabilization using linear feedback.

Recently, some researchers have proposed some useful techniques to analyze the asymptotic stability[2]~[4] and to determine a linear stabilizing feedback control law, see e.g. [5] and references therein. Most results of the robust stability criteria are based on the system matrices norm approach and most robust stabilization results obtained via Riccati equation approach are independent of the size of the delays (i.e the time-delay is allowed to be arbitrarily large) and thus, in general , are conservative, especially when practically existing time-delays are small. In addition, a common feature of the methods based on the Riccati or Lyapunov equation is that the tuning of several parameters and/or a symmetric positive definite matrix is required, but no tuning procedure for such parameters and matrix is available.

This paper deals with the problem of robust stability and robust stabilization controller design for a class of uncertain delay systems with multiple state delays. The major contributions are divided into three parts. First, it gives a new criterion of the delay-dependent asymptotic stability using the LMI approach. Second, it treats directly the uncertain linear systems with multiple state delays. It makes it possible to judge the asymptotic stability of uncertain linear systems with multiple state delays. Third, it is shown that the stability can be determined through the feasibility of related LMI and the controller can be designed by a simple procedure so long as the solutions of related LMI are obtained.

## 2 Systems and preliminaries

Consider uncertain time-delay systems described by the following state equations:

$$\begin{cases} \dot{x}(t) = [A_0+\Delta A_0(t)]x(t)+[B+\Delta B(t)]u(t)+ \\ \qquad \sum_{i=1}^{l}[A_i + \Delta A_i(t)]x(t-\tau_i), \\ x(t) = \phi(t), \quad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $A_i (i = 0, 1, \cdots, l)$ and $B$ are known constant matrices with appropriate dimensions, $\Delta A_i(t)$, and $\Delta B(t)$ are ma-

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trix functions representing the uncertainties in $A_i$, $(i = 0,1,\cdots,l)$ and $B$. $\tau_i$ denote the constant time delays satisfying

$$0 \leqslant \tau_i \leqslant \tau, \quad i = 1,\cdots,l. \tag{2}$$

In the following we will let $\tau_0(t) = 0$ to describe the non-delay term. $\phi(t)$ is a smooth vector-valued initial function. In this paper, we assume that the uncertainty can be described by

$$[\Delta A_0(t) \quad \Delta B(t)] = D_0 F_0(t)[E_0 \quad E_b],$$
$$\Delta A_i(t) = D_i F_d(t) E_i, \ i = 1,\cdots,l, \tag{3}$$

where $F_0(t) \in \mathbb{R}^{k_0 \times j_0}$ and $F_d(t) \in \mathbb{R}^{k_d \times j_d}$ are unknown real time-varying matrices with Lebesgue measurable elements bounded by

$$F_0^{\mathrm{T}}(t)F_0(t) \leqslant I, \ F_d^{\mathrm{T}}(t)F_d(t) \leqslant I, \forall\, t. \tag{4}$$

In the following, for simplicity, we will let

$$\tilde{A}_i(t) = A_i + \Delta A_i(t),$$
$$\tilde{B}(t) = B + \Delta B(t), \ i = 0,1,\cdots,l,$$

**Definition 2.1** The uncertain delay system (1) is said to be robust stable if the trivial solution $x(t) \equiv 0$ of the functional differential equation associated to (1) with $u(t) \equiv 0$ is globally uniformly asymptotically stable for all admissible uncertainties $\Delta A_i(t)$, $(i = 0,1,\cdots,l)$. The uncertain delay system(1) is said to be robust stabilizable if a feedback control law can be found such that the resulting closed-loop system is robust stable.

We end this section by recalling several matrix inequalities which will be essential for the proofs in the next section; see, e.g. [6].

**Lemma 2.1** For any $z, y \in \mathbb{R}^n$ and any positive definite matrix $X \in \mathbb{R}^{n\times n}$

$$-2z^{\mathrm{T}}y \leqslant z^{\mathrm{T}}X^{-1}z + y^{\mathrm{T}}Xy.$$

**Lemma 2.2** Let $A$, $D$, $E$ and $F$ be real matrices of appropriate dimensions with $\| F \| \leqslant 1$. Then

a) For any scalar $\varepsilon > 0$,

$$DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} \leqslant \varepsilon^{-1}DD^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E;$$

b) For any matrix $P > 0$ and scalar $\varepsilon > 0$ such that $\varepsilon I - EPE^{\mathrm{T}} > 0$ it holds

$$(A + DFE)P(A + DFE)^{\mathrm{T}} \leqslant$$
$$APA^{\mathrm{T}} + APE^{\mathrm{T}}(\varepsilon I - EPE^{\mathrm{T}})^{-1}EPA^{\mathrm{T}} + \varepsilon DD^{\mathrm{T}};$$

c) For any matrix $P > 0$ and scalar $\varepsilon > 0$ such that $P - \varepsilon DD^{\mathrm{T}} > 0$ it holds

$$(A + DFE)^{\mathrm{T}}P^{-1}(A + DFE) \leqslant$$
$$A^{\mathrm{T}}(P - \varepsilon DD^{\mathrm{T}})^{-1}A + \varepsilon^{-1}EE^{\mathrm{T}}.$$

## 3 Main results

**Theorem 3.1** Consider the uncertain delay system (1) with $u(t) \equiv 0$. Given scalars $\tau_i$, and $\tau$ satisfying $0 < \tau_i \leqslant \tau$, then for any time delays $\tau_i$, it is robust stable if there exist matrices $X > 0$ and $X_{ij} > 0$ and scalars $\eta_{ij} > 0, \sigma_i > 0, \alpha_j > 0$ satisfying the following LMI:

$$S_1(\tau_i) = \begin{bmatrix} S & H_1 & H_2 & G \\ H_1^{\mathrm{T}} & -J_1 & & 0 \\ H_2^{\mathrm{T}} & 0 & -J_2 & 0 \\ G^{\mathrm{T}} & 0 & 0 & -L \end{bmatrix} < 0, \tag{5}$$

$$i = 1,2,\cdots,l; \ j = 0,\cdots,l,$$

where

$$T_i = \sum_{j=0}^{l} X_{ij},$$
$$S = \sum_{i=0}^{l} A_i X + X\sum_{i=0}^{l} A_i^{\mathrm{T}} + \alpha_0 D_0 D_0^{\mathrm{T}} + \sum_{i=1}^{l}(\alpha_i + \sigma_i)D_i D_i^{\mathrm{T}} + \sum_{i=1}^{l} A_i T_i A_i^{\mathrm{T}},$$
$$H_1 = [XE_0^{\mathrm{T}} \quad XE_1^{\mathrm{T}} \quad \cdots \quad XE_l^{\mathrm{T}}],$$
$$J_1 = \mathrm{diag}(\alpha_0 I, \alpha_1 I, \cdots, \alpha_l I),$$
$$H_2 = [A_1 T_1 E_1^{\mathrm{T}} \quad A_2 T_2 E_2^{\mathrm{T}} \quad \cdots \quad A_l T_l E_l^{\mathrm{T}}]$$
$$J_2 = \mathrm{diag}[(\sigma_1 I - E_1 T_1 E_1^{\mathrm{T}}), (\sigma_2 I - E_2 T_2 E_2^{\mathrm{T}}), \cdots, (\sigma_l I - E_l T_l E_l^{\mathrm{T}})],$$
$$G_i = [XA_0^{\mathrm{T}} \quad XE_0^{\mathrm{T}} \quad XA_1^{\mathrm{T}} \quad XE_1^{\mathrm{T}} \cdots XA_l^{\mathrm{T}} \quad XE_l^{\mathrm{T}}],$$
$$L_i = \mathrm{diag}(X_{i0} - \eta_{i0}D_0 D_0^{\mathrm{T}}, \eta_{i0} I, \cdots, X_{il} - \eta_{il} D_l D_l^{\mathrm{T}}, \eta_{il} I), \tag{6}$$
$$G = [\tau_1 G_1 \quad \tau_2 G_2 \quad \cdots \quad \tau_l G_l],$$
$$L = \mathrm{diag}(L_1, L_2, \cdots, L_l).$$

Proof Consider the unforced time-delay system of (1) ~ (4) with $u(t) \equiv 0$. Let $x(t), t \geqslant 0$ be the solution of linear time-delay system (1) if the initial time and state are 0 and $\phi(t)$, respectively. Since $x(t)$ is continuously differentiable for $t \geqslant 0$, using the Leibniz-Newton formula, one can write

$$x(t - \tau_i) = x(t) - \int_{-\tau_i}^{0} \dot{x}(t + \theta)\mathrm{d}\theta =$$
$$x(t) - \int_{-\tau_i}^{0} \Big\{\sum_{j=0}^{l} \tilde{A}_j(t + \theta)x(t - \tau_j + \theta)\Big\}\mathrm{d}\theta.$$

for $t \geqslant \tau_i$. Then the unforced system (1) is equivalent to the following system[1]:

$$\dot{x}(t) = \sum_{i=0}^{l} \tilde{A}_i(t)x(t) - \sum_{i=1}^{l}\int_{-\tau_i}^{0} \tilde{A}_i(t)\cdot$$
$$\Big\{\sum_{j=0}^{l} \tilde{A}_j(t + \theta)x(t - \tau_j + \theta)\Big\}\mathrm{d}\theta,$$

$$x(t) = \phi(t), \quad t \in [-2\tau, 0],$$

where $\phi(t)$ is a smooth vector-value initial function. Hence, the global uniform asymptotic stability of last systems will ensure the global uniform asymptotic stability of the original system (1); see, e.g. [5]. Choose the Lyapunov functional candidate as

$$V(x,t) = x^{\mathrm{T}}(t)Px(t) + W(x,t), \tag{7}$$

where $P$ is a symmetric positive definite matrix and

$$W(x,t) = \sum_{i=1}^{l}\sum_{j=0}^{l}\int_{-\tau_i}^{0}\int_{t-\tau_j+\theta}^{t} x^{\mathrm{T}}(s)Q_{ij}x(s)\mathrm{d}s\mathrm{d}\theta.$$

Then the time derivative of $V(x,t)$ along the trajectory of the above system is given by

$$\dot{V}(x,t) = x^{\mathrm{T}}[P\sum_{i=0}^{l}\tilde{A}_i + (\sum_{i=0}^{l}\tilde{A}_i)^{\mathrm{T}}P]x - \sum_{i=1}^{l}\sum_{j=0}^{l}\int_{-\tau_i}^{0} 2x^{\mathrm{T}}P\tilde{A}_i(t)\tilde{A}_j(t+\theta)x(t-\tau_j+\theta)\mathrm{d}\theta + \dot{W}(x,t) \tag{8}$$

where

$$\dot{W}(x,t) = \sum_{i=1}^{l}\sum_{j=0}^{l}\tau_i x^{\mathrm{T}}(t)Q_{ij}x(t) - \sum_{i=1}^{l}\sum_{j=0}^{l}\int_{-\tau_i}^{0} x^{\mathrm{T}}(t-\tau_j+\theta)Q_{ij}x(t-\tau_j+\theta)\mathrm{d}\theta.$$

Using the lemmas given in Section 2, we have

$$-\int_{-\tau_i}^{0} 2x^{\mathrm{T}}P\tilde{A}_i\tilde{A}_j(t+\theta)x(t-\tau_j+\theta)\mathrm{d}\theta \leqslant \tau_i x^{\mathrm{T}}P\tilde{A}_iP_{ij}\tilde{A}_i^{\mathrm{T}}Px + \int_{-\tau_i}^{0} x^{\mathrm{T}}(t-\tau_j+\theta)\tilde{A}_j^{\mathrm{T}}(t+\theta)P_{ij}^{-1}\tilde{A}_j(t+\theta)x(t-\tau_j+\theta)\mathrm{d}\theta.$$

Let $W_i = \sum_{j=0}^{l}P_{ij}$. Using Lemma 2.2, we obtain

$$\tilde{A}_i(t)W_i\tilde{A}_i^{\mathrm{T}}(t) \leqslant A_iW_iA_i^{\mathrm{T}} + \varepsilon_iD_iD_i^{\mathrm{T}} + A_iW_iE_i^{\mathrm{T}}(\varepsilon_iI - E_iW_iE_i^{\mathrm{T}})^{-1}E_iW_iA_i^{\mathrm{T}}$$

for any $\varepsilon_i > 0$ with

$$\varepsilon_iI - E_iW_iE_i^{\mathrm{T}} > 0, \quad i = 1,\cdots,l \tag{9}$$

and

$$\tilde{A}_j^{\mathrm{T}}(t+\tau_j)P_{ij}^{-1}\tilde{A}_j(t+\tau_j) \leqslant \tilde{A}_j^{\mathrm{T}}(P_{ij} - \xi_{ij}D_jD_j^{T})^{-1}A_j + \xi_{ij}^{-1}E_j^{\mathrm{T}}E_j$$

for any $\xi_{ij} > 0$ with

$$P_{ij} - \xi_{ij}D_jD_j^{\mathrm{T}} > 0, i = 1,\cdots,l, j = 0,1,\cdots,l. \tag{10}$$

Let

$$Q_{ij} = A_j^{\mathrm{T}}(P_{ij} - \xi_{ij}D_jD_j^{\mathrm{T}})^{-1}A_j + \xi_{ij}^{-1}E_j^{\mathrm{T}}E_j.$$

Hence we have

$$\dot{V}(x,t) \leqslant x^{\mathrm{T}}(t)\{P\sum_{i=0}^{l}A_i + \sum_{i=0}^{l}A_i^{\mathrm{T}}P + \sum_{i=0}^{l}(\alpha_iPD_iD_i^{\mathrm{T}}P + \alpha_i^{-1}E_i^{\mathrm{T}}E_i) + \sum_{i=1}^{l}\tau_iP(A_iW_iA_i^{\mathrm{T}} + \varepsilon_iD_iD_i^{\mathrm{T}})P + \sum_{i=1}^{l}\tau_iPA_iW_iE_i^{\mathrm{T}}(\varepsilon_iI - E_iW_iE_i^{\mathrm{T}})^{-1}E_iW_iA_i^{\mathrm{T}}P + \sum_{i=1}^{l}\sum_{j=0}^{l}\tau_i[A_j^{\mathrm{T}}(P_{ij} - \xi_{ij}D_jD_j^{\mathrm{T}})^{-1}A_j + \xi_{ij}^{-1}E_j^{\mathrm{T}}E_j]\}x(t) = x^{\mathrm{T}}(t)Mx(t), \tag{11}$$

where $\alpha_i > 0, i = 0,\cdots,l$. Let $X = P^{-1}$ we have

$$XMX = \sum_{i=0}^{l}A_iX + X\sum_{i=0}^{l}A_i^{\mathrm{T}} + \sum_{i=0}^{l}(\alpha_iD_iD_i^{\mathrm{T}} + \alpha_i^{-1}XE_i^{\mathrm{T}}E_iX) + \sum_{i=1}^{l}\tau_i(A_iW_iA_i^{\mathrm{T}} + \varepsilon_iD_iD_i^{\mathrm{T}}) + \sum_{i=1}^{l}\tau_iA_iW_iE_i^{\mathrm{T}}(\varepsilon_iI - E_iW_iE_i^{\mathrm{T}})^{-1}E_iW_iA_i^{\mathrm{T}} + \sum_{i=1}^{l}\tau_i\sum_{j=0}^{l}XA_j^{\mathrm{T}}(P_{ij} - \xi_{ij}D_jD_j^{\mathrm{T}})^{-1}A_jX + \sum_{i=1}^{l}\tau_i\sum_{j=0}^{l}\xi_{ij}^{-1}XE_j^{\mathrm{T}}E_jX.$$

Setting $X_{ij} = \tau_iP_{ij}, \eta_{ij} = \tau_i\xi_{ij}, \delta_i = \tau_i\varepsilon_i, i = 1,\cdots,l$; $j = 0,1,\cdots,l$, and using Schur complements, we can find that $M < 0$ and inequalities (9) and (10) hold if the inequality (5) holds. So, if LMI (5) holds, then the uncertain time-delay systme (1) is asymptotically stable.

Q.E.D.

This theorem provides a delay-dependent criteria for robust stability of uncertain linear systems with multiple state delays in terms of the solvability of linear matrix inequalities. In paper [1 ~ 4], only the asymptotic stability of the systems with a single time-delay can be determined. Based on this LMI based approach, it becomes possible to determine the asymptotic stability of time-delay systems with multiple state delays.

**Corollary 3.1** Consider the unforced time-delay system (1) with $\Delta A_i(t) = 0, i = 0,1,\cdots,l, u(t) \equiv 0$. Given scalars $\tau_i$, and $\tau$ satisfying $0 < \tau_i \leqslant \tau$, then for any time delays $\tau_i$, it is robust stable if there exist matrices $X > 0$ and $X_{ij} > 0, i = 1,2,\cdots,l; j = 0,1,$

$\cdots, l$, satisfying the following LMI:

$$\begin{bmatrix} \sum_{i=0}^{l} A_i X + X \sum_{i=0}^{l} A_i^{\mathrm{T}} + \sum_{i=1}^{l}\sum_{j=0}^{l} A_i X_{ij} A_i^{\mathrm{T}} & G \\ G^{\mathrm{T}} & -L \end{bmatrix} < 0, \quad (12)$$

where

$$G_i = [XA_0^{\mathrm{T}} \quad XA_1^{\mathrm{T}} \quad \cdots \quad XA_l^{\mathrm{T}}],$$
$$L_i = \mathrm{diag}(X_{i0}, X_{i1}, \cdots, X_{il}),$$
$$G = [\tau_1 G_1 \quad \tau_2 G_2 \quad \cdots \quad \tau_l G_l],$$
$$L = \mathrm{diag}(L_l, L_2, \cdots, L_l).$$

**Theorem 3.2** Consider the uncertain time-delay system(1). Given scalars $\tau_i$, and $\tau$ satisfying $0 < \tau_i \leqslant \tau$, then for any time delays $\tau_i$, this system is roubst stabilizable if there exist matrices $X > 0, Y$, and $X_{ij} > 0$ scalars $\eta_{ij} > 0, \sigma_i > 0, \alpha_j > 0$ satisfying the following LMI:

$$\begin{cases} S_2(\tau_i) = \begin{bmatrix} S & H_1 & H_2 & G \\ H_1^{\mathrm{T}} & -J_1 & 0 & 0 \\ H_2^{\mathrm{T}} & 0 & -J_2 & 0 \\ G^{\mathrm{T}} & 0 & 0 & -L \end{bmatrix} < 0, \\ i = 1, 2, \cdots, l; \ j = 0, \cdots, l, \end{cases} \quad (13)$$

where $T_i$ is defined as (5) and

$$S = \sum_{i=0}^{l} A_i X + X \sum_{i=0}^{l} A_i^{\mathrm{T}} + BY + Y^{\mathrm{T}} B^{\mathrm{T}} + \alpha_0 D_0 D_0^{\mathrm{T}} + \sum_{i=1}^{l} (\alpha_i + \sigma_i) D_i D_i^{\mathrm{T}} + \sum_{i=1}^{l} A_i T_i A_i^{\mathrm{T}},$$
$$H_1 = [XE_0^{\mathrm{T}} + Y^{\mathrm{T}} E_b^{\mathrm{T}} \quad XE_1^{\mathrm{T}} \quad \cdots \quad XE_l^{\mathrm{T}}],$$
$$J_1 = \mathrm{diag}(\alpha_0 I, \alpha_1 I, \cdots, \alpha_l I),$$
$$H_2 = [A_1 T_1 E_1^{\mathrm{T}} \quad A_2 T_2 E_2^{\mathrm{T}} \quad \cdots \quad A_l T_l E_l^{\mathrm{T}}],$$
$$J_2 = \mathrm{diag}[\sigma_1 I - E_1 T_1 E_1^{\mathrm{T}}, \sigma_2 I - E_2 T_2 E_2^{\mathrm{T}}, \cdots, \sigma_l I - E_l T_l E_l^{\mathrm{T}}],$$
$$G_i = [XA_0^{\mathrm{T}} + Y^{\mathrm{T}} B^{\mathrm{T}} \quad XE_0^{\mathrm{T}} + Y^{\mathrm{T}} E_b^{\mathrm{T}} \quad XA_1^{\mathrm{T}} \quad XE_1^{\mathrm{T}} \quad \cdots \quad XA_l^{\mathrm{T}} \quad XE_l^{\mathrm{T}}],$$
$$L_i = \mathrm{diag}(X_{i0} - \eta_{i0} D_0 D_0, \eta_{i0} I, \cdots, X_{il} - \eta_{il} D_l D_l^{\mathrm{T}}, \eta_{il} I),$$
$$G = [\tau_1 G_1 \quad \tau_2 G_2 \quad \cdots \quad \tau_l G_l],$$
$$L = \mathrm{diag}(L_1, L_2, \cdots, L_l).$$

Moreover, a suitable stabilizing control law is given by

$$u(t) = YX^{-1} x(t).$$

Proof With the control $u(t) = Kx(t)$, the systems (1) becomes

$$\dot{x}(t) = [A_C + D_0 F(t) E_C] x(t) + \sum_{i=1}^{l} [A_i + D_i F(t) E_i] x(t - \tau_i) \quad (14)$$

where

$$A_C = A + BK, \quad E_C = E_0 + E_b K.$$

So, from the Theorem 3.1, we can immediately gain this theorem. Q.E.D.

This theorem provides a delay-dependent condition for robust stabilizability of uncertain linear time-delay systems with multiple state delays in terms of the solvability of linear matrix inequalities. This is contrast with the results such as in [5]; which developed delay-independent criteria for robust stabilizability of time-delay systems in terms of the solution of a modified parametric Lyapunov or Riccati equation. But those equations can not be directly solved and the parameters need to be tuned, the tuning is very difficult when there are more than two parameters. However, LMI can be numerically solved very efficiently and thus no parameter needs to be tuned. Moreover, the time-delays are always bounded, so our results may be less conservative than the results which are delay-independent.

From these theorems, the upper bound of time delay $\tau_i$ can be determined such that the uncertain time-delay system is stabilizable when all other time-delays $\tau_i, i = 1, \cdots, l, i \neq j$, are known.

In fact, the largest value of $\tau_j$ can be computed by solving the following LMI problem when all other time-delays $\tau_i, i = 1, \cdots, l, i \neq j$, are known.

maximize $\tau_j$

subject to $S_2(\tau_i) < 0, X > 0, X_{ij} > 0$

and $\eta_{ij} > 0, \sigma_i > 0, \alpha_j > 0$ .

which is a quasi-convex optimization problem and can be solved using LMI Lab.

We can also determine the upper bound of $\tau$ by solving the following LMI problem

maximize $\tau$

subject to $S_2(\tau_i = \tau) < 0, X > 0, X_{ij} > 0, Y_i > 0$

and $\eta_{ij} > 0, \sigma_i > 0, \alpha_j > 0$

which is also a quasi-convex optimization problem.

## 4 Examples

Consider the uncertain time-delay system with

$$\bar{A}_0 = \begin{bmatrix} -1+0.16\sin t & 2 \\ 0 & 1-0.16\cos t \end{bmatrix},$$
$$\bar{A}_1 = \begin{bmatrix} 0.6 & -0.4 \\ 0 & -0.04\cos t \end{bmatrix},$$
$$\bar{A}_2 = \begin{bmatrix} 0.04\sin t & 0 \\ 0 & -0.5 \end{bmatrix},$$
$$B = [1 \quad 1]^{\mathrm{T}}, \quad x(t) = \phi(t),$$
$$t \in [-\tau, 0], \quad \tau = \max(\tau_1, \tau_2)$$

and the constant time-delays are bounded. The uncertainty can be described by

$$D_0 = E_0 = \begin{bmatrix} 0.4 & 0 \\ 0 & -0.4 \end{bmatrix},$$
$$D_1 = E_1 \begin{bmatrix} 0 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$D_2 = E_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix},$$
$$F_0(t) = F_d(t) = \begin{bmatrix} \sin t & 0 \\ 0 & -\cos t \end{bmatrix}.$$

By applying Theorem 3.2, it is found that this system is stabilizable if $\tau \leqslant 0.5061$. If we know that $\tau_1 = 0.1$, the upper bound of time-delay $\tau_2$ is 1.256 such that this system is robust stabilizable.

## 5 Conclusion

This paper deals with the problem of robust stability and robust stabilization of uncertain linear systems with multiple delays in state. We have developed delay-dependent LMI-based methods of analyzing the robust stability and designing linear memoryless state feedback controllers. A new sufficient delay-dependent condition for the uncertain linear system to be globally uniformly asymptotically stable has been given in terms of an LMI.

## References

1 Hale J. Theory of Functional Differential Equations. New York: Springer, 1977

2 Mori T. Criteria for asymptotic stability of linear time delay systems. IEEE Trans. Automat. Contr., 1985, 30(1): 158 – 161

3 Brierley S D, Chiasson J N, Lee E B and Zak S H. On stability independent of delay for linear systems. IEEE Trans. Automat. Contr., 1982, 27(1): 252 – 254

4 Su J H, Fong I K and Tseng C L. Stability analysis of linear systems with time delay. IEEE Trans. Automat. Contr., 1994, 39(6): 1341 – 1344

5 Su J H and Huang C G. Robust stability of delay dependence for linear uncertain systems. IEEE Trans. Automat. Contr., 1992, 37(10): 1656 – 1659

6 Wang Y, Xie L and de Souza C E. Robust control of a class of uncertain nonlinear systems. Systems and Control Letters, 1992, 19: 139 – 149

## 本文作者简介

**曹永岩** 1968年生.1996年3月在浙江大学获工学博士学位,现在浙江大学工业控制技术研究所从事博士后研究.感兴趣的研究领域为鲁棒控制,时滞系统,容错控制等.

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