

# A Self-Organizing Neural-Network-Based Fuzzy System

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**Abstract:** A self-organizing neural-network-based fuzzy system is proposed in this paper. It can partition the input spaces in a flexible way based on the distribution of the training data. By combining both the nearest neighborhood clustering scheme and the gradient descent method, the learning speed converges much faster than the original back-propagation algorithm. Simulation results suggest that the SONNFS has merits of simple structure, fast learning speed, fewer fuzzy logic rules and relatively high modeling accuracy.

**Keywords:** fuzzy logic; neural network; nearest neighborhood clustering scheme; gradient descent method; back-propagation learning scheme

## 一种基于神经网络的自组织模糊系统

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**摘要:** 提出了一种基于神经网络的自组织模糊系统,它能够根据输入输出数据灵活地划分模糊集合。由于采用模糊聚类方法和梯度下降法分两步对该系统进行训练,其收敛速度要比传统的 BP 算法快得多。仿真结果表明该系统结构简单,学习速度快,规则数少,模型精度高。

**关键词:** 模糊逻辑; 神经元网络; 模糊聚类学习方法; 梯度下降法; BP 学习算法

## 1 Introduction

A self-organizing neural-network-based fuzzy system (SONNFS) is proposed in this paper. This connectionist model is in the form of a feedforward multilayer network. Associated with the SONNFS is a two-phase hybrid learning algorithm which utilizes a nearest neighborhood clustering scheme<sup>[1]</sup> for both structure learning and initial parameters setting and a gradient descent method for fine tuning the parameters of the SONNFS. Since the number of the neurons is determined by the nearest neighborhood clustering scheme instead of given a priori, it is a selforganizing neural network. It can partition the input spaces in a flexible way based on the distribution of the training data, which makes the number of rules reduced greatly compared to the conventional grid partitioned neural fuzzy systems without any loss of modeling accuracy. By combining both the nearest neighborhood clustering scheme and the gradient descent method, the learning speed converges much faster than the original back-propagation learning algorithm. At last, the SONNFS is applied to construct a fuzzy model of

Box's gas furnace and simulation results suggest that the SONNFS has merits of simple structure, fast learning speed, fewer fuzzy logic rules and relatively high modeling accuracy.

## 2 General structure of the neural-fuzzy system

Since an MIMO system can always be separated into a group of MISO systems, we will only consider an MISO system here. The fuzzy IF-THEN rules are of the following form:

$$\begin{aligned} R^j: & \text{IF } x_1 \text{ is } A_{j1} \text{ and } \cdots \text{ and } x_n \text{ is } A_{jn} \\ & \text{THEN } y \text{ is } C^j, \quad j = 1, 2, \cdots, M. \end{aligned} \quad (1)$$

where  $M$  is the total rule number,  $\underline{X} = [x_1 \cdots x_n]^T$  is the input vector with each  $x_i$  normalized to  $[-1, 1]$ ,  $y$  is the output while  $A_{ji}$  and  $C^j$  ( $j = 1, 2, \cdots, M; i = 1, 2, \cdots, n$ ) are input and output fuzzy sets, respectively. The matching degree denoted by  $\Phi_j(\underline{X}) \in [0, 1]$  between  $\underline{X}$  and the  $j$ th rule pattern is measured by

$$\Phi_j(\underline{X}) = \exp \left[ - \sum_{i=1}^n \frac{(x_i - a_{ji})^2}{\sigma_j^2} \right], \quad (2)$$

where  $\lambda_j$  is the center of the fuzzy set  $C^j$ ,  $a_{ji}$  is the cen-

ter of the Gaussian membership function of the fuzzy set  $A_{ji}$  while  $\sigma_j$  is the width of the Gaussian membership function of the fuzzy set  $A_{ji}$  for any  $i$  ( $i = 1, 2, \dots, n$ ). The scalar output of the fuzzy system can then be expressed as

$$y = \frac{\sum_{j=1}^M \lambda_j \Phi_j(\underline{X})}{\sum_{j=1}^M \Phi_j(\underline{X})}. \quad (3)$$

The general structure of the SONNFS is shown in Fig. 1.

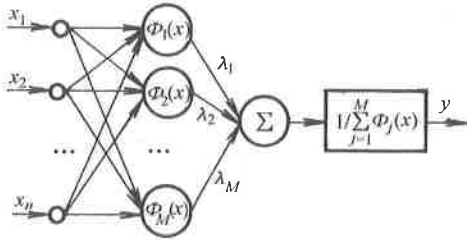


Fig. 1 General structure of the SONNFS

### 3 Learning algorithm

In this section, a two-stage hybrid learning algorithm is presented for the proposed connectionist model. In the first stage, a nearest neighborhood clustering scheme is used to perform a structure learning to determine the number of rules needed according to the training data. The radius  $r$  of the clusters should be carefully selected to ensure both the global property and the local property of the fuzzy system.  $a_{ji}$  and  $\lambda_j$  of each rule are also determined in this stage. The width of each membership function can then be simply determined by the first-nearest-neighbor algorithm as  $\sigma_j = r/s$ , where  $s$  is an overlap parameter which usually satisfies  $s > 1$  to ensure there is an overlap between two neighbor clusters. The larger  $s$  is, the more overlaps between two neighbor clusters there are.

For conventional neural fuzzy systems, when the number of input variables increase, the rule number will grow exponentially. Since the training samples are always finite, many rules in conventional neural fuzzy systems are not trained. By introducing the nearest neighborhood clustering scheme for structure learning, the SONNFS can partition the input spaces in a more flexible way based on the distribution of the training data so that these unnecessary fuzzy logic rules do not exist anymore, which makes the number of rules reduced greatly without any loss of modeling accuracy.

In the second stage, a gradient decent method is employed for fine tuning the fuzzy logic rules and the membership functions so that the modeling accuracy of the SONNFS is improved. The goal is to minimize the error function

$$E = \frac{1}{2}(y - d)^2, \quad (4)$$

where  $d$  is the desired output. With (2), (3) and (4), we can get the training algorithm for  $\lambda_j$ ,  $\sigma_j$ , and  $a_{ji}$ :

$$\begin{aligned} \lambda_j(k+1) &= \lambda_j(k) - \eta \frac{\partial E}{\partial \lambda_j} = \\ &= \lambda_j(k) - \eta(y - d) \tilde{\Phi}_j(\underline{X}), \end{aligned} \quad (5a)$$

$$\begin{aligned} \sigma_j(k+1) &= \sigma_j(k) - \eta \frac{\partial E}{\partial \sigma_j} = \\ &= \sigma_j(k) - \frac{2\eta}{\sigma_j^3}(y - d)(\lambda_j - \gamma) \tilde{\Phi}_j(\underline{X}) \sum_{i=1}^n (x_i - a_{ji})^2, \end{aligned} \quad (5b)$$

$$a_{ji}(k+1) = a_{ji}(k) - \eta \frac{\partial E}{\partial a_{ji}} =$$

$$a_{ji}(k) - \frac{2\eta}{\sigma_j^2}(y - d)(\lambda_j - \gamma)(x_i - a_{ji}) \tilde{\Phi}_j(\underline{X}), \quad (5c)$$

where

$$\tilde{\Phi}_j(\underline{X}) = \Phi_j(\underline{X}) / \sum_{l=1}^M \Phi_l(\underline{X}). \quad (6)$$

and  $\eta$  is a monotonically decreasing scalar learning rate.

It is trivial to say that the closer a point to a solution, the better is the convergence. Since the near optimal parameters have been found through the nearest neighborhood clustering scheme, the learning speed will certainly converge much faster than the original back-propagation learning algorithm.

### 4 Simulation results

Here, we will apply the proposed modeling approach to a well-known problem of modeling a gas furnace system introduced by Box and Jenkins<sup>[2]</sup>. In [2], Box gave 296 pairs of data measured from a gas furnace system with a single input being gas flow rate and a single output being CO<sub>2</sub> concentration in outlet gas.

We will consider three cases regarding this problem. In all cases, the mean-square-error (MSE) is used to evaluate the modeling performance. Set  $r = 0.2$ ,  $s = 2$ . The comparison between our results (case 1 and case 2) and some of those obtained by others is shown in Table 1. Obviously, our fuzzy model is the best among them.

Table 1 Comparative results for different modeling approaches

Reference Number	Number of Inputs	Number of Rules	MSE value
[3]	2	81	0.320
[4]	3	6	0.190
[5]	2	19	0.469
[6]	2	25	0.328
[7]Case 1	2	31	0.212
[7]Case 2	4	45	0.169
[7]Case 3	2	35	0.197
SONNFS Case 1	2	29	0.1371
SONNFS Case 2	4	41	0.1285

In case 3, the first 150 data pairs are used to construct the fuzzy model. The model structure are the same as in case 1. And then all 296 pairs of data are used to test the generalizing capability of the fuzzy model in response to novel situations. The result is shown in Fig. 2.

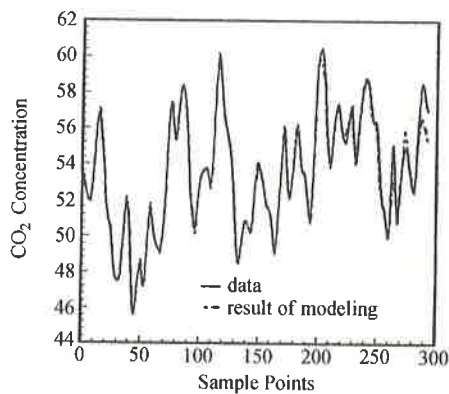


Fig. 2 Desired (solid line) and model (dashed line) CO<sub>2</sub> concentrations (case 3)

## 5 Conclusion

In this paper a self-organizing neural-network-based fuzzy system is proposed. It can partition the input spaces

in a flexible way based on the distribution of the training data which make the number of rules reduced greatly without any loss of modeling accuracy. A training algorithm combining the nearest neighborhood clustering scheme and the gradient descent method is also proposed to build a fuzzy model from numerical data pairs. By combining the above two methods, the learning speed converges much faster than the original back-propagation algorithm. Simulation results suggest that the SONNFS has merits of simple structure, faster learning speed, fewer fuzzy logic rules and relatively high modeling accuracy.

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