Generalized Pole Assignment Self-Tuning Controller

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Abstract: In this paper, we derive predictive models of system and auxiliary system from double constant alterations respectively. The controller based on d-step ahead predictor can locate the closed loop poles at desired positions, whose parameters are adjusted by estimations of plant parameters that are separately estimated. An auxiliary estimator is developed to avoid ill-condition in solving Diophantine equation. Simulations show that these control systems have better dynamic responses under existence of measurable disturbance.

Key words: pole assignment; self-tuning controller; d-step ahead Predictor; generalized minimum variance

广义极点配置自校正控制器

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摘要: 引入双恒等变换推导出系统及辅助系统的的 d 步预测模型, 基于估计器的自校正控制器能将闭环极点配置在所希望的位置, 它的参数是由与其相互独立的对象参数调节. 提出使用辅助估计器克服丢番方程的病态问题. 仿真表明在存在可测干扰的情况下该系统具有良好的动态性能.

关键词: 极点配置; 自校正控制器; d 步前向预报; 广义最小方差

1 Introduction

Pole assignment and generalized minimum variance are still two fundamental methods of self-tuning controller, which easily deals with non-minimal phase systems. Pole assignment self-tuning controller (STC) can permit users to pre-assign closed-loop poles in the desired positions to guarantee a good performance for the control system. Generalized minimum variance (GMV) STC has improved some performance of minimum variance controller at the expense of control accuracy. Among the self-tuning controllers addressed in the papers[1][2], there exists the influence each other between the choosing controller weighting factors and the identification of plant parameters. In this paper, we derive predictive models of system and auxiliary system from double constant alterations respectively. The controller based on predictor can locate the closed loop poles at desired positions and has capability of feedback and feedforward control. An auxiliary estimator is developed to avoid ill-condition in solving Diophantine equation. Simulations show that these control systems have better dynamic responses under existence of measurable disturbance.

2 Description of system

Assume that a process is characterized as follows

$$A(z^{-1})y(k) = B(z^{-1})u(k-d) + B_2(z^{-1})\cdot v(k-d_2) + C(z^{-1})\xi(k), \tag{1}$$

where $y(k)$, $u(k)$ and $v(k)$ are the output, input and measurable disturbance sequences of system respectively, d and d_2 are the delay of control model and disturbance model respectively (assume $d_2 \geqslant d$), $\xi(k)$ is assumed tobe a zero mean sequence with finite variance polynomiasl.

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{na} z^{-na},$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{nb} z^{-nb},$$
$$B_2(z^{-1}) = b'_0 + b'_1 z^{-1} + b'_2 z^{-2} + \cdots + b'_{nb'} z^{-nb'},$$
$$C(z^{-1}) = 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{nc} z^{-nc}.$$

The objective of controller is to minimize the cost function:

$$J = \mathrm{E}\{P(z^{-1})y(k+d) + S(z^{-1})v(k+d-d_2) - R(z^{1})w(k) + Q(z^{-1})u(k)\}^2 = \mathrm{E}\{\Phi_1(k+d)\}^2. \tag{2}$$

where $\Phi_1(k+d) = P(z^{-1})y(k+d) + S(z^{-1})v(k+d-d_2) - R(z^{-1})w(k) + Q(z^{-1})u(k)$ and $P(z^{-1})$,

Manuscript received Dec. 15, 1997, revised Nov. 16, 1998.

$S(z^{-1}), R(z^{-1})$ and $Q(z^{-1})$ are the weighting factors of output $y(k)$, measurable disturbance $v(k)$, setpoin $w(k)$ and input $u(k)$ respectively.

3 Establishment of self-tuning models based on d- step-ahead prediction

Define an auxiliary system as

$$\Phi(k+d) = P(z^{-1})y(k+d), \tag{3}$$

Assume

$$C(z^{-1}) = A(z^{-1})F_1(z^{-1}) + z^{-d}G_1(z^{-1}), \tag{4}$$

$$P(z^{-1})C(z^{-1}) = A(z^{-1})F_2(z^{-1}) + Z^{-d}G_2(z^{-1}), \tag{5}$$

where

$$F_i(z^{-1}) = 1 + f_1^i z^{-1} + f_2^i z^{-2} + \cdots + f_{nf}^i z^{-nfi},$$

$$G_i(z^{-1}) = g_0 + g_1^i z^{-1} + g_2^i z^{-2} + \cdots + g_{ng}^i z^{-ng}, \quad (i = 1,2)$$

$$\deg F_1(z^{-1}) = \deg F_2(z^{-1}) = d-1,$$

$$\deg G_1(z^{-1}) = na,$$

$$\deg G_2(z^{-1}) = \max\{np + nc - d, na - 1\}.$$

Equations (3),(5) and equations (1),(4) can be used to derive the system and auxiliary system based on d-step ahead output predictions respectively.

$$y(k+d) = \frac{B(z^{-1})F_1(z^{-1})}{C(z^{-1})}u(k) + \frac{B_2(z^{-1})F_1(z^{-1})}{C(z^{-1})}\cdot v(k+d-d_2) + \frac{G_1(z^{-1})}{C(z^{-1})}y(k) + F_1(z^{-1})\xi(k+d), \tag{6}$$

$$\Phi(k+d) = \frac{B(z^{-1})F_2(z^{-1})}{C(z^{-1})}u(k) + \frac{B_2(z^{-1})F_2(z^{-1})}{C(z^{-1})}\cdot v(k+d-d_2) + \frac{G_2(z^{-1})}{C(z^{-1})}y(k) + F_2(z^{-1})\xi(k+d). \tag{7}$$

their optimal predictors and errors as follows are

$$y^*(k+d) = B(z^{-1})F_1(z^{-1})u(k) + B_2(z^{-1})F_1(z^{-1})\cdot v(k+d-d_2) + G_1(z^{-1})y(k) + C'(z^{-1})y^*(k+d), \tag{8}$$

$$\Phi^*(k+d) = B(z^{-1})F_2(z^{-1})u(k) + B_2(z^{-1})F_2(z^{-1})\cdot v(k+d-d_2) + G_2(z^{-1})y(k) + C'(z^{-1})\Phi^*(k+d) \tag{9}$$

where $C'(z^{-1}) = 1 - C(z^{-1})$.

Let

$$e_y(k+d) = F_1\xi(k+d), \tag{10}$$

$$e_\Phi(k+d) = F_2\xi(k+d), \tag{11}$$

$$y(k+d) = B(z^{-1})F_1(z^{-1})u(k) + B_2(z^{-1})F_1(z^{-1})\cdot v(k+d-d_2) + G_1(z^{-1})y(k) + C'(z^{-1})y^*(k+d) + e_y(k+d), \tag{12}$$

$$\Phi(k+d) = B(z^{-1})F_2(z^{-1})u(k) + B_2(z^{-1})F_2(z^{-1})\cdot v(k+d-d_2) + G_2(z^{-1})y(k) + C(z-1)\Phi^*(k+d) + e_\Phi(k+d), \tag{13}$$

$$y(k) = X_y^{\mathrm{T}}(k-d)\Theta_y(k) + e_y(k) \tag{14}$$

where

$$X_y^{\mathrm{T}}(k-d) = [y(k-d), y(k-d-1), \cdots, u(k-d), u(k-d-1), \cdots, v(k-d_2), v(k-d_2-1), \cdots, y^*(k-1), y^*(k-2), \cdots],$$

$$\Theta_y(k) = [g_0^1, g_1^1, g_2^1, \cdots, bf_0^1, bf_1^1, \cdots, b_2f_0^1, b_2f_1^1, \cdots, -c_1, -c_2, \cdots]^{\mathrm{T}}$$

while system is slow time variable, the recursive least square method (RLS) is adopted as follows.

$$Z(k) = X(k-d)^{\mathrm{T}}\Theta(k) + e(k), \tag{15}$$

$$\Theta(K) = \Theta(k-1) + K(k)\cdot [Z(k) - X^{\mathrm{T}}(k-d)\Theta(k-1)], \tag{16}$$

$$K(k) = P(k-1) - X(k-d)\beta + X^{\mathrm{T}}(k-d)P(k-1)X(k-d)]^{-1}, \tag{17}$$

$$P(k) = [P(k-1) - K(k)X(k-d)\cdot P(k-1)]/\beta. \tag{18}$$

4 Design of controller

From the auxiliary system Eq. (7) to Eq. (2)

$$J = \mathrm{E}\{\frac{B(z^{-1})F_2(z^{-1})}{C(z^{-1})}u(k) + \frac{B_2(z^{-1})F_2(z^{-1})}{C(z^{-1})}v(k+d-d_2) + \frac{G_2(z^{-1})}{C(z^{-1})}y(k) - R(z^{-1})w(k) + S(z^{-1})v(k+d-d_2) + Q(z^{-1})u(k)\}^2 + \mathrm{E}\{F_1(z^{-1})\xi(k+d)\}^2 = J_1 + J_2 \tag{19}$$

where

$$J_1 = \mathrm{E}\{\frac{B(z^{-1})F_2(z^{-1})}{C(z^{-1})}u(k) + \frac{B_2(z^{-1})F_2(z^{-1})}{C(z^{-1})}v(k+d-d_2) + \frac{G_2(z^{-1})}{C(z^{-1})}y(k) - R(z^{-1})w(k) + S(z^{-1})v(k+d-d_2) + Q(z^{-1})u(k)\}^2,$$

$$J_2 = \mathrm{E}\{F_2\xi(k+d)\}^2.$$

The first term J_1 is the controllable part, and the second one J_2 is uncontrollable.

To minimize J, let $J_1 = 0$. Then optimal control law can be derived from

$$\Phi_1^*(k+d) + C(z^{-1})[S(z^{-1})v(k+d-d_2) - R(z^{-1})w(k) + Q(z^{-1})u(k)] = 0, \quad (20)$$

or

$$[B(z^{-1})F_2(z^{-1}) + C(z^{-1})Q(z^{-1})]u(k) + [B_2(z^{-1})F_2(z^{-1}) + C(z^{-1})S(z^{-1})]v(k+d-d_2) - C(z^{-1})R(z^{-1})w(k) + G_2(z^{-1})y(k) = 0. \quad (21)$$

Eqs.(20),(21) are the optimal control law by minimizing the cost function(2).

5 Determination of weighting factors and realization of pole placement

The control law Eq.(20) generates the closed loop equation

$$(PB + AQ)y(k+d) = BRw(k) + (-BS + B_2Q)v(k+d-d_2) + (B_1F_2 + CQ)\xi(k+d). \quad (22)$$

Let

$$P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1}) = \lambda T(z^{-1}), \quad (23)$$

where $T(z^{-1})$ is the closed loop poles polynomial which is desired and λ is an adjustable constant (for simplification, let $\lambda = 1$). Deformation of Eq.(23) can be

$$P(z^{-1})B(z^{-1})F_1(z^{-1}) + Q(z^{-1})[C(z^{-1}) - Z^{-d}G_1(z^{-1})] = \lambda T(z^{-1})F_1(z^{-1}). \quad (24)$$

Eqs.(23) and (24) are generally termed as Diophantine equations. It is clear that Eq.(23) cannot be solved in general unless A, B are coprime polynomials. Using an auxiliary estimator and considering the control in the case of real-time and estimation on-line of the plant, we substitute the recursive least square estimation method for directly solving Gauss equation.

Define a new variable

$$Z_m(k) = T(z^{-1})e(k), \quad (25)$$

where $e(k)$ can be a known white noise sequence.

Thus Eq.(22) becomes

$$Z^\wedge(k) = B^\wedge(z^{-1})e(k)P(z^{-1}) + A^\wedge(z^{-1})e(k)Q(z^{-1}), \quad (26)$$

$$Z^\wedge(k) = X^{\wedge\mathrm{T}}(k-1)\Theta(k) \quad (27)$$

where

$$X(k-1) = [B^\wedge(z^{-1})e(k), B^\wedge(z^{-1})e(k-1), \cdots, A^\wedge(z^{-1})e(k), A^\wedge(z^{-1})e(k-1), \cdots,]^{\mathrm{T}},$$

$$\Theta(k) = [p_0, p_1, p_2, \cdots, q_0, q_1, \cdots]$$

and $\quad Z_m(k) = Z^\wedge(k).$

Using the recursive least square (RLS) estimation method, P, Q can be obtained indirectly.

For overcoming effects arising from measurable disturbance, let disturbance's coefficient is zero.

$$B(z^{-1})S(z^{-1}) + B_2(z^{-1})Q(z^{-1}) = 0. \quad (28)$$

Customarily

$$S(z^{-1}) = \frac{B_2(1)Q(1)}{B(1)}, \quad (29)$$

or

$$S(z^{-1}) = \frac{B_2(1)F_1(z^{-1})Q(1)}{B(1)F_1(z^{-1})}. \quad (30)$$

For correct tracking, R is chosen as

$$R(1) = P(1) + A(1)/B(1), \quad (31)$$

or

$$R(1) = P(1) + \frac{Q(1)[C(1) - G_1(1)]}{B(1)F_1(1)}. \quad (32)$$

6 Simulation

Consider a non-minimal phase system

$$y(k) - 0.618y(k-1) - 0.36y(k-2) = 0.09u(k-2) + 0.1166u(k-3) + 0.6v(k-3) + 0.4v(k-4) + \xi(k). \quad (33)$$

Assume that Stochastic white noise is $\mathrm{E}\{\xi(k)\} = 0$, $\mathrm{E}\{\xi^2(k)\} = 0.4$, the measurable disturbance $v(k)$ is square wave whose period is 140 and the closed loop polynomials is $T(z^{-1}) = 1 - 0.5z^{-1}$. Simulation is shown in Fig.1 and Fig.2, representing as output $y(k)$, control action $u(k)$ respectively.

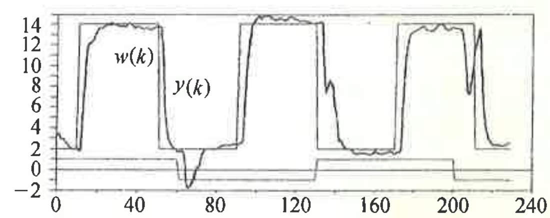

Fig. 1 Reference input $w(k)$ and output $y(k)$

7 Conclusions

To overcome the difficulties in the influence each other between the choosing controller weight factors and the identification of plant parameters, we introduce the double constant alterations into systems and derive the mod-

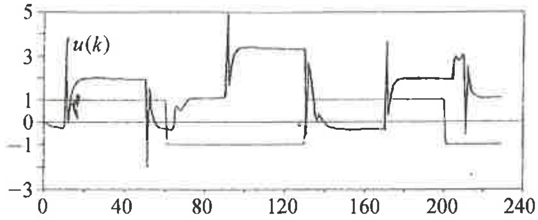

Fig. 2 Control action $u(k)$

els of system and auxiliary system. The controller based on predictior can locate the closed loop poles at desired positions. An auxiliary estimator is developed to avoid ill-condition in solving Diophantine equation. Simulations show that this control system has better dynamic responses under existence of measurable disturbance.

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