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基于高阶统计特性的非高斯 AR 模型的阶次辨识*

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摘要：提出一种基于输入输出信号的三阶累积量(third-order cumulants)对非高斯 AR 模型阶次 p 进行辨识的新算法.该算法依模型阶次递推，通过求解代价函数 $J(\hat{p})$ 的首次最小值，确定 AR 模型的阶次估计值 $\hat{p}$. 理论分析和仿真结果均显示，该算法具有良好的收敛性和精确性.

关键词：随机系统；AR 模型；三阶累积量；系统阶次辨识　系统辨识，　阶次辨识
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Order Identification of Non-Gaussian AR Models Based on Higher-Order Statistics

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Abstract: An order-recursive algorithm for determining the order p of non-Gaussian AR models based on the third-order cumulants is suggested in this paper. The order estimates $\hat{p}$ of AR models can be selected by achieving the first minimum point of a well-defined cost function $J(\hat{p})$. Theoretical analyses and simulation results illustrate that the proposed algorithm, which has a strong convergence behaviour, is accurate and simple.

Key words: stochastic systems; AR models; third-order cumulants; system order identification

1　引言(Introduction)

经典的 AR 模型阶次的估计方法几十年来一直广泛地应用于系统辨识的理论研究和工程实践中，其定阶准则主要包括 FPE, AIC 及 BIC 等.由于模型阶的判定是基于输入或输出信号的二阶统计特性，因而容易产生过高或过低的估计值，导致系统估计性能的下降[1].当输入或输出信号染有高斯噪声时，应用二阶统计特性不能准确地辨识模型[2].另一方面，由于随机信号的高阶统计特性能完全消失信号中包含的高斯噪声，近年来高阶统计理论(higher order statistics)逐渐在系统辨识领域得到广泛的应用.目前已提出了不少基于高阶统计特性对 AR 模型阶次进行辨识的方法[3~7]，主要有：1) 采用修正的 AIC-MAIC 准则辨识 AR 模型的阶次[3,4]；2) 利用三阶或更高阶累积量构成似 Hankel 矩阵，通过确定该矩阵的秩对 AR 模型的阶次进行辨识[5~7].然而这些方法都不具备递推形式，在计算上比较复杂，难以应用于计算机在线辨识.本文提出一种新的基于三阶累积量的 AR 模型阶次的辨识算法，该算法具有完备的递推形式.运算简单，容易在计算机上进行仿真或物理实现.

2　AR 模型及其辨识算法(AR models and an identification algorithm)

设平稳随机过程 $\{y(t)\}$ 的 p 阶 AR 模型为：

$$y(t)+\sum_{i=1}^{p}a(i)y(t-i)=w(t),\tag{1a}$$

式中 t 为离散时间，$a(0)=1, a(p)\neq 0$. $w(t)$ 为可测的零均值的独立同分布非高斯随机过程，满足 $E[w^2(t)]\neq 0, E[w^3(t)]\neq 0$，及 $E[w^6(t)]<\infty$. 当模型的输出染有高斯噪声时，所测得的输出信号序列 $\{y_m(t)\}$ 可表征为

$$y_m(t)=y(t)+n(t).\tag{1b}$$

式中 $\{n(t)\}$ 为独立于模型输入输出的高斯测量噪声序列.本文研究的目标是由测得的模型输入输出序列对 $\{w(t)\}-\{y_m(t)\}$ 来确定模型阶次 p.

由三阶累积量的定义[2]，模型(1a)及(1b)的输出三阶累积量和输入输出三阶协累积量

$$C_3^y(m,n)=E[y_m(t)y_m(t+m)y_m(t+n)]$$

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$$= E[y(t)y(t+m)y(t+n)],\tag{2a}$$

$$C_3^{wy}(m,n) = E[w(t)y_m(t+m)y_m(t+n)] = E[w(t)y(t+m)y(t+n)].\tag{2b}$$

可见三阶累积量能完全消除高斯噪声的影响. 将(1a)式代入(2a)式得:

$$C_3^y(m,n) = C_3^{wy}(m,n) - \sum_{i=1}^{p} a(i)C_3^y(m+i,n+i).\tag{3}$$

设AR模型的阶次估计值为 $\hat{p}$, 则由上式可知, 对 $C_3^y(m,n)$ 的估计为:

$$\hat{C}_3^y(m,n) = C_3^{wy}(m,n) - \sum_{i=1}^{\hat{p}} a(i)C_3^y(m+i,n+i).\tag{4}$$

构造如下代价函数:

$$J(\hat{p}) = \frac{1}{2}\sum_m\sum_n[\hat{C}_3^y(m,n) - C_3^y(m,n)]^2 + \frac{1}{2}\lambda\sum_{t=0}^{N}[-\sum_{i=0}^{\hat{p}} a(i)y_m(t-i) - w(t)]^2.\tag{5}$$

式中 $\lambda \geqslant 0$ 为权常数, N 为 $\{w(t)\}-\{y_m(t)\}$ 的采样周期数. 根据三阶累积量的对称性质[2], 可设 $\max(m) = \max(n) = N$, 将(4)式代入(5)式中写成向量形式为:

$$J(\hat{p}) = \frac{1}{2}[C_3^y(\hat{p})\bar{a}(\hat{p}) - C_3^{wy}]^{\mathrm{T}}[C_3^y(\hat{p})\bar{a}(\hat{p}) - C_3^{wy}] + \frac{1}{2}\lambda[y(\hat{p})\bar{a}(\hat{p}) - w]^{\mathrm{T}}[y(\hat{p})\bar{a}(\hat{p}) - w],\tag{6}$$

上式中

$$C_3^y(\hat{p}) = [c_3^{y\mathrm{T}}(0), c_3^{y\mathrm{T}}(1), \cdots, c_3^{y\mathrm{T}}(N)]^{\mathrm{T}},$$

$$c_3^y(k) = \begin{bmatrix} C_3^y(k,0) & C_3^y(k+1,1) & \cdots & C_3^y(k+\hat{p},\hat{p}) \\ C_3^y(k,1) & C_3^y(k+1,2) & \cdots & C_3^y(k+\hat{p},\hat{p}+1) \\ \vdots & \vdots & \ddots & \vdots \\ C_3^y(k,N) & C_3^y(k+1,N+1) & \cdots & C_3^y(k+\hat{p},\hat{p}+N) \end{bmatrix},$$

$$C_3^{wy} = [c_3^{wy\mathrm{T}}(0), c_3^{wy\mathrm{T}}(1), \cdots, c_3^{wy\mathrm{T}}(N)]^{\mathrm{T}},$$

$$c_3^{wy}(k) = [C_3^{wy}(k,0), C_3^{wy}(k,1), \cdots, C_3^{wy}(k,N)]^{\mathrm{T}},$$

$$a(\hat{p}) = [a(0), a(1), \cdots, a(\hat{p})]^{\mathrm{T}},$$

$$y(\hat{p}) = \begin{bmatrix} y_m(0) & y_m(-1) & \cdots & y_m(-\hat{p}) \\ y_m(1) & y_m(0) & \cdots & y_m(-\hat{p}+1) \\ \vdots & \vdots & \ddots & \vdots \\ y_m(N) & y_m(N-1) & \cdots & y_m(-\hat{p}+N) \end{bmatrix},$$

$$w = [w(0), w(1), \cdots, w(N)]^{\mathrm{T}}.$$

式中 λ 不影响 $J(\hat{p})$ 的收敛性, 仅影响 $J(\hat{p})$ 的收敛精度, 可选 λ 为[4]:

$$\lambda = \lambda_0 \frac{\sum_{m=0}^{N}\sum_{n=0}^{N}[C_3^y(m,n)]^2}{\sum_{t=0}^{N}[w(t)]^2}.\tag{7}$$

其中 $\lambda_0 = 1$ 为基准值. 显然, 使代价函数 $J(\hat{p})$ 取最小值的阶次 $\hat{p}$, 即为AR模型阶次估计的最优解.

由

$$\frac{\partial J(\hat{p})}{\partial \bar{a}(\hat{p})} = 0,$$

得到

$$\bar{a}(\hat{p}) = [C_3^{y\mathrm{T}}(\hat{p})C_3^y(\hat{p}) + \lambda y^{\mathrm{T}}(\hat{p})y(\hat{p})]^{-1}\cdot[C_3^{y\mathrm{T}}(\hat{p})C_3^{wy} + \lambda y^{\mathrm{T}}(\hat{p})w],\tag{8}$$

类似地, 可以写出模型阶次为 $\hat{p}+1$ 时的表达式

$$\bar{a}(\hat{p}+1) = [C_3^{y\mathrm{T}}(\hat{p}+1)C_3^y(\hat{p}+1) + \lambda y^{\mathrm{T}}(\hat{p}+1)\cdot y(\hat{p}+1)]^{-1}[C_3^{y\mathrm{T}}(\hat{p}+1)C_3^{wy} + \lambda y^{\mathrm{T}}(\hat{p}+1)w],\tag{9}$$

其中

$$C_3^y(\hat{p}+1) = [C_3^y(\hat{p}) \vdots c_3^{y\prime}(\hat{p}+1)],$$

$$c_3^{y\prime}(\hat{p}+1) = [C_3^y(\hat{p}+1,\hat{p}+1), C_3^y(\hat{p}+1,\hat{p}+1+1), \cdots, C_3^y(\hat{p}+1,\hat{p}+1+N) \vdots C_3^y(\hat{p}+2,\hat{p}+1), C_3^y(\hat{p}+2,\hat{p}+1+1), \cdots, C_3^y(\hat{p}+2,\hat{p}+1+N) \vdots \cdots \cdots \vdots C_3^y(\hat{p}+N,\hat{p}+1), C_3^y(\hat{p}+N,\hat{p}+1+1), \cdots, C_3^y(\hat{p}+N,\hat{p}+1+N)]^{\mathrm{T}},$$

$$y(\hat{p}+1) = [y(\hat{p}) \vdots \bar{y}(\hat{p}+1)],$$

$$\bar{y}(\hat{p}+1) = [y_m(-\hat{p}-1), y_m(-\hat{p}), \cdots, y_m(-\hat{p}-1+N)]^{\mathrm{T}},$$

则

$$[C_3^{y\mathrm{T}}(\hat{p}+1)C_3^y(\hat{p}+1) + \lambda y^{\mathrm{T}}(\hat{p}+1)y(\hat{p}+1)] =$$

$$\begin{bmatrix} C_3^{\mathrm T}(\hat p) \\ \vdots \\ c_3'^{\mathrm T}(\hat p+1) \end{bmatrix} [C_3(\hat p) \vdots c_3'(\hat p+1)] + \lambda \begin{bmatrix} y^{\mathrm T}(\hat p) \\ \vdots \\ y^{\mathrm T}(\hat p+1) \end{bmatrix} [y(\hat p) \vdots \bar y(\hat p+1)] =$$

$$\begin{bmatrix} C_3^{\mathrm T}(\hat p) C_3(\hat p) + \lambda y^{\mathrm T}(\hat p) y(\hat p) & C_3^{\mathrm T}(\hat p) c_3'(\hat p+1) + \lambda y^{\mathrm T}(\hat p) \bar y(\hat p+1) \\ c_3'^{\mathrm T}(\hat p+1) C_3(\hat p) + \lambda y^{\mathrm T}(\hat p+1) y(\hat p) & c_3'^{\mathrm T}(\hat p+1) c_3'(\hat p+1) + \lambda \bar y_3^{\mathrm T}(\hat p+1) \bar y(\hat p+1) \end{bmatrix}. \qquad (10)$$

令

$$h(\hat p) = \begin{bmatrix} C_3(\hat p) \\ \vdots \\ \sqrt{\lambda} y(\hat p) \end{bmatrix}, \quad v(\hat p+1) = \begin{bmatrix} c_3'(\hat p+1) \\ \vdots \\ \sqrt{\lambda} \bar y(\hat p+1) \end{bmatrix},$$

(10)式可写为

$$[C_3^{\mathrm T}(\hat p+1) C_3(\hat p+1) + \lambda y^{\mathrm T}(\hat p+1) y(\hat p+1)] =$$

$$\begin{bmatrix} h^{\mathrm T}(\hat p) h(\hat p) & h^{\mathrm T}(\hat p) v(\hat p+1) \\ v^{\mathrm T}(\hat p+1) h(\hat p) & v^{\mathrm T}(\hat p+1) v(\hat p+1) \end{bmatrix}. \qquad (11)$$

令

$$G(\hat p) = [C_3^{\mathrm T}(\hat p) C_3(\hat p) + \lambda y^{\mathrm T}(\hat p+1) y(\hat p)]^{-1},$$

则

$$G(\hat p+1) = [C_3^{\mathrm T}(\hat p+1) C_3(\hat p+1) + \lambda y^{\mathrm T}(\hat p+1) y(\hat p+1)]^{-1},$$

于是(11)式可写为

$$G(\hat p+1) = \begin{bmatrix} G^{-1}(\hat p) & h^{\mathrm T}(\hat p) v(\hat p+1) \\ v^{\mathrm T}(\hat p+1) h(\hat p) & v^{\mathrm T}(\hat p+1) v(\hat p+1) \end{bmatrix}^{-1}. \qquad (12)$$

利用分块矩阵求逆定理,(12)式具备如下递推形式

$$G(\hat p+1) = \begin{bmatrix} G(\hat p) + \dfrac{G(\hat p) h^{\mathrm T}(\hat p) v(\hat p+1) v^{\mathrm T}(\hat p+1) h(\hat p) G(\hat p)}{v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1)} & -\dfrac{G(\hat p) h^{\mathrm T}(\hat p) v(\hat p+1)}{v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1)} \\ -\dfrac{v^{\mathrm T}(\hat p+1) h(\hat p) G(\hat p)}{v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1)} & \dfrac{1}{v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1)} \end{bmatrix}, \qquad (13a)$$

上式中

$$R(\hat p) = I - h(\hat p) G(\hat p) h^{\mathrm T}(\hat p), \qquad (13b)$$

再令

$$z = \begin{bmatrix} C_3^{w} \\ \vdots \\ \sqrt{\lambda} w \end{bmatrix},$$

将(13a)式代入(9)式即得到 $a(\hat p)$ 的递推形式

$$a(\hat p+1) = G(\hat p+1) \begin{bmatrix} h^{\mathrm T}(\hat p) \\ \vdots \\ v^{\mathrm T}(\hat p+1) \end{bmatrix} z =$$

$$\begin{bmatrix} \bar a(\hat p) - \dfrac{G(\hat p) h^{\mathrm T}(\hat p) v(\hat p+1) v^{\mathrm T}(\hat p+1) R(\hat p) z}{v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1)} \\ \dfrac{v^{\mathrm T}(\hat p+1) R(\hat p) z}{v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1)} \end{bmatrix}. \qquad (14)$$

考虑到式(6),阶次为 $\hat p+1$ 时的代价函数表达式为

$$J(\hat p+1) = \frac{1}{2}[C_3(\hat p+1) \bar a(\hat p+1) - C_3^{w}]^{\mathrm T} \cdot [C_3(\hat p+1) a(\hat p+1) - C_3^{w}] + \frac{1}{2}\lambda [y(\hat p+1) \bar a(\hat p+1) - w]^{\mathrm T} \cdot [y(\hat p+1) \bar a(\hat p+1) - w], \qquad (15)$$

类似(8)式有下式成立

$$C_3^{\mathrm T}(\hat p+1) C_3(\hat p+1) \bar a(\hat p+1) - C_3^{\mathrm T}(\hat p+1) C_3^{w} + \lambda [y^{\mathrm T}(\hat p+1) y(\hat p+1) \bar a(\hat p+1) - y^{\mathrm T}(\hat p+1) w] = 0, \qquad (16)$$

将(16)式代入(15)式并简化为

$$J(\hat p+1) = \frac{1}{2}[z^{\mathrm T} z - z^{\mathrm T}[h(\hat p) \vdots v(\hat p+1)] a(\hat p+1)], \qquad (17a)$$

(14)式代入(17a)式中并化简为

$$J(\hat p+1) = J(\hat p) - \frac{1}{2} \frac{[v^{\mathrm T}(\hat p+1) R(\hat p) z]^2}{v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1)}. \qquad (17b)$$

上式即构成 AR 模型阶次 $\hat p$ 的辨识递推算法.可以证明(详见附录 A)

$$v^{\mathrm T}(\hat p+1) R(\hat p) v(\hat p+1) > 0, \qquad (17c)$$

于是有

$$J(\hat p+1) \leqslant J(\hat p), \qquad (17d)$$

同时有

$$a(\hat p+1) \leqslant a(\hat p). \qquad (17e)$$

假设 AR 模型参数的真值为

$$a_0 = [-a(1) \quad a(2) \quad \cdots \quad -a(p)]^{\mathrm T},$$

引入零均值误差向量 $e_1(t), e_2(t)$,由式(6)可得

$$C_3^{w} = C_3(p) a_0 + e_1(t), \qquad (18a)$$

$$w = y(p) a_0 + e_2(t). \qquad (18b)$$

当递推阶次 $\hat p$ 等于或大于 AR 模型阶次真值 p 时,取 $\hat p = p+1$,则

$$C_3(p) a_0 + e_1(t) = C_3(p+1) \bar a(p+1) = C_3(p) \bar a(p) + c_3'(p+1) \dot a(p+1),$$

$$\hat{p} = p + 1, \tag{18c}$$

$$\begin{aligned} y(p)a_0+e_2(t) &= y(p+1)\bar{a}(p+1) = \\ &y(p)a(p)+\bar{y}(p+1)\hat{a}(p+1), \\ \hat{p} &= p + 1, \end{aligned} \tag{18d}$$

则有

$$\lim_{\hat{a}(p+1)\to 0} E[\bar{a}(p) - a_0] = 0, \tag{18e}$$

考虑到式(6),有

$$\lim_{\hat{a}(p+1)\to 0} J = J_{\min}. \tag{18f}$$

上式表明,本文提出的辨识递推算法具有良好的收敛性.阶次估计值 $\hat{p}$ 即为代价递推函数 $J(\hat{p})$ 取首次最小值时的递推自变量,即 $\hat{p} = \arg[J(\hat{p})_{\text{first min}}]$.

需要指出的是,三阶累积量及协累积量表达式(2a),(2b)中含有数学期望运算,一般难以从模型输入输出信号中准确获取,但存在其渐近无偏估计值[2]

$$\tilde{C}_3^y(m,n) = \frac{1}{N}\sum_{t=1}^{N} y_m(t)y_m(t+m)y_m(t+n), \tag{19a}$$

$$\tilde{C}_3^{wy}(m,n) = \frac{1}{N}\sum_{t=1}^{N} w(t)y_m(t+m)y_m(t+n). \tag{19b}$$

满足

$$\lim_{N\to\infty} E[\tilde{C}_3^y(m,n) - C_3^y(m,n)]^2 = 0, \tag{20a}$$

$$\lim_{N\to\infty} \tilde{C}_3^y(m,n) = C_3^y(m,n), \tag{20b}$$

及

$$\lim_{N\to\infty} E[\tilde{C}_3^{wy}(m,n) - C_3^{wy}(m,n)]^2 = 0, \tag{21a}$$

$$\lim_{N\to\infty} \tilde{C}_3^{wy}(m,n) = C_3^{wy}(m,n). \tag{21b}$$

因此,在实际应用(17b)递推算法时,应将(19a)~(19b)式中 $\tilde{C}_3^y(m,n)$ 和 $\tilde{C}_3^{wy}(m,n)$ 代替(17b)算法中的 $C_3^y(m,n)$ 和 $C_3^{wy}(m,n)$.

3 仿真及结果分析(Simulation and results analysis)

例 设有如下形式的AR(3)模型:

$$y(t) - 2.2y(t-1) + 1.77y(t-2) - 0.52(t-3) = w(t).$$

其中模型参数真值

$$a_0 = [1, -2.2, 1.77, -0.52],$$

阶次真值 $p = 3$,模型的极点为0.8,0.7±j0.4.模型的输入为单边指数分布的零均值随机序列,模型的输出信号 $\{y(t)\}$ 中包含一高斯噪声 $\{n(t)\}$,其信噪比设为

$$\text{SNR} = 10\log\frac{E[y^2(t)]}{E[n^2(t)]} = 10\text{dB}.$$

设阶次递推的上界 $\max(\hat{p}) = 7$,输入输出信号的采样数 $N = 1024$,并取权 $\lambda = 1$.为了获得准确的模型阶次估计值,仿真实验在50 Monte-Carlo条件下进行.图1显示该模型输出信号的三阶累积量估计值 $\tilde{C}_3^y(m,n)$ 与其真值 $C_3^y(m,n)$ 之间存在较小的误差.采用本文所述算法,相应的代价函数的收敛过程如图2所示.从图2中可以看出,尽管存在高斯噪声,$\arg[J(\hat{p})_{\text{first min}}] = 3$,完全与AR模型的阶次真值吻合.

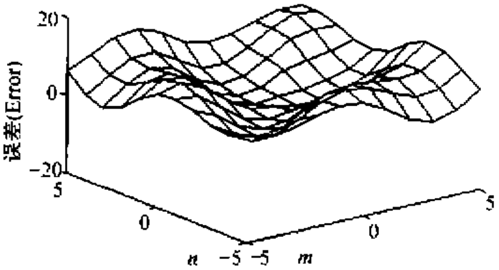

图1 误差:$[\hat{C}_3^y(m,n)-\hat{C}_3^y(m,n)]$,其中 $m,n\in[-5,5]$

Fig. 1 Error: $[\hat{C}_3^y(m,n)-\hat{C}_3^y(m,n)]$, where $m,n\in[-5,5]$

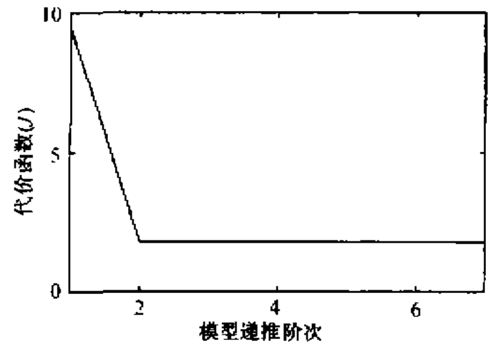

图2 代价函数 J 随AR(3)模型阶次递推的收敛过程

Fig. 2 The cost function $J(\hat{p})$ can be selected by AR(3) model order convergence process

4 小结(Conclusion)

本文提出了一种新的基于三阶累积量的非高斯AR模型阶次辨识的递推算法.这种算法的主要特点是以检验代价函数 $J(\hat{p})$ 的收敛性为确定阶次估计值 $\hat{p}$ 的标准.具有递推形式的阶次辨识算法 $J(\hat{p}+1) = f[J(\hat{p})]$,其代价函数的递推过程是完全收敛的.当代价函数 $J(\hat{p})$ 取首次最小值时的递推次数 $\hat{p}$,正是模型阶次的辨识值,即 $\arg[J(\hat{p})_{\text{first min}}] = \hat{p}$.从给出的仿真结果可以看出,该算法具有良好的收敛性和精确度,并且算法本身也较简单.

附录 A(Appendix A)

对式(17c)的证明.

证　由式(13b)知

$$R(\hat{p}) = I - h(\hat{p})G(\hat{p})h^{\mathrm{T}}(\hat{p}).$$

则

$$R^2(\hat{p}) = R(\hat{p})R(\hat{p}) = [I - h(\hat{p})G(\hat{p})h^{\mathrm{T}}(\hat{p})][I - h(\hat{p})G(\hat{p})h^{\mathrm{T}}(\hat{p})]. \tag{A1}$$

再由式(12)可知

$$G(\hat{p}) = [h^{\mathrm{T}}(\hat{p}) \cdot h(\hat{p})]^{-1}. \tag{A2}$$

将(A2)代入(A1)式并化简得

$$R^2(\hat{p}) = R(\hat{p}), \tag{A3}$$

同时

$$R^{\mathrm{T}}(\hat{p}) = [I - h(\hat{p})G(\hat{p})h^{\mathrm{T}}(\hat{p})]^{\mathrm{T}} = I - h(\hat{p})G(\hat{p})h^{\mathrm{T}}(\hat{p}) = R(\hat{p}). \tag{A4}$$

由(A3),(A4)式知

$$R^{\mathrm{T}}(\hat{p})R(\hat{p}) = R(\hat{p}). \tag{A5}$$

于是

$$v^{\mathrm{T}}(\hat{p}+1)R(\hat{p})v(\hat{p}+1) = v^{\mathrm{T}}(\hat{p}+1)R^{\mathrm{T}}(\hat{p})R(\hat{p})v(\hat{p}+1) = [R(\hat{p})v(\hat{p}+1)]^{\mathrm{T}}[R(\hat{p})v(\hat{p}+1)], \tag{A6}$$

上式中, $R(\hat{p})v(\hat{p}+1)$ 为 $[(N+1)^2+(N+1)]\times 1$ 维向量,显然有

$$v^{\mathrm{T}}(\hat{p}+1)R(\hat{p})v(\hat{p}+1) > 0.$$

证毕

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