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# Robust Adaptive Control for a Class of Uncertain Time-Delay Systems *

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**Abstract**: Robust adaptive control for a class of uncertain time-delay systems is presented in this paper. First, the robust control law is derived based on Lyapnuov stability theory, linear matrix inequality and variable structure control. Then, the adaptive control law is obtained with the estimation for the upper norms of matched uncertainties. The obtained results are illustrated by a numerical example.

**Key words**: uncertain time-delay system; robust control; variable structure control; adaptive control

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# 不匹配不确定线性时滞系统的鲁棒自适应控制

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**摘要**: 对一类同时具有匹配不确定性及结构确定不匹配不确定性的不确定时滞系统进行鲁棒自适应控制.首先,采用李雅谱诺夫函数方法,结合基于线性矩阵不等式的鲁棒控制器设计方法和变结构控制方法,设计鲁棒控制器,保证闭环系统的二次渐近稳定.利用自适应参数估计方法,设计具有匹配不确定性范数界估计能力的鲁棒自适应控制器,保证闭环系统的一致终结有界.结合算例,进行控制器的设计和仿真研究,验证所提出的设计方法的有效性.

**关键词**: 不确定性时滞系统; 鲁棒控制; 变结构控制; 自适应控制

## 1 Introduction

Many researchers have done a lot of works on the topic of the robust control for the matching uncertainties based on variable structure control (VSC)[1~3]. On the other hand, for uncertain systems with the mismatching uncertainties, the property of the variable structure system, which is not sensitive to uncertainties, is not satisfied[4,5] such that using ordinary variable control method can not solve these uncertain systems. Although linear matrix inequality (LMI) is an effective method for the robust control for uncertain systems with structured uncertainties[6], the obtained controller is very conservative. If the uncertainties are divided into two independent parts named the matching part and the mismatching part, the corresponding control action should be designed for the matching part, and the robust control method should be used to overcome the affection of the mismatching uncertainties based on LMI. Meanwhile, the control effectiveness may be widely improved based on the fact that the various effective control strategies are taken for various uncertainties with different property.

A robust controller is proposed in [7] based on VSC method and LMI approach, but the controller is very complex to guarantee the existence of sliding-mode. On the other hand, variable structure control system need not have sliding-mode only if the obtained closed-loop system is stable. Thus, it is very easy to combine the idea of VSC with LMI approach in the design of the robust control system.

With the development of variable structure control theory and time-delay theory, there are many works in the time-delay systems based on VSC method. In [8], a VSC controller is proposed for a class of linear time-delay systems with perfect model, and in [9] for a class

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of linear time-delay systems with matching uncertainties.

In our paper, for a class of uncertain linear time-delay systems with the matching and the mismatching uncertainties, a robust control is developed by using Lyapunov stability theory, and by combining VSC method and LMI approach. For the mismatching uncertainties with known structure, the robust controller is derived based on LMI approach without considering matching uncertainies. For the matching uncertainties, an additional control term with relay-type is constructed based on the idea of VSC method. The obtained controller has the ability to overcome the action of the matching and the mismatching uncertainties. To improve the effectiveness of the developed controller, the adaptive controller is deduced by the estimation for the upper norms of uncertainties such that obtained closed-loop system is uniformly ultimately bounded. Furthermore, the controller is modified with an approximate variable structure control algorithm.

## 2 Problem formulation and assumption

Consider the uncertain linear system with time-delay as follows,

$$\dot{x}(t) = A_1x(t)+M_1\delta_1(x(t))+A_2x(t-\tau(t))+M_2\delta_2(x(t-\tau(t)))+B(u + F(x,t(t-\tau(t)),t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^m$ is the control input, $n \geqslant m \geqslant 1$. The functions $\delta_1(x(t))$, $\delta_2(x(t - \tau(t))$ are uncertainties under the known structure $M_1, M_2$, respectively. The function $F(x(t), x(t-\tau(t)),t)$ is matching uncertainty under the control input. Function of time $\tau(t)$ is time-delay in system state, and there is a positive constant $\sigma$ such that $1 - \dot{\tau}(t) \geqslant \sigma^2$. Matrices $A, B, M_1, M_2$ are constant ones with proper dimensions. The following assumptions are introduced for system (1):

**Assumption 1** For the structured uncertainty in system (1), there are constant matrices $N_1, N_2$ such that

$$\|\delta_1(x(t))\| \leqslant \|N_1x(t)\|,$$
$$\|\delta_2(x(t-\tau(t))\| \leqslant \|N_2x(t-\tau(t))\|, \tag{2}$$

respectively.

**Assumption 2** For the matching uncertainty in system (1), there are positive constants $c_0, c_1, c_2$ and a known function matrix $F_0(x(t), x(t-\tau), t)$ such that

$$\|F(x(t),x(t-\tau(t),t)-F_0(x(t),x(t-\tau(t)),t)\| \leqslant c_0 + c_1\|x(t)\| + c_2\|x(t-\tau(t))\|. \tag{3}$$

Our aim may be described as follows:

Under the assumptions of 1 and 2, derive a robust quadratic stability controller if the parameters $c_0, c_1$ and $c_2$ are known previously; derive a robust controller with the property of the uniformly ultimate bound for the obtained closed-loop system if the parameters $c_0, c_1$ and $c_2$ are unkonwn.

## 3 Robust controller for the systems with matching uncertainties and its known-norm-bound

**Theory 1** For uncertain liner time-delay system with assumptions 1 and 2

$$\dot{x}(t) = A_1x(t)+A_2x(t-\tau(t))+M_1\delta_1(x(t))+M_2\delta_2(x(t-\tau(t)))+Bu. \tag{4}$$

Introduce a set $\Omega$ with the positive-defined matrices $P = P^{\mathrm{T}} > 0, Q = Q^{\mathrm{T}}, R = R^{\mathrm{T}} > 0$, the positive $\varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0$ and the matrix $K$:

$$(P,Q,\varepsilon_1,\varepsilon_2,\varepsilon_3,K,R):$$
$$\begin{cases} P(A_1+BK)+(A_1+BK)^{\mathrm{T}}P + \\ \dfrac{1}{\varepsilon_1}PM_1M_1^{\mathrm{T}}P + \varepsilon_1N_1^{\mathrm{T}}N_1 + \\ \dfrac{1}{\varepsilon_2}PM_2M_2^{\mathrm{T}}P + \dfrac{1}{\varepsilon_3}PA_2A_2^{\mathrm{T}}P + \\ Q + R < 0, \\ \varepsilon_2N_2^{\mathrm{T}}N_2 + \varepsilon_3I_n - \sigma^2R < 0. \end{cases} \tag{5}$$

Assume that the set $\Omega$ is non-empty, then the obtained closed-loop system under the controller $u = Kx(t)$ is quadratic stable for any elements in the set $\Omega$.

Proof Choose Lyapunov function as follows:

$$V = x^{\mathrm{T}}(t)Px(t) + \int_{t-\tau(t)}^{t} x^{\mathrm{T}}(p)Rx(p)\mathrm{d}p.$$

Under the controller $u = Kx(t)$, we have

$$\dot{V} \leqslant x^{\mathrm{T}}(t)\{P(A_1+BK)+(A_1+BK)^{\mathrm{T}}P\}x(t) + 2x^{\mathrm{T}}PM_1\delta_1(x(t)) + 2x^{\mathrm{T}}(t)PM_2\delta_2(x(t-\tau(t))) + x^{\mathrm{T}}Rx(t) - \sigma^2x^{\mathrm{T}}(t-\tau(t))Rx(t-\tau(t)) + 2x^{\mathrm{T}}PA_2x(t-\tau(t)).$$

Consider the following facts,

$$2x^{\mathrm{T}}(t)PM_1\delta_1(x(t)) \leqslant$$
$$\frac{1}{\varepsilon_1}x^{\mathrm{T}}(t)PM_1M_1^{\mathrm{T}}Px(t) + \varepsilon_1\|\delta_1(x(t))\|^2 \leqslant$$
$$\frac{1}{\varepsilon_1}x^{\mathrm{T}}(t)PM_1M_1^{\mathrm{T}}Px(t) + \varepsilon_1 x^{\mathrm{T}}(t)N_1^{\mathrm{T}}N_1x(t),$$
$$2x^{\mathrm{T}}(t)PM_2\delta_2(x(t-\tau(t)) \leqslant$$
$$\frac{1}{\varepsilon_2}x^{\mathrm{T}}(t)PM_2M_2^{\mathrm{T}}Px(t)+\varepsilon_2\|\delta_2(x(t-\tau))\|^2 \leqslant$$
$$\frac{1}{\varepsilon_2}x^{\mathrm{T}}(t)PM_2M_2^{\mathrm{T}}Px(t)+\varepsilon_2x^{\mathrm{T}}(t-\tau))N_2^{\mathrm{T}}N_2(x(t-\tau(t))$$
$$2x^{\mathrm{T}}(t)PA_2x(t-\tau(t)) \leqslant$$
$$\frac{1}{\varepsilon_3}x^{\mathrm{T}}(t)PA_2A_2^{\mathrm{T}}Px(t)+\varepsilon_3x^{\mathrm{T}}(t-\tau(t))x(t-\tau(t)).$$

Thus,

$$\dot V \leqslant x^{\mathrm{T}}(t)\left\{\begin{matrix} P(A_1+BK)+(A_1+BK)^{\mathrm{T}}P \\ +\frac{1}{\varepsilon_1}PM_1M_1^{\mathrm{T}}P+\varepsilon_1N_1^{\mathrm{T}}N_1+ \\ \frac{1}{\varepsilon_2}PM_2M_2^{\mathrm{T}}P+\frac{1}{\varepsilon^2}PA_2A_2^{\mathrm{T}}P+R \end{matrix}\right\}x(t)+$$
$$x^{\mathrm{T}}(t-\tau(t))\{\varepsilon_2N_2^{\mathrm{T}}N_2+\varepsilon_3I_n-\sigma^2R\}x(t-\tau(t)).$$

Because $P,K,\varepsilon_1,\varepsilon_2,\varepsilon_3,Q,R$ belong to the set $\Omega$, we have,

$$\dot V \leqslant -x^{\mathrm{T}}(t)Qx(t) \leqslant -\lambda_{\min}(Q)\|x(t)\|^2.$$

So, the obtained closed-loop system under the controller $u = Kx(t)$ is quadratic stable for any elements in the set $\Omega$. Then Theory 1 is proved.

**Theory 2** For the uncertain system (1) with assumptions 1,2, and the set $\Omega$ being non-empty, introduce the element in the set $\Omega,P,K,\varepsilon_1,\varepsilon_2,\varepsilon_3,Q,R$, then the following VSC controller is applied to system (1),

$$u = Kx(t)-F_0(x(t),x(t-\tau(t)),t)-(c_0+c_1\|x(t)\|+ c_2\|x(t-\tau(t))\|)\mathrm{sgn}(B^{\mathrm{T}}Px(t)). \qquad (6)$$

Then, the closed-loop systems (1) and (6) are quadratic stable.

Proof Choose Lyapunov function,

$$V = x^{\mathrm{T}}(t)Px(t)+\int_{t-\tau(t)}^{t}x^{\mathrm{T}}(p)Rx(p)\mathrm{d}p.$$

Under controller (6), we may have

$$\dot V \leqslant -\lambda_{\min}(Q)\|x(t)\|^2 +$$
$$2x^{\mathrm{T}}(t)PB(F(x(t),x(t-\tau(t)),t) -$$
$$F_0(x(t),x(t-\tau(t)),t)) -$$
$$2(c_0+c_1\|x(t)\| + c_2\|x(t-\tau(t))\|),$$
$$x^{\mathrm{T}}(t)PB\mathrm{sgn}(B^{\mathrm{T}}Px(t)) \leqslant$$
$$-\lambda_{\min}(Q)\|x(t)\|^2 + 2(c_0+c_1\|x(t)\| +$$
$$c_2\|x(t-\tau(t))\|)\|PBx^{\mathrm{T}}(t)\| -$$
$$2(c_0+c_1\|x(t)\| + c_2\|x(t-\tau(t))\|),$$
$$x^{\mathrm{T}}(t)PB\mathrm{sgn}(B^{\mathrm{T}}Px(t)) =$$
$$-\lambda_{\min}(Q)\|x(t)\|^2.$$

Thus, the closed-loop systems (1) and (6) are quadratic stable.

## 4 Robust controller for uncertain systems with the estimation for the bound-norms of uncertainties

When the parameters $c_0,c_1$ and $c_2$ are unknown, the gains of relay-type terms in the controller may be too big or too small. In the case of too big parameters, the effect of the controller may be blow. On the other hand, the robustness of the control system may be decreased in the case of too small parameters. Thus, some VSC controllers with the estimations of the parameters for the upper norms of the uncertainties are proposed in [10,11]. By applying the results in [10,11], the following result is obtained for the system (1).

**Theory 3** For the linear uncertain time-delay system (1) under assumptions 1,2, the parameters $c_0,c_1,c_2$ being unknown, and the set $\Omega$ being non-empty, introduce the element in the set $\Omega$: $P,K,\varepsilon_1,\varepsilon_2,\varepsilon_3,Q,R$, and use the VSC controller,

$$u =$$
$$Kx(t) - F_0(x(t),x(t-\tau(t)),t) -$$
$$(\hat c_0+\hat c_1\|x(t)\|+\hat c_2\|x(t-\tau(t))\|)\mathrm{sgn}(B^{\mathrm{T}}Px(t)), \qquad (7)$$

where $\hat c_0,\hat c_1,\hat c_2$ are the estimations of $c_0,c_1,c_2$, respectively,

$$\begin{cases}\dot{\hat c} = q_0(-\varphi_0\hat c_0+\|B^{\mathrm{T}}Px(t)\|),\\ \dot{\hat c}_1 = q_1(-\varphi_1\hat c_1+\|B^{\mathrm{T}}Px(t)\|\,\|x(t)\|),\\ \dot{\hat c}_2 = q_2(-\varphi_2\hat c_2+\|B^{\mathrm{T}}Px(t)\|\,\|x(t-\tau(t))\|),\end{cases} \qquad (8)$$

where $q_0,q_1,q_2,\varphi_0,\varphi_1,\varphi_2$ are any positive value ($>0$). Then the closed-loop systems (1),(7) and (8) are uniformly ultimately function,

$$V = x^{\mathrm{T}}Px + q_0^{-1}(\hat c_0-c_0)^2 + q_1^{-1}(\hat c_1-c_1)^2 +$$
$$q_2^{-1}(\hat c_2-c_2)^2 + \int_{t-\tau(t)}^{t}x^{\mathrm{T}}(p)Rx(p)\mathrm{d}p.$$

Under controller (7), we have,

$$\begin{aligned}\dot V \leqslant\ & \\ & -\lambda_{\min}(Q)\|x(t)\|^2 + \\ & 2x^{\mathrm T}(t)PB(F(x(t),x(t-\tau(t)),t) - \\ & F_0(x(t),x(t-\tau(t)),t)) - 2(\hat c_0 + \hat c_1\|x(t)\| + \\ & \hat c_2\|x(t-\tau(t))\|)x^{\mathrm T}(t)PB\,\mathrm{sgn}(B^{\mathrm T}Px(t)) + \\ & 2q_0^{-1}(\hat c_0 - c_0)\dot{\hat c}_0 + 2q_1^{-1}(\hat c_1 - c_1)\dot{\hat c}_1 + 2q_1^{-1}(\hat c_2 - c_2)\dot{\hat c}_2 \leqslant \\ & -\lambda_{\min}(Q)\|x(t)\|^2 - 2(\hat c_0 + \hat c_1\|x(t)\| + \\ & \hat c_2\|x(t-\tau(t))\|)\|B^{\mathrm T}Px(t)\| + \\ & 2(c_0 + c_1\|x(t)\| + c_2\|x(t-\tau(t))\|)\|B^{\mathrm T}Px(t)\| + \\ & 2q_0^{-1}(\hat c_0 - c_0)\dot{\hat c}_0 + 2q_1^{-1}(\hat c_1 - c_1)\dot{\hat c}_1 + 2q_1^{-1}(\hat c_2 - c_2)\dot{\hat c}_2.\end{aligned}$$

Applying the estimation $\hat c_0, \hat c_1, \hat c_2$ for $c_0, c_1, c_2$, i.e. (8), we have

$$\begin{aligned}\dot V \leqslant & -\lambda_{\min}(Q) - 2\varphi_0(\hat c_0 - c_0)\hat c_0 - \\ & 2\varphi_1(\hat c_1 - c_1)\hat c_1 - 2\varphi_2(\hat c_1 - c_2)\hat c_0.\end{aligned}$$

Considering the facts,

$$\begin{cases} -2(\hat c_0 - c_0)\hat c_0 \leqslant -(\hat c_0 - c_0)^2 + c_0^2, \\ -2(\hat c_1 - c_1)\hat c_1 \leqslant -(\hat c_1 - c_1)^2 + c_1^2, \\ -2(\hat c_2 - c_2)\hat c_2 \leqslant -(\hat c_2 - c_2)^2 + c_2^2. \end{cases}$$

We have

$$\begin{aligned}\dot V \leqslant & -\lambda_{\min}(Q) - \varphi_0(\hat c_0 - c_0)^2 - \varphi_1(\hat c_1 - c_1)^2 - \\ & \varphi_2(\hat c_2 - c_2)^2 + \varphi_0 c_0^2 + \varphi_1 c_1^2 + \varphi_2 c_2^2 \leqslant \\ & -\min(\lambda_{\min}(Q), \varphi_0, \varphi_1, \varphi_2)\|Y\| + \Delta,\end{aligned}$$

where

$$Y^{\mathrm T} = (x^{\mathrm T}, \hat c_0 - c_0, \hat c_1 - c_1, \hat c_2 - c_2),$$

$$\Delta = \varphi_0 c_0^2 + \varphi_1 c_1^2 + \varphi_2 c_2^2.$$

Based on the fact $\Delta > 0$ and considering the definition and judgement of uniformly ultimate bound in [12,13], the closed-loop systems (1), (7) and (8) are uniformly ultimately bouned. This theory is proved.

## 5 Improved Algorithm

The sign functions in controllers (7) and (8) are the sources of chatting in the controller signals, thus the approximate VSC algorithm[11] is used in our study. The improved controller algorithm may be described as follows:

Let

$$B^{\mathrm T}Px(t) = [s_1 \ \cdots \ s_m]^{\mathrm T}, \zeta > 0,$$

$$\begin{aligned}u = & Kx(t) - F_0(x(t),x(t-\tau(t)),t) - \\ & \rho_0(\hat c_0 + \hat c_1\|x(t)\| + \hat c_2\|x(t - \\ & \tau(t))\|)\mathrm{ch}(B^{\mathrm T}Px(t)),\end{aligned} \tag{9}$$

$$\dot{\hat c}_0 = q_0(-\varphi_0\hat c_0 + \rho_0 B^{\mathrm T}Px(t)\mathrm{ch}(B^{\mathrm T}Px(t)),$$

$$\dot{\hat c}_1 = q_1(-\varphi_1\hat c_1 + \rho_0\|x(t)\|B^{\mathrm T}Px(t)\mathrm{ch}(B^{\mathrm T}Px(t)), \tag{10}$$

$$\begin{aligned}\dot{\hat c}_2 = & q_2(-\varphi_2\hat c_2 + \rho_0\|x(t - \\ & \tau(t))\|B^{\mathrm T}Px(t)\mathrm{ch}(B^{\mathrm T}Px(t)),\end{aligned}$$

where

$$\mathrm{ch}(S) = \begin{cases} \begin{bmatrix} \dfrac{1-\exp(-\mu s_1)}{1+\exp(-\mu s_1)} \\ \vdots \\ \dfrac{1-\exp(-\mu s_m)}{1+\exp(-\mu s_m)} \end{bmatrix}, & \|S\| < \zeta, \\ \dfrac{1}{\rho_0}\mathrm{sgn}S, & \|S\| < \zeta. \end{cases}$$

The uniformly ultimate bound of the closed loop systems (1), (9) and (10) may be proved in the similar way of the proof for the Theory 3.

**Remark 1**

$$-(\hat c_0 + \hat c_1\|x(t)\| + \hat c_2\|x(t-\tau(t))\|)\mathrm{sgn}(B^{\mathrm T}Px(t))$$

and

$$-\rho_0(\hat c_0 + \hat c_1\|x(t)\| + \hat c_2\|x(t-\tau(t))\|)\mathrm{ch}(B^{\mathrm T}Px(t))$$

are called the VSC adaptive term fo (7) and (9), respectively. The controller

$$u_{\mathrm{nvsc}} = Kx(t) - F_0(x(t),x(t-\tau(t)),t)$$

is called the controller without VSC for (7) and (9).

## 6 Calculation Examples

Consider system (1) with the following matrices:

$$A_1 = \begin{bmatrix} -2 & 8 & 0 \\ -2 & -8 & 2 \\ -2 & -8 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5 & 1.5 & 0 \\ 0 & 0.5 & -1.5 \\ -1 & 0 & 0 \end{bmatrix},$$

$$M_1 = M_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},$$

$$N_1 = 0.1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, N_2 = 0.1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\begin{aligned} & F(x(t),x(t-\tau(t)),t) = \\ & -5\sin t + 0.2x_1(t) + 0.6x_2(t-\tau), \\ & \tau(t) = 3(s). \end{aligned}$$

Let

$F_0(x(t), x(t-\tau(t)), t) = 0, Q = 0.5I_3, R = I_3,$

By solving LMI (5), we have,

$$P = \begin{bmatrix} 0.6229 & 0.7996 & 0.3720 \\ 0.7996 & 2.3286 & 0.6065 \\ 0.3720 & 0.6065 & 0.7718 \end{bmatrix},$$

$$\varepsilon_1 = 2.5133,\ \varepsilon_2 = 2.4998,\ \varepsilon_2 = 0.7741$$

$$K = [-1.1950 \quad -4.2404 \quad -3.0914],$$

$$s = [0.3720 \quad 0.6065 \quad 0.7718]x.$$

Furthermore, let

$$q_0 = 0.5,\ q_1 = 0.0025,\ q_2 = 0.0025,$$

$$\varphi_0 = 1,\ \varphi_1 = \varphi_2 = 2,$$

the robust adaptive controller should be

$$u = Kx(t) - (\hat{c}_0 + \hat{c}_1 \| x(t) \| + \hat{c}_2 \| x(t-\tau(t)) \|)\mathrm{sgn}(s),$$

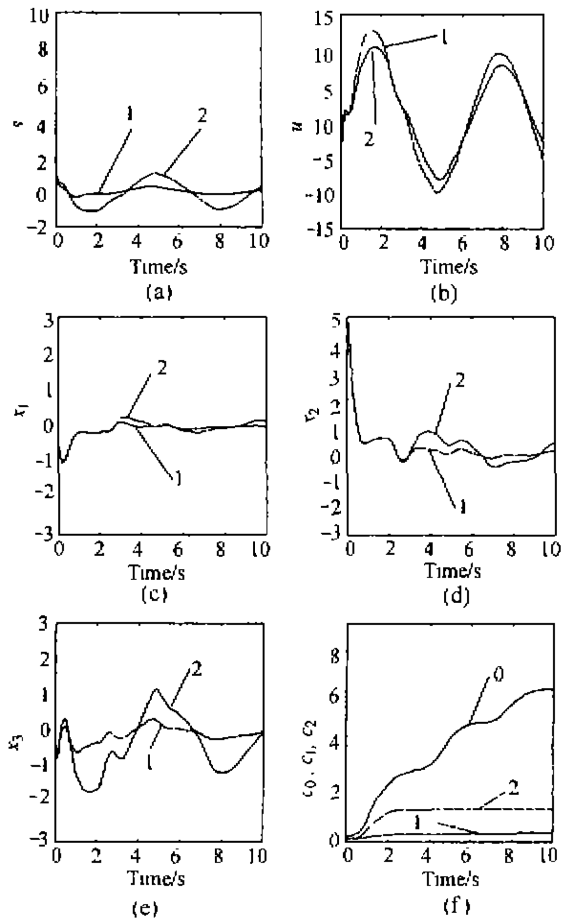
$$\dot{\hat{c}}_0 = q_0(-\varphi_0\hat{c}_0 + \| s \|),$$

$$\dot{\hat{c}}_1 = q_1(-\varphi_1\hat{c}_1 + \| s \| \| x(t) \|),$$

$$\dot{\hat{c}}_2 = q_2(-\varphi_2\hat{c}_2 + \| s \| \| x(t-\tau(t)) \|).$$

In the case of the existence of the disturbance,

$$D(x) = \mathrm{diag}(0.1, 0.1, 0.1)x(t) + [0.1 \quad -0.2 \quad 0.1]^{\mathrm{T}}.$$



1—without VSC, 2—with VSC adaptive term

Fig.1 Simulation results

Fig. 1 (a) ~ (f) show the simulation results for the controllers without VSC and with the adaptive terms, respectively. It shows that the control performance is improved greatly by applying the adaptive term in the controller.

## 7 Conclusion

By applying LMI approach, VSC method and adaptive control theory, a robust controller, a VSC robust controller and a robust VSC adaptive controller are proposed for a class of linear uncertain time-delay systems. The feasible and the effectiveness of our results are demonstrated by a calculation example and its simulation. Our main conclusions are as follows:

1) The designed VSC controller need not satisfy the matching condition of the ordinary VSC controller by using VSC method and LMI approach, this controller can eliminate the actions of the mismatching uncertainties and the matching ones.

2) By applying adaptive control and the estimation for the parameters of the upper norms of the uncertainties, the effect of the controller should be improved.

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tions.

The stability issue of DDP-PTD is an important topic for further investigation. Future work should first introduce appropriate notion of $(A, B)$-controllability subspace, then apply it to the disturbance decoupling problem with stability for system (1).

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