Article ID: 1000-8152(2001)03-0453-03

Robust H_∞ Control of Discrete Time-Delay Linear Systems with Uncertainties in State and Input

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Abstract: We address the problems of robust H_∞ control for a class of uncertain discrete time-delay systems. Using the linear matrix inequality (LMI) approach, both the necessary and sufficient condition for the existence of robust stabilizable controller and a necessary condition for the existence of robust stabilizable with a disturbance attenuation controller were derived. Two types of memoryless state-feedback controller have been designed in terms of the solutions of such LMIs respectively.

Key words: discrete systems; robust stability; LMI; time-delay systems; uncertainty; H_∞ performance

Document code: A

不确定离散时滞线性系统的鲁棒 H_∞ 控制研究

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摘要: 针对一类具有范数有界时变不确定性离散时滞系统,研究其二次稳定化及鲁棒 H_∞ 控制问题.基于线性矩阵不等式方法,推导出了该系统二次稳定的充分必要条件及确保 H_∞ 范数性能的充分条件.利用求解所导出的线性矩阵不等式的可行解,分别构造出这两种无记忆状态反馈控制器.

关键词: 离散系统; 鲁棒稳定; 线性矩阵不等式; 时滞系统; 不确定性; H_∞ 性能

1 Introduction

The time delay is a major source of instability and encountered in various engineering systems. The stabilization and robust H_∞ problems of these systems have attracted many researchers (Choi and Chung[1]; Li and de-Souza[2]; Yu, Chu and Su[3]; Kapila and Haddad[4]), but most of them are about continous systems. On the other hand, the problems of robust stability and H_∞ performance for discrete-time systems with parametric uncertainty have received considerable attention (de-Souza, Fu & Xie[5]; Li Y C and Xu X M[6]; Yu[7] and their references). Despite the significant role of time delays in discrete-time systems, little attention has been paid to the class of uncertain discrete-time systems with delays. Robust stable and robust H_∞ control were considered in Song and Kim[8] and Mahmoud[9] by using Lyapunov functional approach, however the discrete Lyaunov equation is connected in nonlinearity with system state matrices, it leads to more difficult to deal with the uncertainties of discrete systems.

In this paper, for a class of uncertain discrete time-delay systems, the conditions of robust stabilization with a disturbance attenuation γ are investigated. The memoryless state-feedback controllers have been constructed by solving LMIs using the existing efficient convex optimization techniques.

2 Problem statement

We consider a class of uncertain time-delay systems represented by

$$x(k+1) = (A+\Delta A(k))x(k)+A_1x(k-d) + (B+\Delta B(k))u(k) + G\omega(k), \quad (1)$$

$$z(k) = Cx(k), \quad (2)$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $\omega(k) \in \mathbb{R}^p$ is the disturbance input which belongs to $l_2[0,\infty)$, $z(k) \in \mathbb{R}^q$ is the controlled output, d is a known positive integer representing the amount of delay. The time-varying uncertainties $\Delta A(k), \Delta B(k)$ are assumed norm-bounded matrices with

Received date: 2000-07-07; Revised date: 2001-01-02.

$$[\Delta A(k)\ \Delta B(k)] = DF(k)[E_1\ E_2], \tag{3}$$

where D, E_1, E_2 are known constant matrices and the properly dimensioned matrix $F(k)$ is unknown but norm-bounded as

$$F^{\mathrm{T}}(k)F(k) \leqslant I,\ \forall k. \tag{4}$$

In this paper, suppose (A, B) is controllable and the states of system can be measured directly. We will design state feedback control law for system (1) ~ (3) such that the closed-loop system is asymptotically stable and under zero initial conditions, the closed-loop system is robustly stable and guarantees that $\| z(k) \|_2 < \gamma \| \omega(k) \|_2$, $\forall 0 \neq \omega(k) \in l_2[0, \infty)$ independent of delay.

Before main results, we give some lemmas.

Lemma 1[3] Given any constant square matrix A, if there exists a matrix $0 < P = P^{\mathrm{T}}$ such that $A^{\mathrm{T}}PA - P < 0$ holds if and only if there exists a matrix $X > 0$ satisfying

$$\begin{bmatrix} -X & AX \\ XA^{\mathrm{T}} & -X \end{bmatrix} < 0.$$

Lemma 2[2] Given constant matrices W, D, E where $W = W^{\mathrm{T}}$, then

$$W + E^{\mathrm{T}}F^{\mathrm{T}}(k)D^{\mathrm{T}} + DF(k)E < 0,$$

where $F(k)$ satisfied $F^{\mathrm{T}}(k)F(k) \leqslant I$, if and only if for some $\varepsilon > 0$, $W + \varepsilon^{-1}E^{\mathrm{T}}E + \varepsilon DD^{\mathrm{T}} < 0$.

Lemma 3[1] Given constant matrices M, L, W of appropriate dimensions where W is symmetric, for $d > 0, \theta \in \mathbb{R}$, then

$$W + M^{\mathrm{T}}L\mathrm{e}^{\mathrm{j}\theta d} + L^{\mathrm{T}}M\mathrm{e}^{-\mathrm{j}\theta d} < 0,$$

if and only if for a matrix $0 < Q = Q^{\mathrm{T}}$,

$$W + M^{\mathrm{T}}QM + L^{\mathrm{T}}Q^{-1}L < 0.$$

3 Robust stabilization

We firstly consider the unforced disturbance-free portion of the actual system

$$x(k+1) = (A + \Delta A(k))x(k) + A_1x(k-d) + (B + \Delta B(k))u(k). \tag{5}$$

We are concerned with designing a memoryless linear state feedback control law

$$u(k) = K_1x(k), \tag{6}$$

such that the resulting closed-loop system

$$x(k+1) = [A + \Delta A(k) + (B + \Delta B(k))K_1]x(k) + A_1x(k-d) = [A + \Delta A + (B + \Delta B)K_1 + A_1z^{-d}]x(k) \tag{7}$$

is quadratically stable for all admissible uncertainties, where $z = \mathrm{e}^{\mathrm{j}\theta}, \theta \in [0, 2\pi]$.

Definition 1 The system $x(k) = \bar{A}x(k)$ is said to be quadratically stabilizable if the difference of the Lyapunov function $V_k(x) = x^{\mathrm{T}}(k)Px(k)$ satisfies

$$\Delta V = V_{k+1} - V_k = x^{\mathrm{T}}(k)(\bar{A}^{\mathrm{T}}P\bar{A} - P)x(k) \leqslant -\alpha \| x(k) \|^2,$$

for all $x(k) \in \mathbb{R}^n$, where $0 < P = P^{\mathrm{T}}$, $\alpha > 0$.

Theorem 1 Given $\varepsilon > 0$ and $0 < Q = Q^{\mathrm{T}}$, system (7) is quadratically stabilizable if and only if there exist matrix K_1 and symmetric matrix $X > 0$ such that

$$\begin{bmatrix} \varepsilon DD^{\mathrm{T}} + A_1QA_1^{\mathrm{T}} - X & (A + BK_1)X & 0 \\ X(A + BK_1)^{\mathrm{T}} & -X + XQX & X(E_1 + E_2K_1)^{\mathrm{T}} \\ 0 & (E_1 + E_2K_1)X & -\varepsilon I \end{bmatrix} < 0 \tag{8}$$

has a stabilizing solution $X > 0$ and K_1, then $u(k) = K_1x(k)$ is the desired control law.

Proof By Definition 1, (7) is robustly stable if

$$S^{\mathrm{T}}PS - P < 0, \tag{9}$$

where

$$S = A + \Delta A + (B + \Delta B)K_1 + z^{-d}A_1.$$

By Lemma 1, (9) holds if and only if

$$\begin{bmatrix} -X & (H + A_1\mathrm{e}^{-\mathrm{j}\theta d})X \\ X(H^{\mathrm{T}} + A_1^{\mathrm{T}}\mathrm{e}^{\mathrm{j}\theta d}) & X \end{bmatrix} < 0, \tag{10}$$

where

$$H = A + BK_1 + DF(k)(E_1 + E_2K_1).$$

Definite

$$W = \begin{bmatrix} -X & (A + BK_1)X \\ X(A + BK_1)^{\mathrm{T}} & -X \end{bmatrix}.$$

It follows by Schur complement and Lemma 2, 3, for some $\varepsilon > 0$ and $0 < Q = Q^{\mathrm{T}}$, we have

$$\begin{bmatrix} M - X & (A + BK_1)X \\ X(A + BK_1)^{\mathrm{T}} & XNX - X \end{bmatrix} < 0, \tag{11}$$

where

$$M = \varepsilon DD^{\mathrm{T}} + A_1Q^{-1}A_1^{\mathrm{T}},$$

$$N = \varepsilon^{-1}(E_1 + E_2K_1)^{\mathrm{T}}(E_1 + E_2K_1) + Q.$$

Using again Shur complement (11) is equivalent to (9).

Theorem 2 Given a scalar $\varepsilon > 0$ and a matrix $0 < Q = Q^{\mathrm{T}}$, system (7) is quadratically stabilizable if and only if there exist matrix Y and matrix $X > 0$ such that

$$\begin{bmatrix} M-X & AX+BY & 0 & 0 \\ (AX+BY)^{\mathrm{T}} & -X & (E_1X+E_2Y)^{\mathrm{T}} & X \\ 0 & E_1X+E_2Y & -\varepsilon I & 0 \\ 0 & X & 0 & -Q^{-1} \end{bmatrix} < 0. \tag{12}$$

Moreover, a suitable control law is given by $u(k) = YX^{-1}x(k)$.

Proof By Schur complement, (8) can become

$$\begin{bmatrix} M-X & AX+BK_1X & 0 & 0 \\ (AX+BK_1X)^{\mathrm{T}} & -X & (E_1X+E_2K_1X)^{\mathrm{T}} & X \\ 0 & E_1X+E_2K_1X & -\varepsilon I & 0 \\ 0 & X & 0 & -Q^{-1} \end{bmatrix} < 0. \tag{13}$$

Let $Y = KX$, from Theorem 1 we obtain (12).

4 Robust H_∞ control

We are concerned with designing a memoryless linear state feedback control law

$$u(k) = K_2x(k), \tag{14}$$

such that the resulting closed-loop system

$$x(k+1) = [A + \Delta A(k) + (B + \Delta B(k))K_2]x(k) + A_1x(k-d) + G\omega(k) = \bar{A}x(k) + G\omega(k), \tag{15}$$

$$z(k) = Cx(k) \tag{16}$$

is quadratically stabilizable with a disturbance attenuation γ. where

$\bar{A} = A + \Delta A(k) + (B + \Delta B(k))K_2 + A_1z^{-d}$,

$z = e^{j\theta}$, $\theta \in [0, 2\pi]$.

Lemma 4 Let $G(z)$ be a real rational transfer function matrix with nonminimal realization $G(z) = C(zI - A)^{-1}G$. The following statements are equivalent:

a) A is Schur-stable and $\| C(zI - A)^{-1}G \|_\infty < \gamma$;

b) There exists a matrix $0 < P = P^{\mathrm{T}}$ satisfying $\gamma^2 I - G^{\mathrm{T}}PG > 0$ such that the inequality

$$A^{\mathrm{T}}PA - P + A^{\mathrm{T}}PG(\gamma^2 I - G^{\mathrm{T}}PG)^{-1}G^{\mathrm{T}}PA + C^{\mathrm{T}}C < 0.$$

Theorem 3 Given a scalar $\gamma > 0$, system (15), (16) is robustly stable with a disturbance attenuation γ if there exist a scalar $\varepsilon > 0$ and a matrix $0 < Q = Q^{\mathrm{T}}$ such that

$$\begin{bmatrix} M + \gamma^{-2}GG^{\mathrm{T}} - X & (A+BK_2)X & 0 \\ X(A+BK_2)^{\mathrm{T}} & -X + X(C+Q)X & X(E_1+E_2K_2)^{\mathrm{T}} \\ 0 & (E_1+E_2K_2)X & -\varepsilon I \end{bmatrix} < 0 \tag{17}$$

has a stabilizing solution $X^{\mathrm{T}} = X > 0$, then $u(k) = K_2x(k)$ is the desired control law.

Proof By Lemma 4, system (15), (16) is robustly stable with H_∞ performance γ if

$$\bar{A}^{\mathrm{T}}P\bar{A} - P + \bar{A}^{\mathrm{T}}PG(\gamma^2 I - G^{\mathrm{T}}PG)^{-1}G^{\mathrm{T}}P\bar{A} + C^{\mathrm{T}}C < 0. \tag{18}$$

Using Schur complement and Lemma 1,3 and the application of the same technique as Theorem 1, the proof is completed.

Theorem 4 Given a scalar $\gamma > 0$, system (15), (16) is robustly stable with a disturbance attenuation γ if there exist a scalar $\varepsilon > 0$ and a matrix $0 < Q = Q^{\mathrm{T}}$ such that

$$\begin{bmatrix} M + \gamma^{-2}GG^{\mathrm{T}} - X & AX + BZ & 0 & 0 & (AX+BZ)^{\mathrm{T}} \\ 0 & E_1X + E_2Z & -\varepsilon I & 0 & 0 \\ 0 & X & 0 & -Q^{-1} & 0 \\ 0 & CX & 0 & 0 & -I \end{bmatrix} < 0. \tag{19}$$

Moreover, a suitable control law is given by $u(k) = ZX^{-1}x(k)$.

Proof The proof can be completed in a similar way to Theorem 2.

5 Conclusions

For a class of uncertain discrete time-delay systems, this paper has provided conditions such that the closed-loop system is robustly stable with a prescribed H_∞ performance. Two types of memoryless state-feedback controller have been designed by solving certain LMIs.

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bination operator and gene leap operator continually produce new schemata, while selection operator, on one hand, maintains the excellent schemata with high fitness, but on the other hand, falls into disuse bad schemata with low fitness. Similarly with TGA, through genetic operator processing schemata, the individuals in the population continually move towards the optimal individual in PGA, finally the optimal solution can be gained.

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