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Global Regulation for Nonlinear Systems *

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Abstract: We consider the global regulation problem of nonlinear systems. A new definition of the global regulation for nonlinear systems is given. The sufficient and necessary conditions for solving the global regulation problem for nonlinear systems are obtained. Finally an example is given.

Key words: global regulation; global asymptotic stability; state feedback; invariant manifold

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非线性系统的全局调节

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摘要: 研究非线性系统的全局调节问题.给出非线性系统全局调节问题的一种新定义.得到非线性系统全局调节问题可解的充分必要条件.最后给出一个示例.

关键词: 全局调节; 全局渐近稳定性; 状态反馈; 不变流形

1 Introduction

In the past decades, many scholars were interested in the tracking and regulation problem for nonlinear systems[1~6]. In the local case, Isidori and Byrnes[1] have considered the regulation problem of the following systems:

$$\begin{cases} \dot{x} = f(x) + g(x)u + p(x)w, \\ \dot{w} = r(w), \\ e = h(x) + q(w). \end{cases} \tag{1}$$

By assuming that the dynamics $\dot{w} = r(w)$ is Poisson stable and $\dot{\bar{x}} = \frac{\partial f}{\partial x}(0)\bar{x} + g(0)u$ is exponentially stabilizable they have obtained that the local regulation problem of (1) is solvable via the state feedback if and only if there are a neighborhood V of the origin and mapplings $\pi(w)$ and $c(w)$ defining on V, such that on the origin V

$$\begin{cases} \frac{\partial \pi}{\partial w} r(w) = f(\pi(w)) + g((w))c(w) + p(\pi(w))w, \\ h(\pi(w)) + q(w) = 0. \end{cases} \tag{2}$$

In the global case, Dayawansa et al[2] have discussed how to make the systems

$$\begin{cases} \dot{x} = \phi(x \cdot y), \\ \dot{y}_i = y_{i+1}, \ i = 1, \cdots, m-1, \\ \dot{y}_m = \alpha(x, y) + \beta(x, y)u \end{cases} \tag{3}$$

track a reference (or exogenous) signal $r(t)$. Assuming that $|r(t)| \leqslant L$, where L is a constant and $|r^{q}(t)| \leqslant \varepsilon$ with $1 \leqslant q \leqslant p$ and $p \geqslant m+1$, ε is sufficiently small, the tracking problem of (3) is solvable by feedback with data $x(t), r(t), \cdots, r^{(p)}(t)$. If the internal flow of the regulator problem of (1) is C^1 flow equivalent to the exosystems flow, then in [7] there are mappings $c(w)$ and $\pi(w)$ such that (2) holds for all w in the global sense. In this paper, we will prove that (2) is the sufficient and necessary conditions for solving the global regulation problem of (1) under suitable hypotheses.

2 Problem statement

Consider the systems:

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$$\begin{cases} \dot{x} = f(x, w, u), \\ \dot{w} = r(w), \\ e = h(x, w), \end{cases} \quad (4)$$

where $x \in \mathbb{R}^n$ is the internal state, $w \in \mathbb{R}^r$ is the external state, u is the control input and e is the controlled or the tracked output error. The functions appeared in (4) are assumed to be of class C^∞ and without loss of generality, assume that

$$f(0,0,0) = 0, r(0) = 0, h(0,0) = 0.$$

The following assumptions are made in this paper.

H1 For any invariant submanifold $M \subset \mathbb{R}^{n+r}$ of (4) $(x, w) \in M$, $\lim\limits_{|(x,w)| \to \infty} h(x, w) = 0$ implies $h(x, w) = 0, \forall (x, w) \in M$, where the invariant submanifold M of (4) means that $\forall (x_0, w_0) \in M$, the solution $(x(t), w(t))$ of (4) with $x(0) = x_0, w(0) = w_0$ satisfies $(x(t), w(t)) \in M, \forall t \geqslant 0$.

Remark 1 If e is considered as the output of the full systems (4), then H1 implies some observability on the invariant submanifold for nonlinear systems in global sense. In fact, H1 shows that it is practically impossible that as the state of the system approaches large on the invariant submanifold M, but its output conversely approaches small. This may be possible only when the state itself on M is unobservable.

H 2 $\dot{x} = f(x, 0, u)$ is globally asymptotically stabilizable via state feedback, i.e. there is a $k(x)$ such that the zero solution of $\dot{x} = f(x, 0, k(x))$ is globally asymptotically stable (GAS).

H 3 Let a subset $L \subset C^1$ such that for any $l \in L$ the solution $(x(t), w(t))$ of Cauchy problem of the following equations

$$\begin{cases} \dot{x} = f(x, w, l(w)), \\ \dot{w} = r(w), \\ x(0) = 0, \ w(0) = w_0 \in \mathbb{R}^r \end{cases} \quad (5)$$

satisfies the following statement: if we denote $\Phi_\mu(t, s)$, $\Psi_\eta(t, s)$ the fundamental matrix solution of the following partial variation equation of (5)

$$\dot{\mu} = \frac{\partial f}{\partial x}(x(t), w(t), l(w(t)))\mu$$

and

$$\dot{\eta} = \frac{\partial r}{\partial w}(w(t))\eta,$$

then

$$|\Phi_\mu(t, s)| \leqslant \gamma_\mu e^{-\beta(t-s)}, \ t \geqslant s \geqslant 0, \quad (6)$$

$$|\Psi_\eta(t, s)| \leqslant \gamma_\eta e^{-\alpha(t-s)}, \ t \geqslant s \geqslant 0, \quad (7)$$

where $\alpha, \beta, \gamma_\mu, \gamma_\eta$ are positive constants which are independent of l.

We can now state our global regulation problem as follows: To find the state feedback law $u = \alpha(x, w)$, $\alpha(0,0) = 0$ and the open set $\Sigma \subset \mathbb{R}^{n+r}$ such that

i) The zero solution of $\dot{x} = f(x, 0, \alpha(x, 0))$ in (4) is GAS and the solution $(x(t), w(t))$ of Cauchy problem of equations (8) below satisfies (6) and (7)

$$\begin{cases} \dot{x} = f(x, w, \alpha(x, w)), \\ \dot{w} = r(w), \\ x(0) = 0, \ w(0) = w_0. \end{cases} \quad (8)$$

ii) $\forall (x_0, w_0) \in \Sigma$ the solution $(x(t), w(t))$ of the closed loop (4) with $x(0) = x_0, w(0) = w_0$ makes $\lim\limits_{t \to \infty} e(t) = \lim\limits_{t \to \infty} h(x(t), w(t)) = 0$, under the feedback $u = \alpha(x, w)$.

3 Main results

At first, we notice that the dynamics of the exosystems $\dot{w} = r(w)$ in (4) is unforced and is uncontrollable. If we denote its flow by $\Phi_t^r(w)$, $\forall w \in \mathbb{R}^r$ and the collection of all w-limit set of $\Phi_t^r(w)$ by Ω, then any bounded solution $w(t)$ of the exosystem follows $w(t) \to \Omega (t \to \infty)$ and Ω is an invariant submanifold of exosystem. Therefore we first obtain

Proposition 3.1 Assume H2 and H3 hold and $\alpha < \beta$. The global regulation problem of (4) is solvable via state feedback only if there exist mappings $c(w) \in L$ and $S(w) \in C^1$ such that

$$\begin{cases} \dfrac{\partial S(w)}{\partial w} r(w) = f(S(w), w, c(w)), & w \in \mathbb{R}^r, \\ h(S(w), w) = 0, & w \in \Omega. \end{cases} \quad (9)$$

Proof If $u = \alpha(x, w) \in C^1$ and the set $\Sigma \subset \mathbb{R}^{n+r}$ such that (4) satisfies i) and ii) (see Section 2). By H2 and Lemma B3.2 and Thm. B4.1 in [8], there is a $S(w) \in C^1$ so that

$$\frac{\partial S(w)}{\partial w} r(w) = f(S(w), w, \alpha(S(w), w)),$$

$w \in \mathbb{R}^r$, and $\forall (x_0, w_0) \in \mathcal{N}(\varepsilon, \sigma')$, (where $\mathcal{N}(\varepsilon, \sigma')$ is defined as in [8]), $|x(t) - S(w(t))| \to 0 (t \to \infty)$. Obviously, $c(w) = \alpha(S(w), w) \in L$. i.e. the first form of (9) follows. Now we proved that $h(S(w), w) = 0, \forall W \in \Omega$. Since $\lim\limits_{t \to \infty} h(x(t), w(t)) = 0, \forall (x_0, w_0) \in \mathcal{N}(\varepsilon, \sigma')$, if $w(t)$ is bounded, then $w(t) \to \Omega (t \to \infty)$ and there exist $\bar{w} \in \Omega$ and $\{t_k\}, t_k \to \infty (k \to \infty)$, such that $w(t_k) \to \bar{w}(k \to \infty)$. Thus $S(w(t_k)) \to S(\bar{w})(k \to \infty)$ by the continuity of S. Therefore,

$$\lim_{t\to\infty} x(t_k) = \lim_{t\to\infty}(x(t_k) - Sw(t_k)) + \lim_{k\to\infty} S(w(t_k)) = S(\bar{w}).$$

Consequently, $h(S(\bar{w}), \bar{w}) = 0$. Conversely, $\forall w_0 \in \Omega$, there must exist a bounded trajectory $w(t)$ and $\{t_k\}, t_k \to \infty (k \to \infty)$, such that $w(t_k) \to w_0, (k \to \infty)$. It is implied that $h(S(w_0), w_0) = 0$. According to the arbitrarity of w_0, we know that the second form of (9) follows.

Theroem 3.1 Suppose H1 and H3 hold. The global regulation problem of (4) is solvable if and only if there exist $S \in C^1, c \in L, S(0) = 0, c(0) = 0$, such that (9) is valid.

Proof From Proposition 3.1 the necessity is valid. Now we prove the sufficiency. Suppose (9) holds and by H2 there is a $k(x)$ such that the zero solution of $\dot{x} = f(x, 0, k(x))$ is GAS. Let

$$\alpha(x, w) = c(w) + k(x - S(w)),$$

then $\forall (x_0, w_0) \in \mathcal{N}(\varepsilon\sigma')$, from Lemma B3.2 and Thm. B 4.1 in [8], the solution $(x(t), w(t))$ of (4) with $x(0) = x_0, w(0) = w_0$ satisfies $\lim_{t\to\infty} | x(t) - S(w(t)) | = 0$, where we take $\Sigma = \mathcal{N}(\varepsilon\sigma')$. By H1 it only needs to prove that $\lim_{t\to\infty} e(t) = 0$ for the bounded trajectory $w(t)$. If $w(t)$ is bounded, by the continuity of S and $\lim_{t\to\infty} | x(t) - S(w(t)) | = 0$, then $x(t)$ is also bounded. Therefore, the differentiability of h implies that there exists a constant K such that

$$| h(x(t), w(t)) - h(S(w(t)), w(t)) | \leq K | x(t) - S(w(t)) |,$$

and

$$\begin{aligned} &| h(x(t), w(t)) | \leq \\ &| h(x(t), w(t)) - h(S(w(t)), w(t)) | + \\ &| h(S(w(t)), w(t)) | \leq \\ &K | x(t) - S(w(t)) | + | h(S(w(t)), w(t)) |. \end{aligned} \tag{10}$$

Thus in accordance with (9) and (10) and $w(t) \to \Omega(t \to \infty)$ and the continuity of h we gain that $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} h(x(t), w(t)) = 0$.

For the local case, according to Theroem 3.1 we can obtain the more extended result than that of in [1]. i.e. if denote $A = \frac{\partial f}{\partial x}(0,0,0), B = \frac{\partial f}{\partial u}(0,0,0)$, then

Corollary 3.1 Suppose (A, B) be stabilizable. If the dynamics of w is locally stable (not asymptotically stable), then the local regulation problem of (4) is solvable if and only if there are a neighborhood $V \in \mathbb{R}^r$ of the origin $(w = 0)$ and the mappings $\pi(w), c(w) \in C^1, \pi(0) = 0, c(0) = 0$ such that

$$\begin{cases} \frac{\partial \pi(w)}{\partial w} r(w) = f(\pi(w), w, c(w)), \ \forall w \in V, \\ h(\pi(w), w) = 0, \ \forall w \in \Omega \cap V. \end{cases} \tag{11}$$

Example Consider the nonlinear system

$$\begin{cases} \dot{x} = A_1 x + B_1 u_1 + B_2 x u_2 + P(x, w), \\ \dot{w} = A_2 w + \prod_{i=0}^{l} (| w |^2 - r_i^2) w, \\ e = h(x, w) \in \mathbb{R}, \end{cases} \tag{12}$$

where

$$(x^T w^T) = (x_1\ x_2\ x_3\ w_1\ w_2) \in \mathbb{R}^5,$$

$$r_0 > r_1 > \cdots > r_l > 0,$$

l is an even number.

$$A_1 = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \ B_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \ A_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

$$P(x, w) = (p_1 \quad p_2 \quad p_3)^T$$

is continuous function vector, $P(x, 0) \equiv 0, \forall x$ and $\max_{(x,w)\in\mathbb{R}^5} | P | \leq r_0^3$. h is a cotinuous function, $h(0,0) = 0$ and $\frac{\partial h}{\partial x_1}(x, w) \neq 0, \forall (x, w) \in \mathbb{R}^5$.

Obviously, system (12) does not satisfy the condition in [1] in local case, because its interior system is not locally exponentially stabilizable, but only locally asymptotically stabilizable, and also the exterior system is not Poisson stable. As a matter of fact,

$$\forall w_0, | w_0 | < r_1,$$

$$w(t) \to \{w: | w | = r_1\}, (t \to \infty),$$

and

$$\forall w_0 \in \mathbb{R}^2,$$

$$w(t) \to \{w: | w | \leq r_0\}, (t \to +\infty),$$

the origin $(w = 0)$ is not stable. Since $\frac{\partial h}{\partial x_1}(x, w) \neq 0$, the coordinate transformation

$$\psi:(x^{\mathrm{T}},w^{\mathrm{T}})^{\mathrm{T}} \mapsto (e,x_2,x_3,w_1,w_2)^{\mathrm{T}}$$

is a local differomorphism.

From the inversability of ψ there is a function α such that in inverse transformation

$$\psi^{-1}:(e,x_2,x_3,w_1,w_2) \mapsto (x,w),$$
$$x_1 = \alpha(e,x_2,x_3,w).$$

Let

$$u_2 = -(x_2^2 + x_3^2),$$
$$u_1 = \alpha - 2x_3 - p_1(x,w) - \left(\frac{\partial h}{\partial x_1}\right)^{-1}\Big[\frac{\partial h}{\partial x_2}(x_3 + x_2u_2 + p_2(x,w)) + \frac{\partial h}{\partial x_3}(-x_2 + x_3u_2 + p_3(x,w)) + \frac{\partial h}{\partial w}\dot{w} + e\Big],$$

then the closed-loop system is

$$\begin{cases} \dot{e} = -e, \\ \dot{x}_2 = x_3 - x_2(x_2^2 + x_3^2) + \bar{p}_2(e,x_2,x_3,w), \\ \dot{x}_3 = -x_2 - x_3(x_2^2 + x_3^2) + \bar{p}_3(e,x_2,x_3,w), \\ \dot{w} = A_2w + \prod_{i=0}^{l}(|w|^2 - r_i^2)w, \end{cases} \tag{13}$$

where $\bar{p}_i = p_i(\alpha(e,x_2,x_3,w),x_2,x_3,w), i = 2,3$. Therefore, $\forall e_0 \in \mathbb{R}$ we have $|e(t)| \leqslant |e_0|$ and $\lim\limits_{t\to\infty} e(t) = 0$. Since $\forall w_0 \in \Omega_i = \{w: |w| = r_i\}$, $w(t) \in \Omega_i, i = 0,1,\cdots,l$, there must be a mapping S such that $h(S(w),w) = 0, \forall w \in \Omega_i, i = 0,1,\cdots,l$. And from system (12) we have $\Omega = \sum_{i=0}^{l}\Omega_i$, where Ω is the w-limit set of the dynamics w. Therefore, the mapping c also exists. So by Thm. 3.1 the global regulation problem of system (12) is solvable.

4 Conclusion

In this paper, the global regulation problem for general nonlinear systems is discussed. The new definition of the global regulation problem for nonlinear systems is given. The conditions of solvability for global regulation problem of nonlinear systems are obtained by using the invariant manifold theory. In local case, our results extended from that of [1,4]. Finally, one case (tracking a constant signal) in [2] is considered as a particular example in this paper, according to our result the conditions of the solvability are easily obtained and another example is given.

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