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Dissipative Control for Linear Time-Delay Systems

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Abstract: This paper focuses on a class of linear time-delay systems. We are concerned with the design of dissipative static state feedback and dynamic output feedback controllers such that the closed-loop system is quadratically stable and strictly (Q,S,R)-dissipative. Sufficient conditions for the existence of the quadratic dissipative controllers are obtained by using a linear matrix inequality (LMI) approach. Furthermore, we provide a procedure of constructing such controllers from the solutions of LMIs. It is shown that the solvability of dissipative controller design problem is implied by the feasibility of LMIs. The main result of this paper unify the existing results on H_∞ control and passive control for linear time-delay systems.

Key words: dissipative control; time-delay systems; linear matrix inequalities

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线性时滞系统的耗散控制

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摘要: 研究了一类线性时滞系统的二次耗散控制问题,基于线性矩阵不等式(LMI)方法导出了耗散控制器存在的充分条件.通过线性矩阵不等式的可行解构造出耗散态状态反馈和动态输出反馈控制律,相应的闭环系统是二次稳定和严格(Q,S,R)耗散的.本文的主要贡献是统一了线性时滞系统现有的 H_∞ 控制和无源控制结果.

关键词: 耗散控制; 时滞系统; 线性矩阵不等式

1 Introduction

Since the notion of a dissipative dynamical system was introduced by Willems[1], it has played a very important role in system, circuit, network and control engineering and theory. Dissipativeness is a generalization of the passivity in electrical networks and other dynamical systems which dissipate energy in some abstract sense. Applications of dissipativeness in the stability analysis of linear systems with certain nonlinear feedback were first discussed in [1,2]. Subsequently, dissipativeness was crucially used in the stability analysis of nonlinear systems[3,4]. The theory of dissipative systems generalizes basic tools including the passivity theorem, bounded real lemma, Kalman-Yakubovich lemma and the circle criterion.

In the past decade, analysis and synthesis of H_∞ and the passive (or positive real) control of time-delay systems have received remarkable attention[5-8]. In H_∞ control, the small-gain theorem is used to ensure robust stability by requiring that the loop-gain should be less than one at any frequencies by ignoring the phase information. On the other hand, phase information is widely used in the analysis of passive control systems based on positivity theory. In the positivity theorem, when a (strict) positive real system is connected to a positive real plant in a negative-feedback configuration, the (strict) positive real system has its phase less than 90° so that the closed-loop system is stable. But the loop-gain is not used in guaranteeing the stability. Clearly, both the small-gain and positivity theorems deal with gain and phase performances separately and thus may lead to conservative results in applications. An early attempt was reported in [9] to synthesize a controller that achieves desired gain and phase margins by using state feedback. Dissipativeness provides an appropriate framework[10] for a less conservative robust controller design, especially in the applications where both gain and phase performances are considered. This paper is concerned with the probl-

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em of quadratic dissipative control for linear systems with delays. Namely, we design linear static state feedback and linear dynamic output feedback control laws to simultaneously achieve quadratic stability and strict quadratic dissipativeness.

In this paper, we shall use the following notation

$$\langle u,v\rangle_T = \int_0^T u^{\mathrm{T}} v \mathrm{d}t, u,v \in \mathbb{R}^n, T \geqslant 0.$$

2 Problem formulation and preliminaries

Consider the linear time-delay system described by state-space equations of the form

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + B_1 w(t), \\ z(t) = C_1 x(t) + C_{1d} x(t-d) + D_{11} w(t), \\ x(t) = \eta(t), t \in [-d, 0], \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $z(t) \in \mathbb{R}^q$ is the output, $w(t) \in \mathbb{R}^p$ is the exogenous input which belongs to $L_2[0,\infty)$, $d > 0$ is a delay constant, $\eta(t)$ is the initial state vector, and A, A_d, B_1, C_1, C_{1d} and D_{11} are constant matrices with appropriate dimensions.

The quadratic energy supply function E[11] associated with system (1) is defined by

$$E(w,z,T) = \langle z, Qz\rangle_T + 2\langle z, Sw\rangle_T + \langle w, Rw\rangle_T, \quad (2)$$

where Q, S and R are real matrices of appropriate dimensions with Q and R symmetric.

Definition 1 The system (1) is said to be strictly (Q,S,R)-dissipative, if for any $T \geqslant 0$ and some scalar $\alpha > 0$, under zero initial state, the following condition is satisfied

$$E(w,z,T) \geqslant \alpha\langle w,w\rangle_T. \quad (3)$$

Remark 1 The above performance of strict (Q,S,R)-dissipativity includes H_∞ and passivity as special cases.

a) When $Q = -I, S = 0$, and $R = \gamma^2 I$, strict (Q, S, R)-dissipativity (3) reduces to an H_∞ performance requirement[5,6].

b) When $Q = 0, S = I$, and $R = 0$, (3) corresponds to a strict passive problem[8].

Without the loss of generality, we make the following assumption.

Assumption 1 $Q_- = -Q \geqslant 0$.

Remark 2 It can be observed that Assumption 1 holds for both the cases in Remark 1.

Definition 2 Let the Lyapunov functional for system (1) be

$$L(x,t) = x^{\mathrm{T}} P x + \int_{t-d}^{t} x(\tau)^{\mathrm{T}} V x(\tau) \mathrm{d}\tau, \quad (4)$$

where $0 < P \in \mathbb{R}^{n\times n}$ and $0 < V \in \mathbb{R}^{n\times n}$. If there exists a scalar $\varepsilon > 0$ such that the derivative of the Lyapunov functional (4) with respective to time $t (w = 0)$ satisfies

$$\dot{L}(x,t) \leqslant -\varepsilon \| x \|^2, \quad (5)$$

then the time-delay system (1) is said to be quadratically stable.

Theorem 1 Given matrices Q, S and R with Q and R symmetric, consider system (1) subject to Assumption 1. If there exist matrices $P > 0$ and $V > 0$ such that the following LMI holds

$$J = \begin{bmatrix} PA + A^{\mathrm{T}}P + V & PA_d & PB_1 - C_1^{\mathrm{T}}S & C_1^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ A_d^{\mathrm{T}}P & -V & -C_{1d}^{\mathrm{T}}S & C_{1d}^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ B_1^{\mathrm{T}}P - S^{\mathrm{T}}C_1 & -S^{\mathrm{T}}C_{1d} & -(R + D_{11}^{\mathrm{T}}S + S^{\mathrm{T}}D_{11}) & D_{11}^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ Q_-^{\frac{1}{2}}C_1 & Q_-^{\frac{1}{2}}C_{1d} & Q_-^{\frac{1}{2}}D_{11} & -I \end{bmatrix} < 0, \quad (6)$$

then the time-delay system (1) is quadratically stable and strictly (Q,S,R)-dissipative.

Proof Firstly, it is noticed that the condition (6) implies

$$J_1 = \begin{bmatrix} PA + A^{\mathrm{T}}P + V & PA_d \\ A_d^{\mathrm{T}}P & -V \end{bmatrix} < 0. \quad (7)$$

Taking the derivative of the Lyapunov functional (4) along the solution of Equation (1) yields

$$\dot{L}(x,t) = \dot{x}^{\mathrm{T}}Px + x^{\mathrm{T}}P\dot{x} + x^{\mathrm{T}}Vx - x_d^{\mathrm{T}}Vx_d, \quad (8)$$

where $x_d = x(t-d)$. Assuming that $w = 0$, we have

$$\dot{L}(x,t) = [x^{\mathrm{T}} \quad x_d^{\mathrm{T}}] J_1 [x^{\mathrm{T}} \quad x_d^{\mathrm{T}}]^{\mathrm{T}} \leqslant -\varepsilon \left\| \begin{matrix} x \\ x_d \end{matrix} \right\|^2 \leqslant -\varepsilon \| x \|^2, \quad (9)$$

where $\varepsilon = -\lambda_{\max}(J_1) > 0$, $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of square matrix. From Definition 2, the time-delay system (1) quadratically stable.

Secondly, if (6) holds, there exists some sufficiently small scalar $\alpha > 0$ such that

$$J + \mathrm{diag}\{0, 0, \alpha I, 0\} < 0. \quad (10)$$

Hence it follows that

$$z^{\mathrm{T}}Qz + 2z^{\mathrm{T}}Sw + w^{\mathrm{T}}Rw \geqslant \dot{L}(x,t) + \alpha w^{\mathrm{T}}w. \quad (11)$$

Integrating (11) from 0 to T, under zero initial condition we obtain that

$$E(w,z,T) \geqslant \alpha\langle w,w\rangle_T + L(x(T)T) \geqslant$$

$$\alpha\langle w, w\rangle_T, \tag{12}$$

for all $w \in L_2[0, T]$ and all $T \geqslant 0$. Therefore, by using Definition 1, when the condition (6) satisfies, the time-delay system (1) is quadratically stable and strictly (Q, S, R)-dissipative. This completes the proof.

Let us review the following key lemma to get further results.

Lemma 1[12] Given a symmetric matrix Ω and two matrices Σ and Γ with appropriate dimensions, then there exists a matrix K satisfying

$$\Omega + \Sigma^{\mathrm{T}} K\Gamma + \Gamma^{\mathrm{T}} K^{\mathrm{T}} \Sigma < 0,$$

if and only if

$$\Sigma_{\perp}^{\mathrm{T}} \Omega \Sigma_{\perp} < 0,\ \Gamma_{\perp}^{\mathrm{T}} \Omega \Gamma_{\perp} < 0,$$

where $\Sigma_{\perp}$ and $\Gamma_{\perp}$ denote the orthogonal complements of Σ and Γ respectively.

3 Dissipative control of time-delay systems

Consider the following linear time-delay system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d) + B_1 w(t) + B_2 u(t), \\ z(t) = C_1 x(t) + C_{1d} x(t-d) + D_{11} w(t) + D_{12} u(t), \\ y(t) = C_2 x(t) + C_{2d} x(t-d) + D_{21} w(t), \\ x(t) = \eta(t),\ t \in [-d, 0], \end{cases} \tag{13}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control, $w(t) \in \mathbb{R}^p$ is the exogenous input which belongs to $L_2[0, \infty)$, $y(t) \in \mathbb{R}^r$ is the measured output, $z(t) \in \mathbb{R}^q$ is the controlled output, and $A, A_d, B_1, B_2, C_1, C_{1d}, D_{11}, D_{12}, C_2, C_{2d}$ and D_{21} are known matrices with appropriate dimensions.

1) Dissipative control via state feedback.

The dissipative control problem we address here is stated as follows: Design a state feedback controller

$$u(t) = K_s x(t),\ K_s \in \mathbb{R}^{m\times n}, \tag{14}$$

such that the resulting closed-loop system of (13) is quadratically stable and strictly (Q, S, R)-dissipative.

Theorem 2 Given matrices Q, S and R with Q and R symmetric, consider the system (13) subject to Assumption 1. Then there exists a state feedback controller (14) for system (13) such that the closed-loop system is quadratically stable and strictly (Q, S, R)-dissipative if there exist matrices $X_s > 0$, $Y_s > 0$ and W_s satisfying the following LMI:

$$\Phi = \begin{bmatrix} \Phi_{11} & A_d X_s & \Phi_{12} & \Phi_{13} \\ X_s A_d^{\mathrm{T}} & -Y_s & -X_s C_{1d}^{\mathrm{T}} S & X_s C_{1d}^{\mathrm{T}} Q_-^{\frac{1}{2}} \\ \Phi_{12}^{\mathrm{T}} & -S^{\mathrm{T}} C_{1d} X_s & -(R + D_{11}^{\mathrm{T}} S + S^{\mathrm{T}} D_{11}) & D_{11}^{\mathrm{T}} Q_-^{\frac{1}{2}} \\ \Phi_{13}^{\mathrm{T}} & Q_-^{\frac{1}{2}} C_{1d} X_s & Q_-^{\frac{1}{2}} D_{11} & -I \end{bmatrix} < 0 \tag{15}$$

where

$$\Phi_{11} = AX_s + X_s A^{\mathrm{T}} + B_2 W_s + W_s^{\mathrm{T}} B_2^{\mathrm{T}} + Y_s,$$
$$\Phi_{12} = B_1 - (X_s C_1^{\mathrm{T}} + W_s^{\mathrm{T}} D_{12}^{\mathrm{T}}) S,$$
$$\Phi_{13} = (X_s C_1^{\mathrm{T}} + W_s^{\mathrm{T}} D_{12}^{\mathrm{T}}) Q_-^{\frac{1}{2}}.$$

Furthermore, a suitable controller gain is given by $K_s = W_s X_s^{-1}$.

Proof By applying Theorem 1 and Schur complements[13] to the closed-loop system of (13) with $u = Kx$, through straightforward matrix manipulations the theorem is established.

2) Dissipative control via output feedback.

In this subsection, we will provide an LMI approach to the strictly (Q, S, R)-dissipative control via output feedback.

Let the system (13) be the following dynamic output feedback controller

$$\begin{cases} \dot{x}_K(t) = A_K x_K(t) + B_K y(t), \\ u(t) = C_K x_K(t) + D_K y(t), \end{cases} \tag{16}$$

where $x_K(t) \in \mathbb{R}^{n_K} (0 \leqslant n_K \leqslant n)$, and A_K, B_K, C_K and D_K are appropriate matrices to be determined. The extreme case $n_K = 0$ represents static gain output feedback. By introducing the augmented state vector $\bar{x} = [x^{\mathrm{T}}\ \ x_K^{\mathrm{T}}]^{\mathrm{T}}$, we can obtain the following closed-loop system

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_d x(t-d) + \bar{B} w(t), \\ z(t) = \bar{C}\bar{x}(t) + \bar{C}_d x(t-d) + \bar{D} w(t), \end{cases} \tag{17}$$

where

$$\begin{cases} \bar{A} = \hat{A} + \hat{B}_2 K \hat{C}_2,\ \bar{A}_d = \hat{A}_d + \hat{B}_2 K \hat{C}_{2d}, \\ \bar{B} = \hat{B}_1 + \hat{B}_2 K \hat{D}_{21}, \\ \bar{C} = \hat{C}_1 + \hat{D}_{12} K \hat{C}_2,\ \bar{C}_d = C_{1d} + \hat{D}_{12} K \hat{C}_{2d}, \\ \bar{D} = D_{11} + \hat{D}_{12} K \hat{D}_{21}, \\ K = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix},\ \hat{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix},\ \hat{A}_d = \begin{bmatrix} A_d \\ 0 \end{bmatrix}, \\ \hat{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix},\ \hat{B}_2 = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix}, \\ \hat{C}_2 = \begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix},\ \hat{C}_{2d} = \begin{bmatrix} 0 \\ C_{2d} \end{bmatrix},\ \hat{D}_{21} = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix}, \\ \hat{C}_1^{\mathrm{T}} = \begin{bmatrix} C_1^{\mathrm{T}} \\ 0 \end{bmatrix},\ \hat{D}_{12}^{\mathrm{T}} = \begin{bmatrix} 0 \\ D_{12}^{\mathrm{T}} \end{bmatrix}. \end{cases} \tag{18}$$

Let's define a Lyapunov functional for (17)

$$L(x, x_K, t) = \bar{x}^{\mathrm{T}}\bar{P}\bar{x} + \int_{t-d}^{t} x(\tau)^{\mathrm{T}} V x(\tau)\mathrm{d}\tau, \quad (19)$$

where $0 < \bar{P} \in \mathbb{R}^{(n+n_k)\times(n+n_k)}$ and $0 < V \in \mathbb{R}^{n\times n}$. Using Theorem 1, the closed-loop system (17) is quadratically stable and strictly (Q, S, R)-dissipative if there exist matrices $\bar{P} > 0$ and $V > 0$ such that

$$\Lambda + F^{\mathrm{T}}G^{\mathrm{T}}KH + H^{\mathrm{T}}K^{\mathrm{T}}GF < 0, \quad (20)$$

where

$$\Lambda = \begin{bmatrix} \bar{P}\hat{A}+\hat{A}^{\mathrm{T}}\bar{P}+\bar{V} & \bar{P}\hat{A}_d & \bar{P}\hat{B}_1 - \hat{C}_1^{\mathrm{T}}S & \hat{C}_1^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ \hat{A}_d^{\mathrm{T}}\bar{P} & -V & -C_{1d}^{\mathrm{T}}S & C_{1d}^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ \hat{B}_1^{\mathrm{T}}\bar{P}-S^{\mathrm{T}}\hat{C}_1 & -S^{\mathrm{T}}C_{1d} & -(R+D_{11}^{\mathrm{T}}S+S^{\mathrm{T}}D_{11}) & D_{11}^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ Q_-^{\frac{1}{2}}\hat{C}_1 & Q_-^{\frac{1}{2}}C_{1d} & Q_-^{\frac{1}{2}}D_{11} & -I \end{bmatrix},$$

$$F = \begin{bmatrix} \bar{P} & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \quad G^{\mathrm{T}} = \begin{bmatrix} \hat{B}_2 \\ 0 \\ -S^{\mathrm{T}}\hat{D}_{12} \\ Q_-^{\frac{1}{2}}\hat{D}_{12} \end{bmatrix},$$

$$H^{\mathrm{T}} = \begin{bmatrix} \hat{C}_2^{\mathrm{T}} \\ \hat{C}_{2d}^{\mathrm{T}} \\ \hat{D}_{21}^{\mathrm{T}} \\ 0 \end{bmatrix}, \quad \bar{V} = \mathrm{diag}\{V, 0\}.$$

Hence, by Lemma 1 the inequality (20) is solvable for some K if and only if

$$G_\perp^{\mathrm{T}}(F^{-1}\Lambda F^{-1})G_\perp < 0, \quad (21)$$

$$H_\perp^{\mathrm{T}}\Lambda H_\perp < 0, \quad (22)$$

where $G_\perp$ and $H_\perp$ are the orthogonal complements of G and H, respectively. We can choose $[V_1^{\mathrm{T}} \quad V_2^{\mathrm{T}} \quad V_3^{\mathrm{T}}]^{\mathrm{T}}$ and $[V_4^{\mathrm{T}} \quad V_5^{\mathrm{T}} \quad V_6^{\mathrm{T}}]^{\mathrm{T}}$ which are orthogonal complements of $[B_2^{\mathrm{T}} \quad -D_{12}^{\mathrm{T}}S \quad D_{12}^{\mathrm{T}}Q_-^{\frac{1}{2}}]$ and $[C_2 \quad C_{2d} \quad D_{21}]$ respectively, then

$$G_\perp = \begin{bmatrix} V_1 & 0 \\ 0 & 0 \\ 0 & I \\ V_2 & 0 \\ V_3 & 0 \end{bmatrix}, \quad H_\perp = \begin{bmatrix} V_4 & 0 \\ 0 & 0 \\ V_5 & 0 \\ V_6 & 0 \\ 0 & I \end{bmatrix}. \quad (23)$$

To simplify (21) and (22), we partition $\bar{P}$ and $\bar{P}^{-1}$ as

$$\bar{P} = \begin{bmatrix} Y & N \\ N^{\mathrm{T}} & * \end{bmatrix}, \quad \bar{P}^{-1} = \begin{bmatrix} X & M \\ M^{\mathrm{T}} & * \end{bmatrix}, \quad (24)$$

where $0 < X \in \mathbb{R}^{n\times n}$, $0 < Y \in \mathbb{R}^{n\times n}$, $M \in \mathbb{R}^{n\times n_K}$, $N \in \mathbb{R}^{n\times n_K}$, and $*$ means irrelevant. Inequalities (21) and (22) are simplified to the following LMIs (25) and (26) by straightforward matrix manipulations.

$$\Theta_1^{\mathrm{T}}\begin{bmatrix} AX+XA^{\mathrm{T}} & XC_1^{\mathrm{T}}Q_-^{\frac{1}{2}} & B_1-XC_1^{\mathrm{T}}S & A_d & X \\ Q_-^{\frac{1}{2}}C_1X & -I & Q_-^{\frac{1}{2}}D_{11} & Q_-^{\frac{1}{2}}C_{1d} & 0 \\ B_1^{\mathrm{T}}-S^{\mathrm{T}}C_1X & D_{11}^{\mathrm{T}}Q_-^{\frac{1}{2}} & -\Theta & -S^{\mathrm{T}}C_{1d} & 0 \\ A_d^{\mathrm{T}} & C_{1d}^{\mathrm{T}}Q_-^{\frac{1}{2}} & -C_{1d}^{\mathrm{T}}S & -V & 0 \\ X & 0 & 0 & 0 & -V^{-1} \end{bmatrix}\Theta_1 < 0, \quad (25)$$

$$\Theta_2^{\mathrm{T}}\begin{bmatrix} YA+A^{\mathrm{T}}Y+V & YA_d & YB_1-C_1^{\mathrm{T}}S & C_1^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ A_d^{\mathrm{T}}Y & -V & -C_{1d}^{\mathrm{T}}S & C_{1d}^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ B_1^{\mathrm{T}}Y-S^{\mathrm{T}}C_1 & -S^{\mathrm{T}}C_{1d} & -\Theta & D_{11}^{\mathrm{T}}Q_-^{\frac{1}{2}} \\ Q_-^{\frac{1}{2}}C_1 & Q_-^{\frac{1}{2}}C_{1d} & Q_-^{\frac{1}{2}}D_{11} & -I \end{bmatrix}\Theta_2 < 0, \quad (26)$$

where

$$\Theta = R + D_{11}^{\mathrm{T}}S + S^{\mathrm{T}}D_{11},$$

$$\Theta_1 = \begin{bmatrix} V_1 & 0 & 0 \\ V_3 & 0 & 0 \\ V_2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \Theta_2 = \begin{bmatrix} V_4 & 0 \\ V_5 & 0 \\ V_6 & 0 \\ 0 & I \end{bmatrix}.$$

Theorem 3 Given matrices Q, S and R with Q and R symmetric, consider the system (13) subject to Assumption 1. Let $[V_1^{\mathrm{T}} \quad V_2^{\mathrm{T}} \quad V_3^{\mathrm{T}}]^{\mathrm{T}}$ and $[V_4^{\mathrm{T}} \quad V_5^{\mathrm{T}} \quad V_6^{\mathrm{T}}]^{\mathrm{T}}$ are orthogonal complements of $[B_2^{\mathrm{T}} \quad -D_{12}^{\mathrm{T}}S \quad D_{12}^{\mathrm{T}}Q_-^{\frac{1}{2}}]$ and $[C_2 \quad C_{2d} \quad D_{21}]$, respectively. Then there exists an output feedback controller (16) for system (13) such that the closed-loop system is quadratically stable and strictly (Q, S, R)-dissipative if there exist matrices $X > 0$ and $Y > 0$ satisfying the LMIs (25) and (26), respectively, and

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geqslant 0, \quad (27)$$

for some $V > 0$.

Proof There exists a matrix $\bar{P} > 0$ satisfying (24) if and only if inequality $X - Y^{-1} \geqslant 0$ holds. This inequality is equivalent to (27). The rest of the proof is mentioned previously. This completes the proof.

Remark 3 Let $V = \delta I$. Note that given a positive scalar δ, (25) ~ (27) are linear with respect to matrices X and Y. Thus, the existing LMI tool[13] can be applied to find a feasible solution if it exists.

Remark 4 Note that Theorem 3 does not present the computation of the controllers itself, but the existence conditions of controllers is deduced. The computation of a dissipative output feedback controller can be carried out by the procedure proposed in [12]. Indeed, assuming that the conditions (25) ~ (27) are satisfied for some matrices $X > 0$ and $Y > 0$ (not necessarily unique), a suitable controller may be found as follows:

a) Compute two full-column rank matrices $M \in \mathbb{R}^{n\times n_K}$ and $N \in \mathbb{R}^{n\times n_K}$ such that

$$MN^{\mathrm{T}} = I - XY. \tag{28}$$

b) Find the unique solution $\bar{P} > 0$ of the linear equation

$$\begin{bmatrix} Y & I \\ N^{\mathrm{T}} & 0 \end{bmatrix} = \bar{P}\begin{bmatrix} I & X \\ 0 & M^{\mathrm{T}} \end{bmatrix}. \tag{29}$$

Note that (29) is always solvable when $Y > 0$ and M has full-column rank[14].

c) Given the matrix $\bar{P}$, controller parameters A_K, B_K, C_K and D_K can be computed as any solution of LMI (20).

Because the order of the controller depends on the dimension of $\bar{P}$, from the Lemma 7.5 in [14] we can establish the following corollary.

Corollary 1 There exists a reduced-order controller ($n_K < n$) that solves the quadratic dissipative output feedback control problem for the system (13) if, in addition to (25) ~ (27), X and Y also satisfy the rank constraint:

$$\mathrm{rank}(I - XY) \leqslant n_K. \tag{30}$$

Remark 5 In the above Theorem 3, when $Q = -I$, $S = 0$, and $R = \gamma^2 I$, through some slight modification, the LMI-based H_∞ control result of delayed systems in [5] can be deduced. When $Q = 0, S = I$, and $R = 0$, i.e. in the case of strict passive control, this result complements the passive control result of state feedback based on a Riccati equation approach given by Yu et al in [8].

4 Conclusion

In this paper, we have proposed the dissipative controller design method for a class of state delayed systems. A dissipative state feedback or output feedback controller could be obtained by using LMI Toolbox because sufficient condition for the existence of controller is LMI form in terms of related variables. The dissipative feedback control laws guarantee not only the quadratic stability of the closed-loop system but also the strict dissipativeness. Our design results have less conservativeness as it allows a better trade-off between gain and phase performances. The proposed controller design method can be easily extended to the problem of dissipative feedback controller design method for linear time-varying time-delay systems.

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