Article ID: 1000-8152(2001)06-0857-04

A Parameter Learning Method for Stochastic Fuzzy Neural Network *

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Abstract: A stochastic fuzzy logic system and its neural network parameter learning method are developed for the case of noise pollution both in input and output signals. The corresponding parameter learning formulas are also presented. Simulation results indicate that the parameter learning method proposed in this paper is evidently superior to the fuzzy logic neural network methods that do not take noise pollution into consideration.

Key words: stochastic fuzzy logic system; neural network; parameter learning method
Document code: A

随机模糊神经网络的参数学习算法

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摘要: 在输入和输出信号均有噪声污染的情况下,提出随机模糊逻辑神经网络系统及其参数学习算法,给出了参数学习公式;仿真计算表明本文所提出的算法的有效性,它明显优于没有考虑噪声的模糊逻辑神经网络系统。

关键词: 随机模糊逻辑系统; 神经网络; 参数学习方法

1 Introduction

Fuzzy control[1] can simulate brain's thinking ability to describe control systems. The core of fuzzy control is to set up control rules that are presented by human's language, not by accurate equations. So fuzzy control is a good way to solve complex control problems that are difficult to establish mathematics models. Recently, parameter self-learning and self-tuning in fuzzy logic field have been taken into great consideration, and the neural network and fuzzy theory have been combined to form fuzzy-neural network. A parameter learning method of fuzzy-neural network based on singleton fuzzier and singleton defuzzier is presented in [2]. A parameter learning method in which the inputs are nonsingleton fuzzier is also presented in [3]. The fuzzy logic system which has nonsingleton fuzzier is systematically studied in [4]. In this paper, a new parameter learning method for stochastic fuzzy neural network with nonsingleton fuzzier and nonsingleton difuzzier is proposed, and simulation results indicate that the method can accelerate learning rate. In practical applications, input and output signals are often polluted by noises, therefore, the method presented in this paper are valuable both in theory and in practice.

2 Stochastic fuzzy logic system

Consider a fuzzy logic system with multi-input and single output. Assume that the fuzzy system has M if-then rules:

R^l: if x_1 is F_1^l, and $\cdots$, and x_n is F_n^l; then y is G^l.

$$\mu_{B^l}(y) = \sup_{x\in U}[\mu_{F_1^l\times\cdots\times F_n^l\to G^l}(x,y) * \mu_{A_x}(x)]. \quad (1)$$

In (1), F_i^l and G^l denote fuzzy sets based on the sets $U_i \subset \mathbb{R}(i=1,\cdots,n)$ and $V \subset \mathbb{R}$, respectively, where $l = 1,2,\cdots,M$. At the same time, $x = (x_1,\cdots,x_n)^T \in U_1 \times U_2 \times \cdots \times U_n = U$ and $y \in V$ are linguistic variables. Suppose that the fuzzy set A_x is fuzzy input, and M is the rule number of fuzzy logic system.

The rules of (1) define a kind of fuzzy relation: $F_1^l \times \cdots \times F_n^l \to G^l$, whose membership functions of fuzzy sets based on the input and output spaces $U \times V$ are de-

* Foundation item: supported by the Aviation Science Foundation of China(99F5305).
Received date: 1999-05-28; Revised date: 2001-03-09.

cided by $\mu_{F_1^l\times\cdots\times F_n^l\to G^l}(x,y)$. A_x is defined as a fuzzy set of the input space U, and R^l presents the combined fuzzy inference relation of the fuzzy set $B^l = A_x \circ R^l$, which is based on the output space, where we adopt sup – * as the combined calculation.

The purpose of fuzzier is to map the accurate point $x = (x_1,\cdots,x_n) \in U$ into a fuzzy set A_x that is based on the set U. Consider the nonsingleton fuzzier, if $x' = x$, then $\mu_{A_x}(x) = 1$; however, as x' is apart from x, the value of $\mu_{A_x}(x')$ will decrease from 1 to 0. To analyze the stochastic fuzzy system with Gaussian noise, we define $\mu_{A_i^l}(x_i)$ and $\mu_{F_i^l}(x_i)$ as the following Gaussian functions:

$$\mu_{A_i^l}(x_i) = a_{x_i}^l \exp\left[-\left(\frac{x_i - m_{x_i}^l}{\sigma_{x_i}^l}\right)^2\right], \tag{2}$$

$$\mu_{F_i^l}(x_i) = a_{F_i}^l \exp\left[-\left(\frac{x_i - m_{F_i}^l}{\sigma_{F_i}^l}\right)^2\right]. \tag{3}$$

In (2) and (3), we can assume that $a_{x_i}^l = 1$ and $a_{F_i}^l = 1$.

As mentioned above, the membership function $\mu_{G^l}(y)$ of the fuzzy set G^l based on the output space V is defined as the following Guassian function:

$$\mu_{G^l}(y) = b_g^l \exp\left[-\left(\frac{y - m_g^l}{\delta_g^l}\right)^2\right]. \tag{4}$$

And, assume that $b_g^l = 1$.

The purpose of the defuzzier is to map the fuzzy set based on the set V into an accurate point $y \in V$. To solve the influence of the noise, we define a special modified center defuzzier, which is showed as follows:

$$y = \frac{\sum_{l=1}^{M} \bar{y}_g^l [\mu_{B^l}(\bar{y}_g^l)/\delta_g^l]}{\sum_{l=1}^{M} [\mu_{B^l}(\bar{y}_g^l)/\delta_g^l]}, \tag{5}$$

where $\bar{y}_g^l$ is the adjacent value of the central value of Guassian membership function based on the fuzzy set G^l. In (4), $\mu_{B^l}(y)$ is decided by (1).

For the stochastic fuzzy logic system described by (1) ~ (5), we can derive the following theorem that denotes the relationship between input and output.

Theorem The stochastic fuzzy logic system that is composed of the nonsingleton fuzzier, inference rule, modified center defuzzier and Guassian membership functions in (1) ~ (5), the mapping relationship between input and output is as follows:

$$y = f(x) = \frac{\sum_{l=1}^{M} \frac{\bar{y}_g^l}{\delta_g^l}\exp\left[-\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)^2\right] \prod_{i=1}^{n} \exp\left[-\left(\frac{x_{i\max}^l - m_{F_i}^l}{\sigma_{F_i}^l}\right)^2 - \left(\frac{x_{i\max}^l - x_i}{\sigma_{x_i}^l}\right)^2\right]}{\sum_{l=1}^{M} \frac{1}{\delta_g^l}\exp\left[-\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)^2\right] \prod_{i=1}^{n} \exp\left[-\left(\frac{x_{i\max}^l - m_{F_i}^l}{\sigma_{F_i}^l}\right)^2 - \left(\frac{x_{i\max}^l - x_i}{\sigma_{x_i}^l}\right)^2\right]}, \tag{6}$$

where

$$x_{i\max}^l = \frac{\sigma_{x_i}^{2l} m_{F_i}^l + \sigma_{F_i}^{2l} x_i}{\sigma_{F_i}^{2l} + \sigma_{x_i}^{2l}}. \tag{7}$$

Proof Using the inference rule, we can obtain the following formula from (1)

$$\mu_{B^l}(\bar{y}_g^l) = \exp\left[-\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)^2\right] \cdot \sup_{x'\in U} \prod_{i=1}^{n} \exp\left[-\left(\frac{x_i' - m_{F_i}^l}{\sigma_{F_i}^l}\right)^2 - \left(\frac{x_i' - m_{x_i}^l}{\sigma_{x_i}^l}\right)^2\right]. \tag{8}$$

Suppose that $x = (x_1,\cdots,x_n)^{\mathrm{T}}$ is the input signal of fuzzy logic system. Evidently, we should adopt the input signal x_i as the central value $m_{x_i}^l$ of the Guassian membership function in (2). Therefore, (8) can be denoted as follows.

$$\mu_{B^l}(\bar{y}_g^l) = \exp\left[-\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)^2\right] \sup_{x'\in U} \prod_{i=1}^{n} \exp\left[-\left(\frac{x_i' - m_{F_i}^l}{\sigma_{F_i}^l}\right)^2 - \left(\frac{x_i' - x_i}{\sigma_{x_i}^l}\right)^2\right]. \tag{9}$$

After some derivations, we can obtain that the max-value of $x_i' \in U_i$ is denoted by the formula (7).

Therefore, (9) can be denoted as follows.

$$\mu_{B^l}(\bar{y}_g^l) = \exp\left[-\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)^2\right] \cdot \prod_{i=1}^{n} \exp\left[-\left(\frac{x_{i\max}^l - m_{F_i}^l}{\sigma_{F_i}^l}\right)^2 - \left(\frac{x_{i\max}^l - x_i}{\sigma_{x_i}^l}\right)^2\right]. \tag{10}$$

Substituting (10) into (5), we obtain the mapping relationship (6), finally.

It should be pointed out that, in order to deal with stochastic Guassian output signal, the membership function of G^l in (4) is adopted as Guassian function, but $\bar{y}_g^l$ should be adopted as the adjacent value of the center of $\mu_{G^l}(y)$, not as the center value of $\mu_{G^l}(y)$.

Furthermore, from the stochastic fuzzy logic system described by (1) ~ (5), we can further obtain the improved mapping relationship which is shown as follows.

$$y = f(x) = \frac{\sum_{l=1}^{M} \frac{\bar{y}_g^l}{\delta_g^l}\exp\left[-\left(\frac{(\bar{y}_g^l - m_g^l)}{\delta_g^l}\right)^2\right]\prod_{i=1}^{n}\exp\left[-\frac{(x_i - m_{F_i}^l)^2}{\sigma_{F_i}^{2l} + \sigma_{x_i}^{2l}}\right]}{\sum_{l=1}^{M} \frac{1}{\delta_g^l}\exp\left[-\left(\frac{(\bar{y}_g^l - m_g^l)}{\delta_g^l}\right)^2\right]\prod_{i=1}^{n}\exp\left[-\frac{(x_i - m_{F_i}^l)^2}{\sigma_{F_i}^{2l} + \sigma_{x_i}^{2l}}\right]}. \tag{11}$$

Proof Substitute (7) into (6), after some derivations, and we obtain the final result (11).

3 Stochastic fuzzy neural network (SFNN) and its BP learning algorithm for parameters

The mapping relationship between input and output of the stochastic fuzzy logic system (1) ~ (5) is described by (11). In fact, the calculating process of the $f(x)$ in (11) can be expressed as a forward network system.

Therefore, (11) can be analyzed as the following form:

$$f = \frac{A}{B}, \tag{12}$$

$$A = \sum_{l=1}^{M} a_l z_l, \tag{13}$$

$$B = \sum_{l=1}^{M} b_l z_l, \tag{14}$$

$$z_l = \prod_{i=1}^{n}\exp\left[-\frac{(x_i - m_{F_i}^l)^2}{\sigma_{F_i}^{2l} + \sigma_{x_i}^{2l}}\right], \tag{15}$$

$$a_l = \frac{\bar{y}_g^l}{\delta_g^l}\exp\left[-\left(\frac{(\bar{y}_g^l - m_g^l)}{\delta_g^l}\right)^2\right], \tag{16}$$

$$b_l = \frac{1}{\delta_g^l}\exp\left[-\left(\frac{(\bar{y}_g^l - m_g^l)}{\delta_g^l}\right)^2\right], \tag{17}$$

$$a_l = \bar{y}_g^l b_l. \tag{18}$$

At the same time, we can also express the calculating process as a forward network which is shown by Fig.1. The network is a stochastic fuzzy neural network (SFNN) with multi-input and single output.

The SFNN mentioned above can use the back propagation (BP) learning algorithm to train the parameters of neural network.

According to the given input and output data (x, y_d), where $x = (x_1, \cdots, x_n)^T \in U \subset \mathbb{R}^n$ and $y_d \in V \subset U$. The back propagation (BP) learning algorithm is used to train the SFNN model. The corresponding index function is:

$$E = \frac{1}{2}[f(x) - y_d]^2. \tag{19}$$

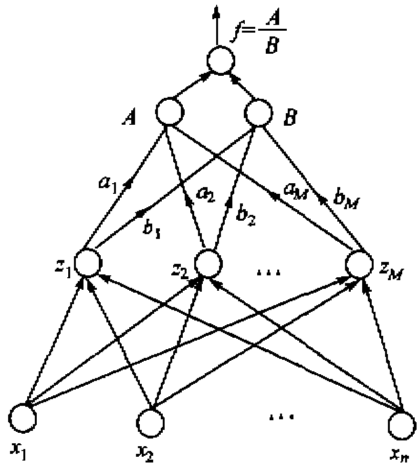

Fig. 1 Forward network for $f(x)$

In order to minimize the error E, we can tune the following parameters, such as $\bar{y}_g^l$, m_g^l, δ_g^l, $m_{F_i}^l$, $\sigma_{F_i}^l$ and x_i^l. Suppose that the weight w is the parameter that can be adjusted, so we can obtain the following rule:

$$\Delta w(k) \propto -\left.\frac{\partial E}{\partial w}\right|_k, \tag{20}$$

$$w(k+1) = w(k) + \alpha\Delta w(k) + \eta\Delta w(k-1), \tag{21}$$

where $\alpha \in [0,1]$ is the learning coefficient, and $\eta \in [0,1]$ is the momentum factor. In terms of the index function E, after some derivations, the partial derivations of parameters are shown as follows:

$$\frac{\partial E}{\partial m_{F_i}^l} = (f - y_d)\frac{(\bar{y}_g^l - f)b_l}{B}\cdot\frac{2(x_i - m_{F_i}^l)}{\sigma_{F_i}^{2l} + \sigma_{x_i}^{2l}}\cdot z_l, \tag{22}$$

$$\frac{\partial E}{\partial \sigma_{F_i}^l} = (f - y_d)\frac{(\bar{y}_g^l - f)b_l}{B}\cdot\frac{2\sigma_{F_i}^l(x_i - m_{F_i}^l)^2}{(\sigma_{F_i}^{2l} + \sigma_{x_i}^{2l})^2}\cdot z_l, \tag{23}$$

$$\frac{\partial E}{\partial \sigma_{x_i}^l} = (f - y_d)\frac{(\bar{y}_g^l - f)b_l}{B}\cdot\frac{2\sigma_{x_i}^l(x_i - m_{F_i}^l)^2}{(\sigma_{F_i}^{2l} + \sigma_{x_i}^{2l})^2}\cdot z_l, \tag{24}$$

$$\frac{\partial E}{\partial m_g^l} = (f - y_d)\frac{b_l}{B}\cdot\frac{(\bar{y}_g^l - f)}{\delta_g^l}\cdot 2\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)\cdot z_l \tag{25}$$

$$\frac{\partial E}{\partial \bar{y}_g^l} = (f - y_d)\frac{b_l}{B}\left[1 - \frac{(\bar{y}_g^l - f)}{\delta_g^l}\cdot 2\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)\right]z_l, \tag{26}$$

$$\frac{\partial E}{\partial \delta_g^l} = (f - y_d)\frac{b_l}{B}\frac{(\bar{y}_g^l - f)}{\delta_g^l}\left[2\left(\frac{\bar{y}_g^l - m_g^l}{\delta_g^l}\right)^2 - 1\right] z_l. \tag{27}$$

4 Simulations

Consider the Example 3.2 in [2], we simulate it by the mentioned algorithms. That is a two-order differential equation presented as follows:

$$y(k+1) = g[y(k), y(k-1)] + u(k). \tag{28}$$

In equation (28)

$$g[y(k), y(k-1)] = \frac{y(k)y(k-1)[y(k)+2.5]}{1 + y^2(k) + y^2(k-1)} \tag{29}$$

and $u(k) = \sin(2\pi k/25)$.

Series-parallel identifier is described by the following equation:

$$\hat{y}(k+1) = f[y(k), y(k-1)] + u(k). \tag{30}$$

Using the method in [2] and our method, respectively, where

$$u'(k) = u(k) + 0.2\ \text{randn},$$
$$g'(k) = g(k) + 1.0\ \text{randn}.$$

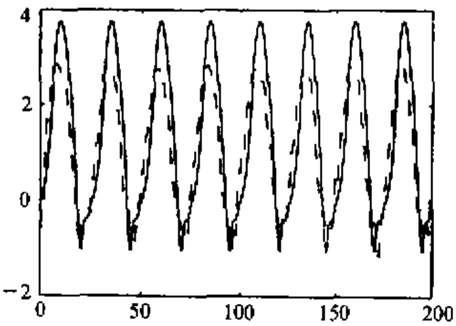

Fig. 2 Results from [2]

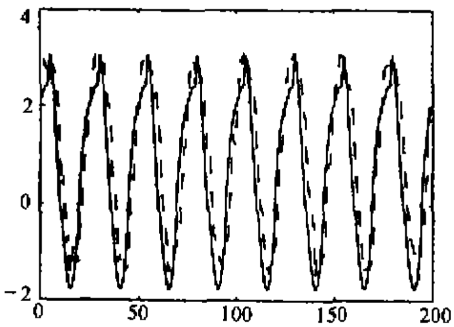

Fig. 3 Result from this paper

Under the above conditions, we can obtain 200 pairs of input-output data with noise, and then train parameters. Parameters are selected randomly at the following range:

$$\bar{y}_g:(0 \sim 4),\ m_F:(-1.35 \sim 1.35);$$
$$m_g:(0 \sim 4),\ \sigma_x:(2\ \text{randn});$$
$$\delta_g:(4\ \text{randn}),\ \sigma_F:(2\ \text{randn});$$

According to the mentioned conditions, we choose the data randomly to train the stochastic fuzzy neural network, and calculate the output value. The results are showed by the following figures (the dotted line demonstrates the simulated output, while the solid line demonstrates the output data without noise).

The simulation results indicate, when the input and output signals are polluted by noise, the parameter learning algorithms presented above are better than algorithms presented in [2].

5 Conclusion

The situation where the input and output signals are polluted by stochastic noises is a common matter in the applications of fuzzy neural network. In this paper, we propose a new parameter learning algorithm to overcome the training error which is often caused by input and output nosie. Simulation results indicate that the parameter learning algorithm of stochastic fuzzy neural network presented by the authors is more effective and better than that in [2]. Because the stochastic noise always exists in practical projects, the theory of stochastic fuzzy neural network will be more important and useful in practical projects.

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