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A Class of Nonlinear H_∞ Controller Design via State Feedback for Disturbance Attenuation *

LI Xi, TANG Xiaoqi, ZHOU Yunfei and CHEN Jihong
(National Engineering & Technology Research Center of Numerical Control · Wuhan, 430074, P.R. China)

Abstract: A class of disturbance attenuation approach is proposed for affine nonlinear systems, which yields the local stable systems of these problems via state feedback standard H_∞ control for unknown finite bounded periodical disturbance. The method for these problems is showed by using dissipation inequality, and the parameters of controller are derived.

Key words: nonlinear system; H_∞-controller; state feedback; disturbance attenuation

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一类抑制扰动的 H_∞ 状态反馈非线性控制器设计

李 曦 唐小琦 周云飞 陈吉红
(国家数控系统工程技术研究中心·武汉, 430074)

摘要: 研究了一类状态可测仿射非线性系统的扰动抑制问题,针对局部渐近稳定的非线性系统受到未知有界的周期性扰动,以 H_∞ 状态反馈标准控制方法为基础,给出这类问题的设计方法,得到了控制器的参数化描述.

关键词: 非线性系统; H_∞ 控制器; 状态反馈; 干扰抑制

1 Introduction

A popular method to solve the standard H_∞ control problem is the state feedback approached and developed in [1]. Recently, attention was extended to H_∞ robust performance problem for nonlinear systems and there has been considerable progress in the design of robust H_∞ controller for nonlinear systems. The H_∞ optimal control or sub-optimal control was developed by several authors[1-10]. The discussion of these problems was almost focused on output-feedback or measurement feedback[2-4] and state-feedback[1,5,6]. Specially, [3] shows the concept of disturbance attenuation in the sense of L_2 norms and gives a set of sufficient condition for the problem of local disturbance attenuation with internal stability in a nonlinear affine system via neasurement feedback, that is when the set of measured variables is just a function of the state of plant and of the disturbance input (also see [2]), and in terms of the solution of a pair of Hamilton-Jacobi inequality, which is the nonlinear version of the Riccati inequality considered in analysis of the H_∞ (sub) optimal control problem for linear systems. [2] presents a necessary condition for the existence of a solution to the problem of (local) disturbance attenuation in the case of measurement feedback, and yields the construction of a feedback law for disturbance attenuation. The necessity of these sufficient conditions has also been discussed in [4] and [9]. [10] discuses the H_∞-controller design problem in case of decentralized large scale nonlinear systems by using the Hamilton-Jacobi inequality approach via measurement feedback. The solution to the output feedback H_∞ control for nonlinear systems was obtained in [4] and [9]. The state feedback H_∞ control has also been investigated in [5] and [6].

More recent contributions to this area of research are the works [6] and [10]. In particular, [6] presents the problem of H_∞ control for nonlinear systems with a known Lyapunov function, and using dissipation inequality a state feedback controller is designed to grant that the closed-loop system is globally asymptotically stable. The most advantage of this method is not rely on solution of any HJIE's(Hamilton-Jacobi-Isaacs equation).

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In this paper we addressed, along the research line of [6] and [10], nonlinear H_∞-control problem for periodic disturbance attenuation (or, periodical nonlinear H_∞-control problem PNHP). Using the notion of dissipativeness and Lyapunov function, some disturbance attenuation for the solution of H_∞ control problem are obtained and the suitable state controller is constructed explicitly.

2 Notation

Definition 2.1 If the signal $w(t)$ satisfying

$$w(t) = a_0/2 + \sum_{i=1}^{\infty}(a_i\cos i\omega t + b_i\sin i\omega t) = a_0/2 + \sum_{i=1}^{N}(a_i\cos i\omega t + b_i\sin i\omega t) + R(i\omega t) \approx a_0/2 + \sum_{i=1}^{N}(a_i\cos i\omega t + b_i\sin i\omega t), \tag{1}$$

where $\lim_{t\to\infty} R(i\omega t) = 0$, we call $w(t)$ is a periodical signal.

For all internal stable systems, the system gain for $w(t) = \sum_{i=0}^{l} w_i(t)$ as input signal can be approximately expressed as follows[10]

$$\gamma_{\max} = \max_{t\geq 0}\sqrt{\frac{\int_0^t[z_1^2(\tau) + \cdots z_l^2(\tau)]\mathrm{d}\tau}{\int_0^t[w_0^2(\tau) + \cdots w_l^2(\tau)]\mathrm{d}\tau}}, \tag{2}$$

where $z_i(i = 0,1,\cdots,l)$ are the output of disturbance.

Consider nonlinear system Σ

$$\Sigma:\ \dot{x} = f(x) + \sum_{i=0}^{l} g_{w_i}(x)w_i + g(x)u, \tag{3}$$

$$z = h(x) + d(x)u, \tag{4}$$

where $x \in \mathbb{R}^n$ stands for the state, $u \in \mathbb{R}^m$ is the local control input, $w_i \in \mathbb{R}^r(i = 0,\cdots,l)$ as Definition 2.1, are the square-integrable exogenous disturbances, $z \in \mathbb{R}^q$ is an output to be regulated, $f(x), g(x)$ are smooth vector fields in $\mathbb{R}^n$ and $\mathbb{R}^m$. $g_{w_i}(x)$ is a matrix for weight of exogenous disturbances, $h(x), d(x)$ are all known smooth mapping vector functions of appropriate dimensions, and satisfying $f(0) = 0, h(0) = 0$. For convenience, we denote $g_w(x) = [g_{w_0},\cdots,g_{w_l}]$, $w = [w_0^{\mathrm{T}},\cdots,w_l^{\mathrm{T}}]^{\mathrm{T}}$, such that Σ can be written as

$$\Sigma:\ \dot{x} = f(x) + g_w(x)w + g(x)u, \tag{5}$$

$$z = h(x) + d(x)u. \tag{6}$$

Definition 2.2 Suppose for the initial state of system Σ is $x_0 = 0$ and a neighborhood of $X \subseteq \mathbb{R}^n$ of x_0, for any $\hat{x} \in X$; there exist a t_1 and a $w(t)$ such that $\hat{x} = \Phi(t_1, 0, x, w(t))$. If this happens, then called that the Σ is state reachable at x_0. Where the function $\Phi(t_1,0,x,w(t))$ is the solution of equation $\dot{x} = f(x) + g_w(x)w + g(x)u$ at $x_0 = 0$ and $u = 0$.

Definition 2.3 The pair $\{f, h\}$ is said to be locally detectable if there exists a neighborhood X of the point $x = 0$ such that, if $x(t)$ is any integral curve of $\dot{x} = f(x)$ satisfying $f(0) = 0, h(0) = 0$.

The PNHP to be addressed in this paper is as follows.

Given a scalar $\gamma \geq \gamma_{\max} > 0$, design a locally smooth nonlinear state feedback control game $u = \alpha(x)$ for Σ, with $\alpha(0) = 0$, and such that:

1) The origin is locally asymptotically stable equilibrium point of the closed-loop system;

2) the L_2-gain of the closed-loop system from w to z is not larger than γ, i.e., satisfies the following inequality

$$\int_0^t z^{\mathrm{T}}(\tau)z^{\mathrm{T}}(\tau)\mathrm{d}\tau \leq \gamma^2\int_0^t w^{\mathrm{T}}(\tau)w^{\mathrm{T}}(\tau)\mathrm{d}\tau, \text{ for all } t \geq 0.$$

3 Main result

Lemma 3.1 If system Σ at $x_0 = 0$ is state reachable, and there exists a differentiable smooth positive definite matrix function $V(x)$, satisfying $V(0) = 0$, so

$$(\partial V/\partial x)f(x) + h^{\mathrm{T}}(x)h(x) + (\partial V/\partial x)g_w(x)g_w^{\mathrm{T}}(x)(\partial V/\partial x)^{\mathrm{T}}/(4\gamma^2) \leq 0. \tag{7}$$

Proof Since the L_2-gain of the system is less than or equal to γ and the system is reachable, for any $T_1 \geq t_0$, we have

$$0 \leq \int_{t_0}^{T_1}(\gamma^2 w^{\mathrm{T}}(\tau)w(\tau) - z^{\mathrm{T}}(\tau)z(\tau))\mathrm{d}\tau < \infty. \tag{8}$$

That is

$$\int_{t_0}^{0}(\gamma^2 w^{\mathrm{T}}(\tau)w(\tau) - z^{\mathrm{T}}(\tau)z(\tau))\mathrm{d}\tau \geq -\int_0^{T_1}(\gamma^2 w^{\mathrm{T}}(\tau)w(\tau) - z^{\mathrm{T}}(\tau)z(\tau))\mathrm{d}\tau. \tag{9}$$

We order

$$V(x) = -\lim_{T_1\to\infty}\inf_{w\in L_2[0,T_1]}\int_0^{T_1}(\gamma^2 w^{\mathrm{T}}(\tau)w(\tau) - z^{\mathrm{T}}(\tau)z(\tau))\mathrm{d}\tau, \tag{10}$$

consider the nonlinear system is reachable at $x_0 = 0$, when $w(t) \equiv 0$, the $x(t) = x_0$ is obviously the solution of $z(t)\equiv 0$, so $V(x)\geqslant 0$.

Obviously

$$V(x)\leqslant\int_{t_0}^{0}(\gamma^2 w^{\mathrm T}(\tau)w(\tau)-z^{\mathrm T}(\tau)z(\tau))\mathrm d\tau<\infty, \tag{11}$$

when $x_0=\mathbf 0$, $V(\mathbf 0)=\mathbf 0$.

Consider the optimal problem for system Σ

$$W(x)=\inf_{w(\tau)}\int_0^{\infty}(\gamma^2 w^{\mathrm T}(\tau)w(\tau)-z^{\mathrm T}(\tau)z(\tau))\mathrm d\tau. \tag{12}$$

It can be known from optimal control theory that $W(x(t),t)$ satisfies

$$\frac{\partial W}{\partial x}=-\min_{w(t)}\{\gamma^2 w^{\mathrm T}w-z^{\mathrm T}z+\frac{\partial W}{\partial x}(f(x)+g_w(x)w)\}. \tag{13}$$

It also can be said that $W(x(t),t)$ satisfies

$$\frac{\partial W}{\partial x}=-\min_{w(t)}\{\gamma^2 w^{\mathrm T}w-z^{\mathrm T}z+\frac{\partial W}{\partial x}(f(x)+g_w(x)w)\}. \tag{14}$$

Because, $W(x)=-V(x)$, that is $\frac{\partial W}{\partial x}=-\frac{\partial V}{\partial x}$, and $\frac{\partial W}{\partial t}=0$, so that

$$-\min_{w(t)}\{\gamma^2 w^{\mathrm T}w-z^{\mathrm T}z+\frac{\partial W}{\partial x}(f(x)+g_w(x)w)\}=$$

$$-\min_{w(t)}\{\gamma^2 w^{\mathrm T}w-z^{\mathrm T}z-(\frac{\partial V}{\partial x})f(x)-(\frac{\partial V}{\partial x})g_w(x)w\}=$$

$$-\min_{w(t)}\{\gamma^2 w^{\mathrm T}w-h^{\mathrm T}(x)h(x)-\frac{\partial V}{\partial x}f(x)-\frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_w(x)g_w^{\mathrm T}(x)(\frac{\partial V}{\partial x})^{\mathrm T}-\frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_w(x)g_w^{\mathrm T}(x)(\frac{\partial V}{\partial x})^{\mathrm T}-\frac{\partial V}{\partial x}g_w(x)w\}=$$

$$\min_{w(t)}\{\|\gamma w-\frac{1}{2\gamma}g_w^{\mathrm T}(x)(\frac{\partial V}{\partial x})^{\mathrm T}\|^2+[\frac{\partial V}{\partial x}f(x)+h^{\mathrm T}(x)h(x)+\frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_w(x)g_w^{\mathrm T}(x)(\frac{\partial V}{\partial x})^{\mathrm T}]\}=0.$$

We denote

$$P(x)=\frac{\partial V}{\partial x}f(x)+h^{\mathrm T}(x)h(x)+\frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_w(x)g_w^{\mathrm T}(x)(\frac{\partial V}{\partial x})^{\mathrm T}. \tag{15}$$

Obviously

$$\|\gamma w-\frac{1}{2\gamma}g_w^{\mathrm T}(x)(\frac{\partial V}{\partial x})^{\mathrm T}\|^2\geqslant\mathbf 0,$$

so that

$$P(x)=\frac{\partial V}{\partial x}f(x)+h^{\mathrm T}(x)h(x)+\frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_w(x)g_w^{\mathrm T}(x)(\frac{\partial V}{\partial x})^{\mathrm T}\leqslant 0.$$

Q.E.D.

Lemma 3.2 Suppose that the system Σ is state reachable at $x_0=\mathbf 0$, if there exist a neighborhood $X\subseteq\mathbb R^n$ and a differentiable smooth positive definite matrix function $V(x)$ for $x(\mathbf 0)\in X$, and suppose system Σ satisfies the following hypothesis.

Assumption 1 There exists a smooth function $V(x)\geqslant 0$ in the neighborhood of X such that $\frac{\partial V}{\partial x}\leqslant 0$.

Assumption 2 Matrix function $d(x)$ is of full-column rank for any $x\in\mathbb R^n$.

Assumption 3 Function $V(x)$ is a diagonal matrix.

Then given any positive scalar $\gamma\geqslant\gamma_{\max}>0$, the control law

$$u=-[d^{\mathrm T}(x)d(x)]^{-1}[g^{\mathrm T}(x)(\partial V/\partial x)^{\mathrm T}/2+d^{\mathrm T}(x)h(x)] \tag{16}$$

solves the H_∞ control problem for this system.

Proof For convinece we denote $Q(x)=[d^{\mathrm T}(x)d(x)]^{-1}$, evidently $Q(x)>0$.

We know from Assumption 1, the dissipation inequality of system satisfies

$$V(x(t_1))-V(x(t_0))+\int_{t_0}^{t_1}L(x(\tau),w(\tau),u(\tau))\mathrm d\tau\leqslant 0 \text{ for any } t_1\geqslant t_0,$$

where

$$L(x,w,u)=\|h(x)+d(x)u\|^2-\gamma^2\|w\|^2, \tag{17}$$

$$H(x,w,u)=(\frac{\partial V}{\partial x})\dot x+L(x,w,u)=$$

$$\frac{\partial V}{\partial x}(f(x)+g_w(x)w+g(x)u)+(h(x)+d(x)u)^{\mathrm T}(h(x)+d(x)u)-\gamma^2 w^{\mathrm T}w=$$

$$\frac{\partial V}{\partial x}f(x)+\frac{\partial V}{\partial x}g(x)+\frac{\partial V}{\partial x}g(x)u(x)+h^{\mathrm T}(x)h(x)+u^{\mathrm T}(x)d^{\mathrm T}(x)h(x)+h^{\mathrm T}(x)d(x)u(x)+u^{\mathrm T}(x)d^{\mathrm T}(x)d(x)u(x)-\gamma^2 w^{\mathrm T}w.$$

From Assumption 1, the system satisfies the condition of saddle point, such that for w and u from $\partial H/\partial w=\mathbf 0$, $\partial H/\partial u=\mathbf 0$ we can get the saddle point of $H(x,w,u)$.

$$w_p = \frac{1}{2\gamma^2} g_w^T(x) (\frac{\partial V}{\partial x})^T, \tag{18}$$

$$u_p = -(d^T(x) d(x))^{-1} \cdot \frac{1}{2} g^T(x) (\frac{\partial V}{\partial x})^T + d^T(x) h(x)). \tag{19}$$

Consider (18) and (19), we obtain

$$\begin{aligned}
&H(x, w_p, u_p) = \\
&(\frac{\partial V}{\partial x}) \dot{x} + L(x, w_p, u_p) = \\
&\frac{\partial V}{\partial x} [f(x) + \frac{1}{2\gamma^2} g_w(x) g_w^T(x) (\frac{\partial V}{\partial x})^T - \\
&\frac{1}{2} [d^T(x) d(x)]^{-1} g(x) g^T(x) (\frac{\partial V}{\partial x})^T - \\
&[d^T(x) d(x)]^{-1} g(x) d^T(x) h(x)] + h^T(x) h(x) - \\
&\frac{1}{4\gamma^2} (\frac{\partial V}{\partial x}) g_w(x) g_w^T(x) (\frac{\partial V}{\partial x})^T - \\
&\frac{1}{2} [d^T(x) d(x)]^{-1} [g^T(x) (\frac{\partial V}{\partial x})^T + \\
&d^T(x) h(x)]^T d^T(x) h(x) - \\
&\frac{1}{2} [d^T(x) d(x)]^{-1} d^T(x) h(x) [g^T(x) (\frac{\partial V}{\partial x})^T + \\
&d^T(x) h(x)] + [d^T(x) d(x)]^{-1} [\frac{1}{2} g^T(x) (\frac{\partial V}{\partial x})^T + \\
&d^T(x) h(x)]^T \cdot [\frac{1}{2} g^T(x) (\frac{\partial V}{\partial x})^T + d^T(x) h(x)] = \\
&\frac{\partial V}{\partial x} f(x) + h^T(x) h(x) + \\
&\frac{1}{4\gamma^2} \frac{\partial V}{\partial x} g_w(x) g_w^T(x) (\frac{\partial V}{\partial x})^T - \\
&\frac{1}{4} Q(x) \frac{\partial V}{\partial x} g(x) g^T(x) (\frac{\partial V}{\partial x})^T - \\
&Q(x) \frac{\partial V}{\partial x} g(x) d^T(x) h(x) - \\
&Q(x) h^T(x) d(x) d^T(x) h(x) = \\
&P(x) - Q(x) \| \frac{1}{2} \frac{\partial V}{\partial x} g(x) + d^T(x) h(x) \|^2.
\end{aligned}$$

Since $Q(x) = [d^T(x) d(x)]^{-1} > 0$, so that

$$Q(x) \left\| \frac{1}{2} \frac{\partial V}{\partial x} g(x) + d^T(x) h(x) \right\|^2 \geqslant 0.$$

$$H(x, w_p, u_p) \leqslant \frac{\partial V}{\partial x} f(x) + h^T(x) h(x) + \frac{1}{4\gamma^2} \frac{\partial V}{\partial x} g_w(x) g_w^T(x) (\frac{\partial V}{\partial x})^T.$$

From Lemma 3.1 we can obtain

$$H(x, w_p, u_p) \leqslant 0.$$

Q.E.D.

4 Conclusion

This paper has considered the problem of nonlinear H_∞-control problem for periodic disturbance attenuation. We have developed a methodology for designing nonlinear state feedback controllers that ensure local stability and a prescribed bound for the closed-loop system. The proposed design methodology involves the explicit construction of the control law on a Lyapunov function of system, which avoids the need for solving HJIE's. There is real value in the work of this paper.

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本文作者简介

李 曦 1965年生.讲师.华中理工大学博士生.主要研究方向:非线性控制理论及应用,控制系统设计.

唐小琦 1958年生.副教授.博士.主要研究方向:非线性控制理论及应用,计算机过程控制,控制系统设计.

周云飞 1956年生.教授.博士生导师.主要研究方向:数控加工技术,计算机过程控制.

陈吉红 1965年生.教授.博士.主要研究方向:数控加工技术,计算机过程控制,控制系统设计.