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Cumulant-Based Approach to the Harmonic Retrieval *

YAO Xiaodong, JING Zhongliang and JIN Dekun

(Institute of Aerospace Information and Control, Shanghai Jiaotong University · Shanghai, 200030, P.R.China)

Abstract: In order for cumulant-based approach to be applied to harmonic retrieval and practical dynamic data, the topics of determinate initial phase, multi-harmonic and wide dynamic range are studied, some new characteristics are obtained, and the defects of harmonic retrieval and noise pre-filtering algorithms are pointed out. Finally, the improved algorithm for wide dynamic range signal is given. The simulation results show that the algorithm is effective and satisfactory.

Key words: cumulant; harmonic retrieval; multi-harmonics; wide dynamic range

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累量域谐波恢复方法

姚晓东 敬忠良 金德琨

(上海交通大学航空航天信息处理与控制研究所·上海,200030)

摘要: 累量域谐波恢复算法,通常假定谐波初相在[- π,π]区间上均匀分布,而且大量的仿真实验沿用相关域的分析方法,假定谐波数目为二,其振幅几乎相等.本文在对实际信号模型分析的基础上,从理论上得出了初相确定、谐波数目较多和信号动态范围比较大时累量自身及累量域参数估计方法的一些特点,指出了现有谐波恢复和基于三阶累量的噪声预滤波算法在理论和实际信号处理中的缺陷,并对大动态范围信号谐波恢复算法作了改进,使其理论更加完善、改进算法稳健性增强.

关键词: 累积量; 谐波恢复; 谐波数目; 动态范围

1 Introduction

The harmonic retrieval problem involves estimating the parameters of a harmonic signal with noise. It has wide applications in the field of signal processing, such as radar, sonar and so on. Up to now, most of the harmonic retrieval approaches assumed white noise and utilized second-order statistics. In order for these methods to be applied to colored Gaussian noise, the noise covariance matrix must be known or estimated. Typically, however, the practical noise is colored and its covariance matrix is unavailable. By exploiting the fact that the fourth-order cumulants are zero for Gaussian process, it has been demonstrated that the cumulant-based harmonic retrieval methods can suppress the effect of colored Gaussian noise. The cumulant-based harmonic retrieval algorithms can improve their toleration and moderation theoretically. To be applied to the harmonic retrieval and the noise filtering of the real data, these cumulant-based methods should be studied further. The distinction between the signal model adopted widely in theoretic research and the signal used in practice is also pointed out.

In the research of the cumulant theory, in order to ensure stationarity, it is often assumed that the initial phase of sinusoid is uniformly distributed over $[-\pi,\pi]$ and it is used in convenience. In simulations, it is also assumed that the number of harmonic is two and the amplitudes of sinusoids are almost equal. So the dynamic range of signal is very narrow. But the above assumptions are not consistent with the real signal with determinate initial phase, multi-harmonic and wide dynamic range. Our simulations show that most of the harmonic retrieval algorithms are not effective to the real signal. So, the model that is consistent with real signal is analyzed in detail. The concept of mixed cumulant is introduced, and its estimation and properties are first discussed. Then, the topics of the multi-harmonic and the

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wide dynamic range are introduced. Finally, the defects of common harmonic retrieval and the third-order cumulant-based noise pre-filtering algorithms are pointed out, and the improved algorithm is proved to be effective.

2 Mixed cumulant of harmonic

For the sake of convenience, the contaminated single record signal $x(t)$, which has constant phases, is modeled as

$$s(t) = \sum_{k=1}^{M} a_k \cos(\omega_k t + \varphi_{k0}) + n(t), \ t = 1,\cdots,N, \tag{1}$$

where the amplitudes a_k 's and angular frequencies ω_k 's are unknown constants, $\omega_j \neq \omega_i$ for $j \neq i$, and $n(t)$ is zero-mean additive Gaussian noise with unknown covariance. The amplitudes are assumed to be positive and $\omega_k^1 \in [-\pi,\pi), k = 1,2,\cdots,M$.

If the sinusoid phases in equation (1) are uniformly distributed over $[-\pi,\pi]$, the fourth-order cumulant for $x(t)$ is defined as the real cumulant. Otherwise, the phases are deterministic, and $x(t)$ is non-stationary. Consequently, an alternative statistic is necessary for developing high order statistic-based harmonic retrieval methods. We give the following Definition 1, where only real signal is considered, and the similar result can be obtained for complex signal.

Definition 1 (Mixed fourth-order cumulant) If $x(t)$ is quasistationary up to the fourth order, the fourth-order mixed cumulant is given by

$$\begin{aligned}&\bar{C}_{4x}(\tau_1,\tau_2,\tau_3) = \\ &\bar{cum}[x(t),x(t+\tau_1),x(t+\tau_2),x(t+\tau_3)] = \\ &\bar{E}[x(t)x(t+\tau_1)x(t+\tau_2)x(t+\tau_3)] - \bar{E}[x(t)x(t+\tau_1)]\bar{E}[x(t+\tau_2)x(t+\tau_3)] - \bar{E}[x(t)x(t+\tau_2)]\bar{E}[x(t+\tau_1)x(t+\tau_3)] - \bar{E}[x(t)x(t+\tau_3)]\bar{E}[x(t+\tau_1)x(t+\tau_2)],\end{aligned} \tag{2}$$

where

$$\bar{E}[f(t)] = \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}E[f(t)].$$

The definition of the fourth-order mixed moment is similar to the fourth-order mixed cumulant.

We refer to these statistics as mixed cumulants because they are defined with the specific case of a mixed process. In particular, mixed cumulants are appropriate for mixed processes consisting of deterministic signals with finite, non-zero kth-order mixed cumulants plus stationary random processes. These definitions also mix time averaging and ensemble averaging. Note that for a stationary process, the definition in equation (2) is equivalent to the definition of the true fourth-order cumulants. On the other hand, for a mixed process, the definitions for the mixed cumulants and true cumulants are different. The mixed-cumulant definition includes stationary signals as a special case.

Several important propositions of the mixed cumulant are given as follows.

Proposition 1[1] Consider a signal $x(t) = s(t) + g(t)$, where $s(t)$ is a zero-mean deterministic signal, and $g(t)$ is a zero-mean stationary random process. If $x(t)$ is quasistationary up to the fourth order, the fourth-order mixed cumulant is given by

$$\bar{C}_{4x}(\tau_1,\tau_2,\tau_3) = \bar{C}_{4s}(\tau_1,\tau_2,\tau_3) = C_{4g}(\tau_1,\tau_2,\tau_3). \tag{3}$$

Proposition 2[1] Consider a real signal $s(t) = \sum_{i=1}^{M} a_i\cos(\omega_i t + \varphi_i)$, if ω satisfies

$$\begin{cases}\omega_k + \omega_l = \omega_m + \omega_n, \\ \quad (k = m \cup l = m) \mid (k = n \cup l = m), \\ \omega_k + \omega_l + \omega_m \neq \omega_n + 2\pi r, \ r = 0,1 \\ \omega_k+\omega_l+\omega_m+\omega_n \neq 2\pi, \ k,l,m,n = 1,\cdots,M,\end{cases} \tag{4}$$

the fourth-order mixed cumulant is given by

$$\begin{aligned}&\bar{C}_{4s}(\tau_1,\tau_2,\tau_3 \mid \theta) = \\ &-\frac{1}{8}\sum_{i=1}^{M} a_i^4\{\cos\omega_i(\tau_1-\tau_2-\tau_3) + \\ &\cos\omega_i(\tau_2-\tau_3-\tau_1) + \cos\omega_i(\tau_3-\tau_1-\tau_2)\}.\end{aligned} \tag{5}$$

Proposition 3[5] Consider a real signal $s(t) = \sum_{i=1}^{M} a_i\cos(\omega_i t + \varphi_i)$, if ω satisfies

$$\begin{aligned}&\omega_k + \omega_l \neq \omega_m + 2\pi r, \\ &r = 0,1, \ k,l,m = 1,2,\cdots,M,\end{aligned} \tag{6}$$

the third-order mixed cumulant satisfies

$$\bar{C}_{3s}(\tau_1,\tau_2) = 0.$$

Proposition 1 shows that the fourth-order mixed cumulant is theoretically blind to Gaussian noise.

Propositions 2 and 3 are important in the following discussion.

3 Multi-harmonic retrieval method

In simulations, the number of harmonic assumed as two is effective to the covariance-based harmonic retrieval methods, which can be seen from the covariance deduction process. In the research of the cumulant theory, it is often assumed that the initial phase of sinusoids is uniformly distributed over $[-\pi, \pi]$. This is often used for convenience. But the initial phase of real signal is determinate, so the result has shortcomings in theory and it can not solve the practical problem directly. The concept of mixed cumulant can ensure the harmonic retrieval methods effective to real signal with determinate initial phase. In the deduction process, equation (4) is introduced. If the number of harmonic is two, equation (5) exists and the cumulant algorithms are moderate. When the number of harmonic is equal to or larger than four, equation (5) exists only when equation (4) is satisfied. We discuss the multi-harmonic retrieval method as follows.

For the signal $s(t)$ in equation (1), no matter equation (4) exists or not, we can obtain the following equations by equation (2)

$$\bar{m}_{2s}(\tau) = \frac{1}{2}\sum_{i=1}^{M}\alpha_i^2\cos(\omega_i\tau), \tag{7}$$

$$\begin{aligned}&\bar{m}_{4s}(\tau_1,\tau_2,\tau_3) = \\ &\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}E[s(t)s(t+\tau_1)s(t+\tau_2)s(t+\tau_3)] = \\ &\sum_{k,l,m,n=1}^{M}\alpha_k\alpha_l\alpha_m\alpha_n\{\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}[\cos(\omega_k t+\varphi_k)\cos(\omega_l(t+\tau_1)+\varphi_l)\cdot \\ &\cos(\omega_m(t+\tau_2)+\varphi_m)\cos(\omega_n(t+\tau_3)+\varphi_n)]\} = \\ &\frac{1}{8}\sum_{k,l,m,n=1}^{M}\alpha_k\alpha_l\alpha_m\alpha_n\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\sum_{i=1}^{8}\cos[\lambda_i(t,\bar{\tau},\bar{\omega},\bar{\varphi})],\end{aligned} \tag{8}$$

where $\bar{m}_{2s}(\tau)$ and $\bar{m}_{4s}(\tau_1,\tau_2,\tau_3)$ are the second-order and fourth-order mixed moment of signal $s(t)$, respectively. $\bar{\tau} = (\tau_1,\tau_2,\tau_3)$, $\bar{\omega} = (\omega_k,\omega_l,\omega_m,\omega_n)$, $\bar{\varphi} = (\varphi_k,\varphi_l,\varphi_m,\varphi_n)$, and

$$\lambda_1(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k+\omega_l(t+\tau_1)+\varphi_l+\omega_m(t+\tau_2)+\varphi_m+\omega_n(t+\tau_3)+\varphi_n,$$

$$\lambda_2(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k+\omega_l(t+\tau_1)+\varphi_l-\omega_m(t+\tau_2)-\varphi_m-\omega_n(t+\tau_3)-\varphi_n,$$

$$\lambda_3(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k+\omega_l(t+\tau_1)+\varphi_l-\omega_m(t+\tau_2)-\varphi_m+\omega_n(t+\tau_3)+\varphi_n,$$

$$\lambda_4(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k+\omega_l(t+\tau_1)+\varphi_l+\omega_m(t+\tau_2)+\varphi_m-\omega_n(t+\tau_3)-\varphi_n,$$

$$\lambda_5(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k-\omega_l(t+\tau_1)-\varphi_l+\omega_m(t+\tau_2)+\varphi_m+\omega_n(t+\tau_3)+\varphi_n,$$

$$\lambda_6(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k-\omega_l(t+\tau_1)-\varphi_l-\omega_m(t+\tau_2)-\varphi_m-\omega_n(t+\tau_3)-\varphi_n.$$

$$\lambda_7(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k-\omega_l(t+\tau_1)-\varphi_l+\omega_m(t+\tau_2)+\varphi_m-\omega_n(t+\tau_3)-\varphi_n,$$

$$\lambda_8(t,\bar{\tau},\bar{\omega},\bar{\varphi}) = \omega_k t+\varphi_k-\omega_l(t+\tau_1)-\varphi_l-\omega_m(t+\tau_2)-\varphi_m+\omega_n(t+\tau_3)+\varphi_n.$$

Obviously, if the time variable t can not be vanished in $\lambda_i(t,\bar{\tau},\bar{\omega},\bar{\varphi})$,

$$\lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\cos[\lambda_i(t,\bar{\tau},\bar{\omega},\bar{\varphi})] = 0. \tag{9}$$

Since $k,l,m,n = 1,\cdots,M$, we consider five possible cases for the indices k,l,m and n:

$$\begin{cases}\text{Case 1} & k = l = m = n, \\ \text{Case 2} & \{k = l \neq m = n\}_3, \\ \text{Case 3} & \{k = m = l \neq n\}_4, \\ \text{Case 4} & \{k = l \neq m \neq n\}_6, \\ \text{Case 5} & k \neq l \neq m \neq n.\end{cases}$$

The subscripts indicate the number of possibilities for the indices k,l,m and n. Let

$$\Gamma_j = \frac{1}{8}\sum_{k,l,m,n=1}^{M}\alpha_k\alpha_l\alpha_m\alpha_n\cdot \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\sum_{i=1}^{8}\cos[\lambda_i(t,\bar{\tau},\bar{\omega},\bar{\varphi})],\ j = 1,\cdots,5$$

where k,l,m and n satisfy the conditions of the i-th case.

From equation (7), the second-order terms are given by

$$\begin{aligned}&\bar{m}_{2s}(\tau_1)\bar{m}_{2s}(\tau_3-\tau_2) = \\ &\Big[\frac{1}{2}\sum_{k=1}^{M}\alpha_k^2\cos(\omega_k\tau_1)\Big]\Big[\frac{1}{2}\sum_{l=1}^{M}\alpha_l^2\cos[(\omega_l(\tau_3-\tau_2)]\Big] = \\ &\frac{1}{8}\sum_{k,l=1}^{M}\alpha_k^2\alpha_l^2\{\cos[\omega_k\tau_l+\omega_l(\tau_3-\tau_2)] + \\ &\cos[\omega_k\tau_1+\omega_l(\tau_2-\tau_3)]\}.\end{aligned} \tag{10}$$

$\bar{m}_{2s}(\tau_2)\bar{m}_{2s}(\tau_3-\tau_1)$, $\bar{m}_{2s}(\tau_3)\bar{m}_{2s}(\tau_2-\tau_1)$ and $\bar{m}_{2s}(\tau_1)\bar{m}_{2s}(\tau_3-\tau_2)$, are similar to $\bar{m}_{2s}(\tau_1)\bar{m}_{2s}(\tau_3-\tau_2)$, so omitted.

If equation (4) is satisfied, the time variable t in

$\cos[\lambda_i(t,\bar{\tau},\bar{\omega},\bar{\varphi})]$ can not be vanished. From equations (7) ~ (10), we obtain

$$\Gamma_3 = \Gamma_4 = \Gamma_5 = 0, \tag{11}$$

$$\bar{C}_{4s}(\tau_1,\tau_2,\tau_3 \mid \theta) = -\frac{1}{8}\sum_{i=1}^{M}\alpha_i^4 |\cos\omega_i(\tau_1-\tau_2-\tau_3) + \cos\omega_i(\tau_2-\tau_3-\tau_1) + \cos\omega_i(\tau_3-\tau_1-\tau_2)|. \tag{12}$$

If equation (4) is not satisfied, the time variable t in $\cos[\lambda_i(t,\bar{\tau},\bar{\omega},\bar{\varphi})]$ among Γ_3,Γ_4 and Γ_5 may be vanished completely. So equation (12) can not exist, it means that the harmonic retrieval algorithms obtained by the replaced covariance with cumulant can not exist, either. If so, the cumulant-based retrieval algorithms may result in wrong results.

Considering the real signal $s(t)$, if the number of harmonic is two, the third-order cumulant of the signal is equal to zero. But when the number of harmonic is larger than two, whether the third-order cumulant of the signal is equal to zero or not is determined by

$$\omega_k + \omega_l \neq \omega_m + r2\pi,\ r = 0,1;\ k,l,m = 1,2,\cdots,M. \tag{13}$$

If equation (13) is not satisfied, such as $\omega_k + \omega_l = \omega_m$, the time variable t in $\lambda_i(t,\bar{\tau},\bar{\omega},\bar{\varphi})$ may be vanished completely. The third-order cumulant of the signal is not equal to zero[2], namely

$$\bar{C}_{3s}(\tau_1,\tau_2) \neq 0. \tag{14}$$

To mixed noise, equation (14) shows that the noise prefiltering methods[3,4] based on third-order cumulant can not be adopted blindly, otherwise the available information will be filtered.

In order to prove our viewpoint, we conducted the following computer experiments.

Experiment 1 Consider the signal $x_1(t), x_2(t)$ and $x_3(t)$, respectively, as

$$x_1(t) = \cos(2\pi\times 0.08\times t) + \cos(2\pi\times 0.12\times t + \pi/3),$$

$$x_2(t) = \sum_{i=1}^{4}\alpha_i\cos(2\pi\times 0.12\times t),$$

$$x_3(t) = \cos(2\pi\times 0.04\times t) + \cos(2\pi\times 0.12\times t + \pi/3) + \cos(2\pi\times 0.22\times t) + \cos(2\pi\times 0.38\times t + \pi/3).$$

Assume that $\hat{C}_{2s}(\tau)$, $\hat{C}_{3s}(\tau)$ and $\hat{C}_{4s}(\tau)$ are the estimation values of the signal second-order, third-order cumulant diagonal slice and fourth-order cumulant diagonal slice, respectively, and

$$\bar{C}_{4s}(\tau) = -\frac{3}{8}\sum_{i=1}^{M}\alpha_i^4(\cos\omega_i\tau),$$

$\hat{C}_{2s}(\tau)$, $\hat{C}_{3s}(\tau)$, $\hat{C}_{4s}(\tau)$ and $\bar{C}_{4s}(\tau)$ are estimated from 500 Monte Carlo trials. The results are shown in Table 1.

Table 1 The comparison of multi-harmonic estimated cumulants

N = 2500		τ = 1	2	3	4	5	6	7	8	9	10
$x_1(t)$	$\hat{C}_{2s}(\tau)$	0.8546	0.4947	0.1021	−0.1323	−0.1105	0.1198	0.3926	0.5153	0.3741	−0.0020
	$\hat{C}_{3s}(\tau)$	−0.0023	−0.0028	−0.0019	−0.0007	0.0001	0.0003	−0.0007	−0.0023	−0.0029	−0.0022
	$\hat{C}_{4s}(\tau)$	−0.6419	−0.3708	−0.0758	0.0994	0.0825	−0.0900	−0.2936	−0.3846	−0.2781	0.0032
	$\bar{C}_{4s}(\tau)$	−0.6417	−0.3722	−0.0776	0.0987	0.0830	−0.0894	−0.2942	−0.3868	−0.2816	0.0000
$x_2(t)$	$\hat{C}_{2s}(\tau)$	−0.4177	−0.0742	−0.4384	−0.0390	−0.4996	0.0728	−0.7595	1.5531	0.5267	−0.5008
	$\hat{C}_{3s}(\tau)$	−0.2108	−0.6072	−1.0138	−0.5504	−1.1288	−0.3608	−1.2057	3.6773	1.7347	−1.1288
	$\hat{C}_{4s}(\tau)$	−0.6748	−0.9580	−1.1767	−0.5677	−1.3553	−0.3983	−2.1375	5.2532	2.2862	−1.9175
	$\bar{C}_{4s}(\tau)$	0.3142	0.0555	0.3294	0.0289	0.3750	−0.0554	0.5698	−1.1661	−0.3934	0.3750
$x_3(t)$	$\hat{C}_{2s}(\tau)$	0.5764	0.0354	0.0963	−0.3593	0.5590	−0.3696	−0.5770	0.7878	0.1768	0.0579
	$\hat{C}_{3s}(\tau)$	−0.0021	−0.0006	−0.0001	−0.0010	−0.0019	−0.0021	−0.0021	−0.0040	−0.0021	−0.0007
	$\hat{C}_{4s}(\tau)$	0.8968	0.4141	0.2686	−0.9438	0.4401	−0.4198	−0.4504	1.2880	0.0085	0.0184
	$\bar{C}_{4s}(\tau)$	−0.4335	−0.0270	−0.0724	0.2698	−0.4193	0.2767	0.4335	−0.5903	−0.1330	−0.0443

From Table 1 and the above statement, we can obtain the following conclusion:

1) If the number of harmonic is larger than two and $\omega_k + \omega_l = \omega_m, (k, l, m = 1, 2, \cdots, M)$, third-order cumulant of the signal is not equal to zero, and the noise pre-filtering methods[3,4] based on third-order cumulant can not be adopted blindly, otherwise the available information will be filtered.

2) If the number of harmonic is larger than three and equation (4) can not be satisfied, the fourth-order cumulant diagonal slices can not satisfy the following equation

$$\bar{C}_{4s}(\tau) = -\frac{3}{8}\sum_{i=1}^{M}\alpha_i^4(\cos\omega_i\tau).$$

The wrong results may be obtained by adopting cumulant algorithms blindly.

4 Processing method of wide dynamic range data

In simulations, the harmonic amplitudes are almost assumed equal, so the signal dynamic range is very narrow. However, the real signal has wide dynamic range. Our simulations show that the common harmonic retrieval algorithms are not effective to the real signal. They are not tolerant to the signal dynamic range, in other words, it means that the performance of the harmonic retrieval methods based on fourth-order cumulant declines rapidly with the increase of data dynamic range. The harmonic retrieval methods based on cumulant should be improved so that they are effective for signal with wide dynamic range. It is very valuable in practice.

Supposing the model of the real wide dynamic range signal $s(t)$ as

$$s(t) = \sum_{i=1}^{M}\alpha_i\cos(\omega_i t + \varphi_i), \qquad |\alpha_1| \gg |\alpha_j|,\ j = 2, \cdots, M, \tag{15}$$

when equation (4) is satisfied, we can express the second-order moment and fourth-order cumulant of signal $s(t)$, respectively, as

$$\bar{m}_{2s}(\tau) = \frac{1}{2}\sum_{i=1}^{M}\alpha_i^2\cos(\omega_i\tau), \tag{16}$$

$$\bar{C}_{4s}(\tau) = -\frac{3}{8}\sum_{i=1}^{M}\alpha_i^4(\cos\omega_i\tau). \tag{17}$$

Since $|\alpha_1| \gg |\alpha_j|, j = 2, \cdots, M$, we choose the initial value of α_1 as

$$\alpha_{10} = \frac{\bar{C}_{4s}(\tau)}{\bar{m}_{2s}(\tau)}, \tag{18}$$

the efficient estimation value $\hat{f}_1$ of frequency f_1 can be obtained and the estimation model of the big amplitude harmonic can be expressed as

$$\hat{s}_1'(t) = \alpha_{10}\cos(\hat{\omega}_1 t + \phi), \tag{19}$$

where $\bar{s}(t) = s(t) - \hat{s}_1'(t)$.

Let

$$\rho(\tau) = \left[\frac{-4\bar{C}_{4\bar{s}}(\tau)}{3\bar{m}_{2\bar{s}}(\tau)}\right]^{1/2} \tag{20}$$

and

$$\cos(\omega_1 t + \varphi_1) = \cos(\hat{\omega}_1 t + \phi). \tag{21}$$

When equation (21) exists, ρ gets the minimum value. Then existence of equation (21) does not mean that φ_1 is equal to ϕ, because the following equation is possible

$$\phi = \varphi_1 + \omega_1 n, n = 1, 2, \cdots, N,$$

where $\phi \in [0, 2\pi]$. We can determine the value of ϕ and the restrained signal can be expressed as

$$s_1'(t) = \bar{s}(t) - \rho\cos(\hat{\omega}_1 t + \phi).$$

In order to evaluate the performance of our improved method, the following computer experiments are conducted.

Experiment 2 Consider the following signal

$$x(t) = \alpha_1\cos(2\pi\times0.005\times t) + \cos(2\pi\times0.01\times t+\pi/3) + \cos(2\pi\times0.015\times t) + \cos(2\pi\times0.02\times t+\pi/4) + n(t),$$

where $n(t) = e(t) + 0.8e(t-1) + 0.3e(t-2) - 0.6e(t-3), t = 1, \cdots, N$, and the amplitude α_1 is a variable constant. The length of the data is 500.

Using the common harmonic retrieval algorithms and the improved algorithm respectively, the simulation results show that when α_1 is larger than four, the common harmonic retrieval algorithms are not effective. But the improved algorithm is very effective no matter how much α_1 is. The results of the improved algorithm are shown in Table 2 when α_1 is equal to 400.

From Table 2 and the above analysis, we can obtain a conclusion that by modeling the big amplitude harmonic, the improved algorithm can improve the moderation of the wide dynamic range signal.

Table 2 The retrieval results of wide dynamic range harmonic with the improved algorithm

N = 500		SNR/dB					
		20	15	10	5	2	0
f_1	average value	5.126-E3	5.126E-3	5.127E-3	5.127E-3	5.125E-3	5.125E-3
	average quadrantal error	4.08E-7	1.30E-6	4.16E-6	1.38E-5	2.94E-5	5.03E-5
f_2	average value	1.0163E-2	1.0163E-2	1.0163E-2	1.0164E-2	1.0167E-2	1.0173E-2
	average quadrantal error	4.83E-7	1.53E-6	4.84E-6	1.56E-5	3.26E-5	5.61E-5
f_3	average value	1.5192E-2	1.5192E-2	1.5192E-2	1.5192E-2	1.5189E-2	1.5190E-2
	average quadrantal error	7.01E-7	2.23E-6	7.10E-6	2.32E-5	4.88E-5	8.22E-5
f_4	average value	2.0282E-2	2.0283E-2	2.0284E-2	2.0286E-2	2.0301E-2	2.0316E-2
	average quadrantal error	1.91E-6	6.06E-6	1.91E-5	6.12E-5	1.24E-4	2.02E-4

5 Conclusion

In this paper, the cumulant-based harmonic retrieval approaches of real data are discussed. Topics of determinate initial phase, multi-harmonic and wide dynamic are studied. The defects of harmonic retrieval and noise prefiltering algorithms are pointed out. Finally, The improved algorithm for wide dynamic range signal is given. The simulations show that the improved algorithm is effective and satisfactory. The proper understanding of cumulant-based harmonic retrieval can be obtained from this paper and it is of important engineering guidance value to applications.

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本文作者简介

姚晓东 1970年生.2000年获西北工业大学导航、制导与控制专业博士学位.现在上海交通大学控制科学与工程流动站做博士后研究.研究方向为智能信号处理与信息融合.

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金德琨 1938年生.中国航空工业第二集团科技委常委.博士生导师.上海交通大学兼职教授.研究方向为航空电子系统综合.