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Study on the Passive Stabilization of Motion of a Class of Lagrange System

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Abstract: The idea of the passive stabilization of nonasymptotically stable motion of the dynamical systems by introducing supplementary degrees of freedom was advanced for the first time in paper [1]. The effectiveness of the application to the study of Lagrangian system was shown in the specific example of this paper, which has the unique scientific significance. In the system nonlinear friction and inelastic potential energy are adopted. It was shown that the problem of the passive stabilization can be solved in the nonlinear case [2]. It was shown that the problem of the passive stabilization can be solved in the nonlinear and inelastic potential in this paper.

Key words: passive stabilization; equilibrium; nonlinear; generalized force

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一类 Lagrange 系统的 PSM 问题的进一步研究

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摘要: 文献[1]第一次提出 PSM 思想,得到了广泛的应用.本文根据 PSM 思想,建立一个具有特殊意义模型,继文献[2]在非线性拉格朗日系统中,解决具有非线性摩擦的 PSM 后,进一步研究具有非线性摩擦和非弹性势能的 PSM,解决了模型的 PSM 控制.

关键词: 非自治运动; 平衡稳定性; 非线性; 广义力

1 Statement of the problem

The attitude of a satellite is often controlled by reactive forces requiring some additional energy. But the satellite can also be stabilized by means of the relative motion of some piece of the satellite moving in nonideal fluid as an oscillator with damping. This does not require additional energy and is called "passive stabilization". Here we consider passive stabilization for Largrangian system from the specific example, which has the independent scientific meaning.

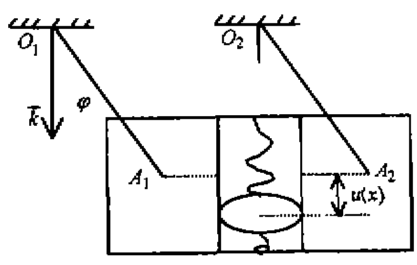

Fig. 1 The special of PSM

Let an absolutely rigid body S with a mass M performs plane-parallel movement under the action of gravity as the parallelogram pendulum (Fig. 1). Vector $\vec{k}$ is parallel to the vector of gravity, lines O_1O_2 and A_1A_2 are vertical to vector $\vec{k}$ during the whole motion. The position of S in space is determined by angle φ (Fig. 1). Moreover, we suppose a block s with mass m is contained in S. Now we consider two cases. In the first case, s is fixed to body S (or is frozen in S). In the second one, s can move with a friction (nonlinear) under the action of some spring fixed to body S and its moving direction is vertical to A_1A_2. The position of the block s to body S is determined by coordinate u (Fig. 1). Thus, we can say, $u = u_0$ = constant in the first situation and $u = u(t)$ in the second one. Obviously, in the first situation the equilibrium state defined by $\varphi = 0$ and $u = u_0$ is stable, yet it is nonasymptotically stable. Is the equilibrium state asymptotically stable when s is defrozen? If it is asymptotically stable, we can say the problem of pass-

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ive stabilization is solved. The following statement will verify that this problem can be solved only in nonlinear case by the method in paper [3].

2 Movement equation and their reduction to special form

Block s moves with an inelastic potential energy $\frac{1}{2}\beta u^2 + \frac{1}{4}\bar{\beta}u^4$, where $\beta > 0, \bar{\beta} > 0$, and with a nonlinear friction $Q_u = -\alpha_1\dot{u} - \alpha_2\dot{u}^3$, namely the generalized force, where $\alpha_1 > 0, \alpha_2 > 0$. Thus, the studied mechanical system has Lagrange's equation of the second type as follows

$$\begin{cases} M_* l^2\ddot{\varphi} - ml\sin\varphi\ddot{u} + M_* gl\sin\varphi = 0, \\ m\ddot{u} - ml\sin\varphi\ddot{\varphi} - ml\dot{\varphi}^2\cos\varphi - mg + \\ \beta u + \bar{\beta}u^3 + \alpha_1\dot{u} + \alpha_2\dot{u}^3 = 0. \end{cases} \tag{1}$$

For the sake of convenience we reduce equation (1) to a undimensional form by introducing undimensional time τ and length u' subject to

$$t = \sqrt{\frac{l}{g}}\tau,\ \mathrm{d}\tau = \sqrt{\frac{g}{l}}\mathrm{d}t,\ u' = \frac{u}{l}.$$

Thus equation (1) can be written as

$$\begin{cases} \ddot{\varphi} = \\ \dfrac{\sin\varphi(-M' + \dot{\varphi}^2\cos\varphi + 1 - \beta' u' - \bar{\beta}' u'^3 - \alpha_1'\dot{u}' - \alpha_2'\dot{u}'^3)}{M' - \sin^2\varphi}, \\ \ddot{u}' = \\ \dfrac{M'(\dot{\varphi}^2\cos\varphi + \cos^2\varphi - \beta' u' - \bar{\beta}' u'^3 - \alpha_1'\dot{u}' - \alpha_2'\dot{u}'^3)}{M' - \sin^2\varphi}, \end{cases} \tag{2}$$

where

$$M' = \frac{M_*}{m} > 1,\ \beta' = \frac{\beta l}{mg} > 0,$$

$$\bar{\beta}' = \frac{\bar{\beta}l^3}{mg},\ \alpha_1' = \frac{\alpha_1}{m}\sqrt{\frac{l}{g}} > 0,$$

$$\alpha_1' = \frac{\alpha_1}{m}\sqrt{\frac{l}{g}} > 0,\ \alpha_2' = \frac{\alpha_2 l\sqrt{lg}}{m}.$$

Equation (2) admits particular solution

$$\varphi = 0,\ u' = u_0,\ 1 - \beta u_0 - \bar{\beta}u_0^3 = 0, \tag{3}$$

corresponding to a equilibrium state of the system.

Supposing $\varphi = x$ and $u' = u_0 + v$ in perturbed movement, we can decompose the right side of equation (2) to series about perturbation $x, \dot{x}, v, \dot{v}$ till terms of the third order smallness (including the third order) as follows:

$$\begin{cases} \ddot{x} = a_1x + b_1x(u_0+v) + b_2x\dot{v} + c_1x^3 + c_2x\dot{x}^2 + \cdots, \\ \ddot{v} = 1 + d_1(u_0 + v) + d_2\dot{v} + a_1x^2 + \dot{x}^2 + \\ \quad b_1(u_0+v)x^2 + b_2x^2\dot{v} + e(u_0 + v)^3 + f\dot{v}^3 + \cdots, \end{cases} \tag{4}$$

where

$$a_1 = \frac{1 - M'}{M'},\ b_1 = -\frac{\beta'}{M'},\ b_2 = -\frac{\alpha_1'}{M'},$$

$$c_1 = \frac{(1 - M')(6 - M')}{6M'^2},\ c_2 = \frac{1}{M'},$$

$$d_1 = -\beta',\ d_2 = -\alpha_1',\ e = -\bar{\beta}',\ f = -\alpha_2'.$$

By introducing variables

$$\dot{x} = y,\ \dot{v} = w, \tag{5}$$

we have the following equation, which consists of four differential equations of the first order

$$\begin{cases} \dot{x} = \lambda y, \\ \dot{y} = -\lambda x + p_1xv + p_2xw + q_1x^3 + q_2xy^2 + \cdots, \\ \dot{v} = w, \\ \dot{w} = \bar{d}_1v + d_2w - \lambda^2x^2 + \lambda^2y^2 + \tau v^2 + b_1x^2v + \\ \quad b_2x^2w + ev^3 + fw^3 + \cdots, \end{cases} \tag{6}$$

where

$$p_1 = \frac{b_1}{\lambda},\ p_2 = \frac{b_2}{\lambda},$$

$$q_1 = \frac{c_1}{\lambda}, q_2 = \lambda c_2,\ \tau = -3\bar{\beta}'u_0,$$

$$\bar{d}_1 = -\beta' - 3\bar{\beta}'u_0^2 < 0,\ \lambda^2 = -(a_1 + u_0b_1) > 0.$$

It is possible to point out that the linearized system of equation (6) falls into two independent linear systems, the first

$$\dot{x} = \lambda y,\ \dot{y} = -\lambda x,$$

corresponds to a pair of purely imaginary roots of its characteristic equation, and the second

$$\dot{v} = w,\ \dot{w} = \bar{d}_1v + d_2w \tag{7}$$

corresponds to a pair of complex roots with negative real parts. By conclusion of known Lyapunov's theorem (see [3]), we have a critical case, from which it is impossible to obtain the conclusion about stability or instability of solution (3) in system (4) without using nonlinear terms.

3 Constructing auxilary function of Lyapunov's type

Since linear system (7) has eigenvalues of negative real part (since $\bar{d}_1 < 0, d_2 < 0$), there exists definitely positive Lyapunov's function $V^{(2)}(v,w)$ such that its time derivative $\dot{V}^{(2)}(v,w)$ along equation (7) is negative definite. Those conditions satisfy function

$$V^{(2)}(v,w) = \frac{1}{2}(m_{11}v^2 + 2m_{12}vw + m_{22}w^2). \tag{8}$$

It satisfies some conditions, make its time derivative

$$\dot{V}^{(2)}(v,w) = -(\lambda^2 v^2 + w^2) < 0. \tag{9}$$

Moreover, according to the methodology of critical case of n pair purely imaginary[4], we construct auxililary function $V(x,y,v,w)$ as follows:

$$V(x,y,v,w) = \frac{p}{2}(x^2+y^2) + qV^{(2)}(v,w) + V^{(3)}(x,y,v,w) + V^{(4)}(x,y,v,w), \tag{10}$$

where $V^{(2)}(v,w)$ is defined by formula (10), and

$$V^{(r)}(x,y,v,w) = \sum_{i+j+k+n=r} a_{ijkn}x^i y^j v^k w^n, \ r = 3,4 \tag{11}$$

are forms of the third and the fourth order about x, y, v, w, here a_{ijkn} are constant coefficients and its algorithm will be given later, p and q are arbitrary constants.

The derivative along equation (6) by virtue of equality (10) has the following form:

$$\dot{V}(x,y,v,w) = -q(\lambda^2v^2+w^2)+\dot{V}^{(3)}(x,y,v,w)+\dot{V}^{(4)}(x,y,v,w)+\cdots,$$

where $\dot{V}^{(3)}$ and $\dot{V}^{(4)}$ are the forms of the third and the fourth order respectively defined by:

$$\dot{V}^{(r)}(x,y,v,w) = \lambda y\frac{\partial V^{(r)}}{\partial x} - \lambda x\frac{\partial V^{(r)}}{\partial y} + w\frac{\partial V^{(r)}}{\partial v} + (\bar{d}_1 v + d_2 w)\frac{\partial V^{(r)}}{\partial w} + W^{(r)}(x,y,v,w), \ r = 3,4, \tag{12}$$

where

$$W^{(3)}(x,y,v,w) = py(p_1xv + p_2xw) + q(m_{12}v + m_{22}w)(-\lambda^2x^2 + \lambda^2y^2 + \tau v^2),$$

$$W^{(4)}(x,y,v,w) = py(q_1x^3+q_2xy^2)+q(m_{12}v+m_{22}w)(b_1x^2v+b_2x^2w+ev^3 + fw^3)+(p_1xv+p_2xw)\frac{\partial V^{(3)}}{\partial y}+(-\lambda^2x^2+\lambda^2y^2+\tau v^2)\frac{\partial V^{(3)}}{\partial w}. \tag{13}$$

We can seek the coefficients of the form $V^{(3)}(x,y,v,w)$ from the condition

$$\dot{V}^{(3)}(x,y,v,w) = \lambda y\frac{\partial V^{(3)}}{\partial x} - \lambda x\frac{\partial V^{(3)}}{\partial y} + w\frac{\partial V^{(3)}}{\partial v} + (\bar{d}_1 v + d_2 w)\frac{\partial V^{(3)}}{\partial w} + W^{(3)} \equiv 0, \tag{14}$$

where $W^{(3)}$ is defined by formula (13).

According to [3], this equation in the form $V^{(3)}(x,y,v,w)$ has a unique solution.

Substituting $V^{(3)}(x,y,v,w)$ defined by equation (12) into equation (14), we can see that all nonzero coefficients of $V^{(3)}$ are function of arbitrary parameters p and q. In

$$W^{(4)}(x,y,v,w) = \sum_{i+j+k+n=4} w_{ijkn}x^iy^jv^kw^n, \tag{15}$$

we only need to find out the coefficients $w_{4000}, w_{0400}, w_{2200}$, which are necessary to the solution of the problem of passive stabilization.

Now we can seek the coefficients of the form $V^{(4)}(x,y,v,w)$ from the condition

$$\dot{V}^{(4)}(x,y,v,w) = \lambda y\frac{\partial V^{(4)}}{\partial x} - \lambda x\frac{\partial V^{(4)}}{\partial y} + w\frac{\partial V^{(4)}}{\partial v} + (\bar{d}_1 v + d_2 w)\frac{\partial V^{(4)}}{\partial w} + W^{(4)}(x,y,v,w) = C(x^2+y^2)^2, \tag{16}$$

where $V^{(4)}$ and $W^{(4)}$ are defined by equations (11) and (15) respectively.

From the lemma of [4], there exists a unique C, in which equation (14) has solution in the form $V^{(4)}$, and also

$$C = \frac{1}{8}(3w_{4000}+w_{2200}+3w_{0400}) = \frac{\lambda^2}{4}(a_{0201}-a_{2001}). \tag{17}$$

In Eqs. (14) and (17). We first suppose $p = 1, q = 0$, then we obtain functions as follows:

$$C_p = C\,|_{q=0}^{p=1} = -\frac{1}{2}\lambda^2 a_{2001} = -\frac{\lambda^2(\beta' + 4\lambda^2 + \bar{d}_1)a_1'}{2M'[(4\lambda^2+\bar{d}_1)^2 + 4\lambda^2 d_2^2]} < 0. \tag{18}$$

Thus we obtain

$$V = \frac{p}{2}(x^2+y^2) + qV^{(2)}(v,w) + V^{(3)}(x,y,v,w) + V^{(4)}(x,y,v,w), \tag{19}$$

where the coefficients of $V^{(4)}$ are linear functions of arbitrary parameters p and q as those of $V^{(3)}$, and its derivative along equation (6) is

$$\dot{V} = -q(\lambda^2 v^2 + w^2) + (pC_p + qC_q)(x^2 + y^2)^2 + \cdots, \tag{20}$$

where the ellipses denote infinitely small terms whose orders are higher than the fourth.

So the particular solution in equation (3) is asymptotically stable. This shows that the problem of passive stabilization about oscillating of body S is solved by introducing supplementary degree of freedom which is determined by coodinate u (namely block s is defrozen).

Remark We point out that if we use a geostationary satellite to take a picture of a certain object on the ground, it is very important for the camera to take aim, namely the motion of the satellite must be asymptotically stable. Usually, the effect brought by a small pertubation is removed by the reaction of a jet stream, which is produced by burning fuel, but the astronautic fuel is very expensive (Fig.2, Fig.3 in the paper [2]). The result obtained by us shows that the satellite can also be stabilized by means of the relative motion of some piece of the satellite moving in a nonideal fluid as an oscillator with damping and this does not require additional energy.

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6 Conclusion

Two dynamical identification schemes of stator flux and rotor speed proposed in this paper are based on special model reference adaptive principle, both of them use obtainable information fully. According to Lyapunov stability theorem, adaptive laws of parameters are synthesized to make state and parameters converge quickly even the parameter variant laws are not sure. Simulation shows their advance.

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