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On the Equivalence Classes of Fusion Rules for Distributed Multisensor Decision Systems *

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Abstract: Under the assumption that the optimal sensor rules can be found for any given fusion rule, we analyze the conditions resulting in the performance equivalence and superiority between fusion rules for general distributed multisensor decision systems. To obtain globally optimal performance of a system, by applying the above results, we can partition all possible fusion rules into equivalence classes of fusion rules and compare performances between some of the equivalence classes; therefore, the number of the valuable fusion rules will be reduced greatly. Moreover, the above analysis does not depend on the statistical properties of the observation data as well as the objective of optimizing a certain system performance.

Key words: data fusion; multisensor decision system; fusion rules

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分布式多传感器决策系统融合律的等价类

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摘要: 在任何融合律给定后最优传感器律能求得的假设下,我们分析了导致融合律之间等价性和优越性的条件.应用如上结果,欲获全局最优的系统性能,我们可以划分所有可能的融合律为若干等价类和比较某些等价类之间的性能,于是有价值的融合律等价类数目将大大减少,而且上面的分析并不依赖于观测数据的统计性质和优化系统性能的目标.

关键词: 数据融合; 多传感器决策系统; 融合律

1 Introduction

Consider a distributed multisensor system making a decision which one occurs among multiple hypotheses. The data collected by the sensors are first processed locally to compress them to one or a few binary digits (information bits), which are then transmitted to the fusion center that makes a decision by combining the received set of local messages under a given fusion (final decision) rule with the objective of optimizing a certain system performance. Therefore, to obtain globally optimal performance of a system, we should find an optimal fusion rule and the corresponding set of optimal local compression rules. However, this is a very difficult task since optimal fusion rule in general depends on the statistical properties of the observation data as well as the objective of optimizing the system performance. To reduce the number of fusion rules which are needed to check if it is optimal, when all sensor rules are given, and sensor observation data are conditionally independent mutually as well as $P_{Di} \geqslant P_{Fi}$, where P_{Di} and P_{Fi} are all the sensor probabilities of detection and false alarm respectively, a monotonic fusion rule was defined and the optimal fusion rule can be found in the class of monotonic fusion rules (see [1] and [2, Remark 3.3.1, Page 63]). In this way, the fusion rules considered can be reduced greatly. However, this result is not suitable to global system optimization because all sensor rules were assumed to be given already.

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In this article, from the point of view of globally optimal performance, we analyze the equivalence of some of the fusion rules for general distributed multisensor multi-hypothesis decision problem. To focus on comparing the performances of different fusion rules, suppose one can always find out optimal local compression rules for any given fusion rule (for a specific algorithm, cf. [3,4]). After making such an assumption, we can concentrate only on fusion rules and show under what condition two different fusion rules are actually equivalent with the same final cost, and one may be superior to another. Thus, by partitioning the set of all fusion rules to equivalence classes, comparing performance between some equivalence classes and dropping off those "worse" fusion rule classes, the number of the valuable fusion rules will be reduced, such as from 16 to 3 for two sensor binary decision system, or from 256 to 32 for three sensor binary decision system. For more general multisensor m-ary decision systems, one can use computer to analyze valuable fusion rules according to the results in this paper.

2 Problem formulation

2.1 Model

We consider m hypotheses: $H_1, H_2, \cdots, H_m$, multisensor (multiple observation data) $y_1, \cdots, y_l$ distributed decision problem. A set of local compression rules:

$$(I_1^{(1)}(y_1), \cdots, I_1^{(r_1)}(y_1); \cdots; I_l^{(1)}(y_l), \cdots, I^{(r_l)}(y_l))$$

compresses data y_i to $r_i (\leqslant l)$ information bits at each sensor, respectively. Obviously, different r_i binary digits can correspond to 2^{r_i} different integers, that is to say, the ith sensor quantizes its observation data y_i to 2^{r_i} different integers.

The fusion rule – final decision rule of the fusion center is given by an m-valued function $F(I_1^{(1)}(y_1), \cdots, I_1^{(r_1)}(y_1); \cdots; I_l^{(1)}(y_l), \cdots, I_l^{(r_l)}(y_l)): \{0,1\}^{\sum_{i=1}^{l} r_i} \mapsto \{0,1,\cdots,m-1\}$. Denote $N = \sum_{i=1}^{l} r_i$. In fact, the above fusion rule divides a set of 2^N different N-tuples $(I_1^{(1)}, \cdots, I_1^{(r_1)}; \cdots; I_l^{(1)}, \cdots, I_l^{(r_l)})$ into m disjoint subsets generally. The number of all the above different partitions is m^{2^N}. In addition, a fixed fusion rule with a variety of local compression rules can produce a variety set of different final decision regions, which is called the *variety set of final decision regions generated by this fusion rule*. The goal of our distributed multisensor decision is to find a globally optimal final decision region of $(y_1, y_2, \cdots, y_l)$ determined by an optimal partition of the set of $\sum_{i=1}^{l} r_i$-tuples $(I_1^{(1)}, \cdots, I_1^{(r_1)}; \cdots; I_l^{(1)}, \cdots, I_l^{(r_l)})$ and the corresponding optimal local compression rules to minimize a desired cost functional. Suppose one can always find out optimal sensor rules solution for any given cost functional and any fixed fusion rule; thus, minimizing cost functional just depends on the variety set of the final decision regions generated by the fusion rule.

2.2 Some definitions

Definition 1 If two fusion rules have the same generated variety set of final decision regions, they are called to be equivalent. Clearly, the minimum costs for the equivalent fusion rules are identical. Collecting all equivalent fusion rules yields a fusion rule equivalence class.

Definition 2 If the variety set of final decision regions generated by the first fusion rule is a real subset of the variety set of final decision regions generated by the second one, we call the latter to be superior to the former.

Obviously, Definition 2 is reasonable because optimization result over larger domain is superior to that over smaller domain.

Definition 3 We call $1 - I_i^{(j)}(y_i)$ the complementary indicator function of $I_i^{(j)}(y_i)$. Obviously, $1 - I_i^{(j)}(y_i)$ is also a compression rule for data y_i, and its 0 compression region is just the 1 compression region of $I_i^{(j)}(y_i)$, i.e.

$$\{y_i : 1 - I_i^{(j)}(y_i) = 0\} = \{y_i : I_i^{(j)}(y_i) = 1\}.$$

3 Propositions

Using the above definitions, we have the following two propositions to classify fusion rules and compare performances between two fusion rules.

Propositions 1 If a fusion rule comes from another fusion rule by changing some of the local compression rules in the latter fusion rule to their complementary indicator functions, the both fusion rules are equivalent.

Now we give an example to show two fusion rules are equivalent. Suppose

$$\mathcal{H}_0^{(1)} = \{(y_1, y_2): (I_1(y_1) = 0, I_2(y_2) = 0)\}$$

and

$$\mathcal{H}_0^{(2)} = \{(y_1, y_2): (I_1(y_1) = 0, I_2(y_2) = 1)\}$$

are two different H_0 decision regions for 2 sensor binary decision problem. In the sequel, for notational simplicity denote the above two fusion rules by

$$\{(0^{(1)}, 0^{(2)})\}_{H_0} \text{ and } \{(1^{(1)}, 1^{(2)})\}_{H_0},$$

respectively. By Proposition 1 the two fusion rules are equivalent.

From Definition 2 we have

Proposition 2 If restricting some of the local compression rules in a fusion rule to be identical to 1 or 0, i.e., only one choice above for those local sensors or rules, can yields another fusion rule, the former should be superior to the latter.

For example, the fusion rule $\{(1^{(1)}, 1^{(2)})\}_{H_0}$ should be superior to the fusion rule $\{(1^{(1)}, 1^{(2)}), (1^{(1)}, 0^{(2)})\}_{H_0}$ because restricting the second local compression rule $I_2(y_2) \equiv 1$ in the first fusion rule yields the second fusion rule just like ignoring the observation data of the second sensor.

In the above argument, only local compression regions and final decision regions are considered, no test type and cost functional are involved; therefore, the above results do not depend on statistical properties of observation data, and hold for all other distributed decision problems, such as Neyman-Pearson test.

4 Applications

We can now apply the above two propositions to analyze any two fusion rules if they are equivalent and if one may be superior to another. As examples, the analysis results for two most popular distributed multisensor decision systems are presented below.

4.1 Two sensor binary decision

Suppose each sensor compresses its observation data to 1 information bit. There exist 4 different sets of local messages as follows:

$$(0^{(1)}, 0^{(2)}), (1^{(1)}, 0^{(2)}), (0^{(1)}, 1^{(2)}), (1^{(1)}, 1^{(2)}).$$

Clearly, they can produce 16 different fusion rules.

Ⅰ 1-set-of-local-message case.

$$\{(0^{(1)}, 0^{(2)})\}_{H_0}, \{(1^{(1)}, 0^{(2)})\}_{H_0},$$

$$\{(0^{(1)}, 1^{(2)})\}_{H_0}, \{(1^{(1)}, 1^{(2)})\}_{H_0},$$

which are all equivalent to each other (OR rule for H_1 decision).

Ⅱ 2-set-of-local-message case.

1) $\{(0^{(1)}, 0^{(2)}), (1^{(1)}, 0^{(2)})\}_{H_0}$,
$\{(0^{(1)}, 0^{(2)}), (0^{(1)}, 1^{(2)})\}_{H_0}$,
$\{(1^{(1)}, 1^{(2)}), (1^{(1)}, 0^{(2)})\}_{H_0}$,
$\{(1^{(1)}, 1^{(2)}), (0^{(1)}, 1^{(2)})\}_{H_0}$.

They are divided into two equivalence classes (ignoring Sensor 1 or Sensor 2) and both are worse than Class Ⅰ.

2) $\{(0^{(1)}, 0^{(2)}), (1^{(1)}, 1^{(2)})\}_{H_0}$,
$\{(0^{(1)}, 1^{(2)}), (1^{(1)}, 0^{(2)})\}_{H_0}$.

They are both equivalent to each other (XOR rule).

Ⅲ 3-set-of-local-message case.

$$\{(0^{(1)}, 0^{(2)}), (0^{(1)}, 1^{(2)}), (1^{(1)}, 0^{(2)})\}_{H_0},$$

$$\{(1^{(1)}, 1^{(2)}), (0^{(1)}, 1^{(2)}), (1^{(1)}, 0^{(2)})\}_{H_0}.$$

They are both equivalent to each other (AND rule for H_1 decision).

Ⅳ 4-set-of-local-message case.

$$\{(0^{(1)}, 0^{(2)}), (0^{(1)}, 1^{(2)}), (1^{(1)}, 0^{(2)}), (1^{(1)}, 1^{(2)})\}_{H_0}.$$

This fusion rule implies that the fusion center always says H_0 occurs no matter what data are observed at the two local sensors. It can also be regarded as a special OR rule with the two particular local compression rules $(I_1(y_1) \equiv 0, I_2(y_2) \equiv 0)$; therefore, it is worse than OR rule as well as Class Ⅱ.

Ⅴ 0-set-of-local-message case.

Here 0-set-of-local-message case means the fusion center always chooses H_1 for any local observation data. Similar to the last case, it is worse than OR rule and Class Ⅱ.

To sum up, in two sensor binary decision case, 16 fusion rules are divided into 7 equivalence classes where only 3 classes are valuable to find global optimum fusion rule and the corresponding 2 compression rules.

4.2 Three sensor binary decision

Still, suppose each sensor compresses its observation data to 1 information bit. There exist 8 different sets of local messages as follows:

$$(0^{(1)}, 0^{(2)}, 0^{(3)}), (1^{(1)}, 0^{(2)}, 0^{(3)}), (0^{(1)}, 1^{(2)}, 0^{(3)}),$$
$$(0^{(1)}, 0^{(2)}, 1^{(3)}), (1^{(1)}, 1^{(2)}, 0^{(3)}), (1^{(1)}, 0^{(2)}, 1^{(3)}),$$
$$(0^{(1)}, 1^{(2)}, 1^{(3)}), (1^{(1)}, 1^{(2)}, 1^{(3)}).$$

Clearly, they can produce 256 different fusion rules. Using the above two propositions and the similar method, we can show there are only the following 32 valuable equivalence classes of the fusion rules in this case. If we know more information about sensors, for example the sensors are all identical, the number of the valuable equivalence classes will be reduced again. In the above example, they will be reduced to 16 valuable fusion rules.

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