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An Adaptive Variable Structure Control Approach for Mobile Robots *

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Abstract: This paper addresses the robust tracking of mobile robots with uncertain parameter variations. Based on the fact that a controllable driftless system with m inputs and at most $m+2$ state variables is a differentially flat system, we develop a robust control approach to the trajectory tracking problem of a three-wheel mobile robot via the dynamical extension approach and adaptive variable structure control technique. By applying the dynamical extension approach to the kinematic model of the three-wheel mobile robot, the exact linearization of the extended dynamic feedback system is first realized under mild regular condition. In designing the robust control law we adopt the control interpolation in a boundary layer and on-line parameter estimation to avoid the control chattering under large parameter uncertainties. Finally, numerical simulations are performed to show the efficiency of the proposed approach.

Key words: mobile robots; underactuated nonlinear systems; dynamic feedback; exact linearization; adaptive variable structure control

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一种关于移动机器人的自适应变结构控制方法

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摘要: 基于具有 m 个输入、至多 $m+2$ 个状态变量的可控无漂移系统为微分平面系统这样一个事实,通过动态扩展原理和自适应控制技术,本文提出了一种针对三轮移动机器人轨迹跟踪问题的鲁棒自适应控制方法. 应用动态扩展原理首先对三轮移动机器人的运动模型实现了精确线性化,设计变结构控制时采用了边界层内插方法和在线参数估计以削弱控制抖动. 仿真结果表明该方法优于一般的变结构控制.

关键词: 移动机器人; 动态反馈; 精确线性化; 自适应滑动控制

1 Introduction

There has been an increasing interest in the design of feedback control laws for nonholonomic mobile robots in recent years due to their wide applications in real world[1-8]. Mobile robots, as typical underactuated control systems, play an important role in the theory and applications of nonholonomic control systems[1,4,7,9-14]. The fact that no time-invariant smooth state feedback laws exist for the stabilization of nonholonomic systems makes the control and motion planning of mobile robots difficult. It is thus appealing for us to explore the applications of other nonlinear control techniques. Various approaches have been proposed for such control problems in recent years, such as time varying feedback[7,13,15], approximate linearization[14], input-output linearization[10,11] and state feedback linearization.

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Variable structure control is a special discontinuous control technique applicable to a broad variety of practical systems due to its easy implementation and good robustness to unmodeled dynamics, parameter variations and external disturbances. Unfortunately chattering phenomenon existing in sliding mode is now one of the main obstacle for wide application of this control strategy, which will excite the unmodeled high-frequency dynamics, lead to large control force and power consumption, and steady state error.

Based on the fact that any controllable driftless system with m inputs and at most $m+2$ state variables is a differentially flat system[16,17] and motivated by the dynamical extension approach in nonlinear control systems[18,19], we develop an adaptive variable structure control approach to the trajectory tracking problem of a three-wheel mobile robot in the present paper. In the next section, the dynamical extension approach is applied to the kinematic model of the three-wheel mobile robot to achieve full state and parameter linearization of the extended system. Section 3 focuses on the design of adaptive variable structure controllers to avoid the control chattering without affecting the tracking performance seriously. We connect on-line parameter estimation with boundary layer in the situations of large parameter uncertainty. Numerical simulation results are given in Section 4 to illustrate the efficiency of the current approach.

2 Problem formulation

2.1 Kinematic model

The kinematics of a two-wheel driven mobile robot as shown in Fig. 1 can be described by

$$\begin{cases} \dot{x} = \dfrac{1}{2}(R\omega_r + R\omega_l)\cos\theta, \\ \dot{y} = \dfrac{1}{2}(R\omega_r + R\omega_l)\sin\theta, \\ \dot{\theta} = \dfrac{1}{2D}(R\omega_r - R\omega_l). \end{cases} \tag{1}$$

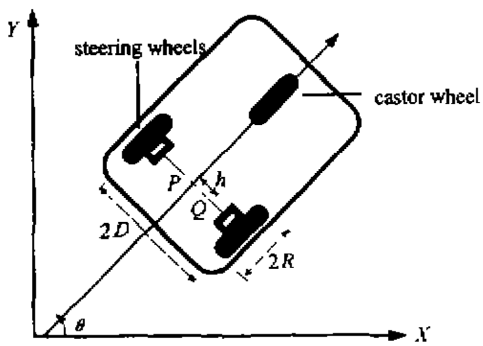

Fig. 1 Two-wheel driven mobile robot

Here x, y and θ are the positions of point P corresponding to the midpoint of the rear axis and orientation relative to the x-axis of the vehicle, ω_r, ω_l are the angular velocity of the two driven motors, and $2R$ and $2D$ are diameter and distance of the two rear wheels respectively.

Applying the following input vector

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{2}(\omega_r + \omega_l) \\ \dfrac{1}{2D}(\omega_r - \omega_l) \end{pmatrix} \tag{2}$$

to (1), then it results in the following simple model

$$\begin{cases} \dot{x} = Ru_1\cos\theta, \\ \dot{y} = Ru_1\sin\theta, \\ \dot{\theta} = Ru_2. \end{cases} \tag{3}$$

The above model (1) or (3), as a kind of underactuated nonlinear systems, allows us to analyze the behavior of the robot within the framework of the theory of nonholonomic systems. However, it should be emphasized that there are two main drawbacks in the existing literature. One is that the model is kinematic (correspondingly the control efforts are actually velocity), but the controls applied to the robot are usually the generalized forces, which makes the implementation difficult. The other one is the sensitivity to the nonidealities such as tire deformation, since almost all the existing controllers heavily depend on the assumption of known parameters. Thus it is imperative for us to develop robust controller design approach for mobile robot systems, as indicated in the recent surveys [3] and [4].

2.2 Dynamic feedback

It is known that nonholonomic systems can not be exactly linearized via state and input transformations. However, through proper output selection, the input-output linearization can be achieved as indicated in [3] and [4]. Moreover, it is possible to realize the exact linearization if we extend the system by proper dynamic compensations.

It has been shown that any controllable driftless system with m inputs and at most $m+2$ states is a differentially flat system which can be fully linearizable by dynamic feedback (c.f., [16] and [17]). Since the kinematic model (3) is of two-input and three states driftless form, it thus can be linearizable via dynamic feedback. First, we define two suitable linearizing output functions which correspond to the position of Q point of the rear axis

$$\begin{cases} z_1 = x + h\sin\theta, \\ z_2 = y - h\cos\theta, \end{cases} \tag{4}$$

then, from (3) and (4), we obtain

$$\begin{cases} \dot{z}_1 = R\cos\theta(u_1 + hu_2), \\ \dot{z}_2 = R\sin\theta(u_1 + hu_2). \end{cases} \tag{5}$$

Introducing the extended state variable

$$z_3 = u_1 + hu_2 \tag{6}$$

and auxiliary input ω_1 such that

$$\dot{z}_3 = \omega_1, \tag{7}$$

then, with the extended state vector $z_e = (x, y, \theta, z_3)^{\mathrm{T}}$, and the associated inputs u_2 and ω_1, we obtain the extended system as follows

$$\begin{cases} \dot{x} = R(z_3 - hu_2)\cos\theta, \\ \dot{y} = R(z_3 - hu_2)\sin\theta, \\ \dot{\theta} = Ru_2, \\ \dot{z}_3 = \omega_1. \end{cases} \tag{8}$$

From (5) and (8) it follows that

$$\begin{cases} \ddot{z}_1 = R\cos\theta\omega_1 - R^2\sin\theta z_3 u_2, \\ \ddot{z}_2 = R\sin\theta\omega_1 + R^2\cos\theta z_3 u_2, \end{cases} \tag{9}$$

where

$$\begin{cases} z_3 = \dfrac{1}{R}\sqrt{\dot{z}_1^2 + \dot{z}_2^2}, \\ \cos\theta = \dfrac{\dot{z}_1}{\sqrt{\dot{z}_1^2 + \dot{z}_2^2}}, \\ \sin\theta = \dfrac{\dot{z}_2}{\sqrt{\dot{z}_1^2 + \dot{z}_2^2}}, \end{cases} \tag{10}$$

then

$$\begin{cases} \ddot{z}_1 = R\left(\dfrac{\dot{z}_1}{\sqrt{\dot{z}_1^2 + \dot{z}_2^2}}\omega_1 - \dot{z}_2 u_2\right), \\ \ddot{z}_2 = R\left(\dfrac{\dot{z}_2}{\sqrt{\dot{z}_1^2 + \dot{z}_2^2}}\omega_1 + \dot{z}_1 u_2\right). \end{cases} \tag{11}$$

Obviously, the new decoupling matrix is singular only when the linear velocity z_3 of Q point is zero. Let

$$\begin{cases} \omega_1 = \dfrac{\dot{z}_1}{\sqrt{\dot{z}_1^2 + \dot{z}_2^2}}v_1 + \dfrac{\dot{z}_2}{\sqrt{\dot{z}_1^2 + \dot{z}_2^2}}v_2, \\ u_2 = -\dfrac{\dot{z}_2}{\dot{z}_1^2 + \dot{z}_2^2}v_1 + \dfrac{\dot{z}_1}{\dot{z}_1^2 + \dot{z}_2^2}v_2, \end{cases} \tag{12}$$

then it results in (11) or the following linear dynamic system

$$\ddot{Z} = Rv, \tag{13}$$

where $Z = [z_1\ z_2]^{\mathrm{T}}$ and $v = [v_1\ v_2]^{\mathrm{T}}$. So the extended system (8) is linearized by input transformation (12) and diffeomorphism (14) under regular condition $z_3 \neq 0$,

$$\Psi = (z_1\ \dot{z}_1\ z_2\ \dot{z}_2)^{\mathrm{T}}. \tag{14}$$

Consequently, the exact linearization which has some advantages compared with input-output linearization is realized via the dynamic extension algorithm. From the control standpoint, the above conclusion is meaningful since the resulted system is no longer in kinematic form, but in dynamic form. Thus one of the designed controls is the generalized force which is commonly used in current actuators.

3 Adaptive variable structure control

In the above section we assume that the related parameter radius R is exactly known. In fact the parameter radius may be varied to some extent mainly due to the wearing between the wheel and the ground. To obtain the robustness of the proposed control algorithm against parameter variation and to avoid control chattering, we will apply an adaptive robust control law here rather than classical sliding mode control to the trajectory tracking problem of this system.

To develop the adaptive robust control approach[20] we first define the switching surface $S = 0$ as

$$S = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right)\cdot \tilde{Z} = \dot{\tilde{Z}} + \lambda \cdot \tilde{Z}, \tag{15}$$

where $\tilde{Z} = Z(t) - P_d(t)$ denotes the tracking error, $S \in \mathbb{R}^2$ and λ is a positive constant. Again we define a boundary layer to smooth out the control discontinuities:

$$B(t) = \{X: |S(X,t)| \leqslant \phi\}. \tag{16}$$

In the paper we choose ϕ to be time-invariant, to derive a control law that ensures convergence to the boundary layer,. we first define a Lyapunov function candidate $V(t)$ as

$$V(t) = \frac{1}{2}[S_\Delta^{\mathrm{T}}S_\Delta + R(\hat{R}^{-1} - R^{-1})^2], \tag{17}$$

where

$$S_\Delta = S - \phi\,\mathrm{sat}(S/\phi) \tag{18}$$

is a measure of the algebraic distance of the current state to the boundary layer, $\hat{R}^{-1}$ can be thought of as the estimate of R^{-1}, we then select control law v as

$$v = -\hat{R}^{-1}(v^* + \epsilon \cdot \mathrm{sat}(S/\phi)), \tag{19}$$

where

$$v^* = -\ddot{P}_d(t) + \lambda \cdot \dot{\tilde{Z}} \tag{20}$$

can be thought of as the control when there is no adaptation error. since

$$\dot{V}(t) = S^{\mathrm{T}}\dot{S}_{\Delta} + \dot{\hat{R}}^{-1}(R\hat{R}^{-1} - 1), \tag{21}$$

using the fact that $\dot{S}_{\Delta} = \dot{S}$ outside the boundary layer and

$$\dot{S} = \ddot{Z} - \ddot{P}_d + \lambda \cdot \dot{\tilde{Z}} = \ddot{Z} + v^*, \tag{22}$$

connected with equation (13) we have

$$\dot{S}_{\Delta} = -R\hat{R}^{-1}(v^* + \varepsilon \cdot \mathrm{sat}(S/\phi)) + v^*, \tag{23}$$

then after computing

$$\dot{V}(t) = -R\hat{R}^{-1}[S_{\Delta}^{\mathrm{T}} \cdot v^* - \dot{\hat{R}}^{-1} + S_{\Delta}^{\mathrm{T}} \cdot \varepsilon \cdot \mathrm{sat}(S/\phi)] + S_{\Delta}^{\mathrm{T}} \cdot v^* - \dot{\hat{R}}^{-1}. \tag{24}$$

Selecting the adaptation laws to be:

$$\dot{\hat{R}}^{-1} = S_{\Delta}^{\mathrm{T}} \cdot v^* + S_{\Delta}^{\mathrm{T}} \cdot \varepsilon \cdot \mathrm{sat}(S/\phi) \tag{25}$$

leads to

$$\dot{V}(t) = S_{\Delta}^{\mathrm{T}} \cdot v^* - \dot{\hat{R}}^{-1} = -S_{\Delta}^{\mathrm{T}} \cdot \varepsilon \cdot \mathrm{sat}(S/\phi). \tag{26}$$

So that

$$\dot{V}(t) \leqslant -\varepsilon \cdot |S_{\Delta}|. \tag{27}$$

From the above deduction we can conclude that control law (19) and (25) can guarantee the trajectory eventually converge to the boundary layer and then tend to the prescribed trajectory along the sliding surface $S = 0$.

4 Simulation results

In order to show the efficiency of the adaptive variable structure control, we have performed numerical simulations, let the reference output trajectory $P_d(t)$ be given as

$$\begin{cases} x_d(t) = \sin(t), \\ y_d(t) = 1 - \cos(t). \end{cases} \tag{28}$$

The related control parameters are selected as $\lambda = 1$, $\varepsilon = 1$ and the thickness of the boundary layer $\phi = 0.01$ respectively, and the reciprocal of the wheel radius as $R^{-1} = 0.3$ but may be variable to some extent mainly due to the wearing between the wheels and the ground. With the estimated parameter $\hat{R}^{-1}$ fixed at 0.2, Fig. 2 shows the simulation results that there are large control chattering when the adaptive law is not adopted and possibly fail to track when the parameter uncertainty is big enough. With the adaptive law used (initial value of $\hat{R}^{-1}$ is 0.2), Fig. 3 shows that the control chattering is quickly weakened even for large parameter variation.

The regular condition ($Z_3 \neq 0$) should be satisfied in the control process, singularity of the controller is determined by the reference trajectory or the initial state of the system. Even if the desired trajectory does not result in the singularity, suitable initial state of the system can possibly do it but just for a few isolated points, a simple way to avoid the last situation is to force $Z_3 \neq 0$ in the singular points.

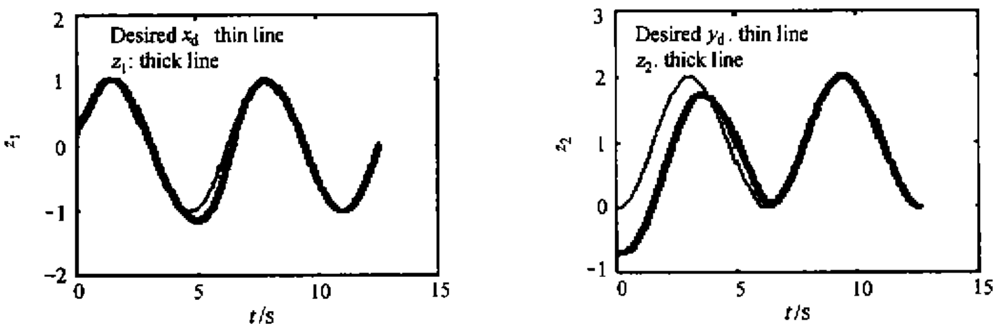

Fig. 2a Time curves of the output function without adaptation

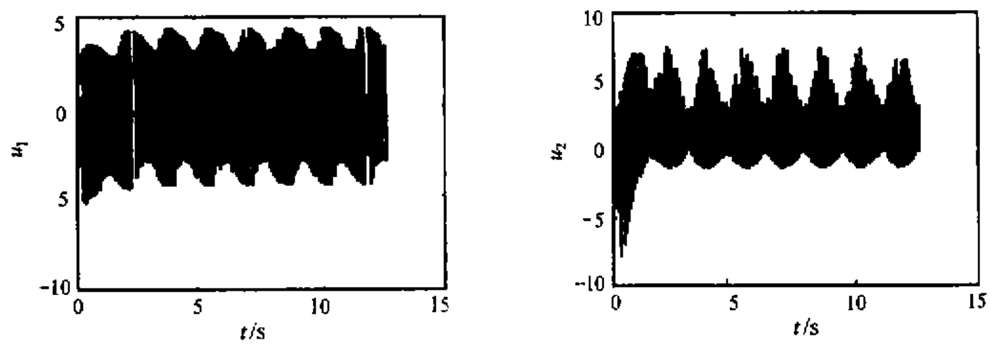

Fig. 2b Chattering of control u_1 and u_2 without adaptation

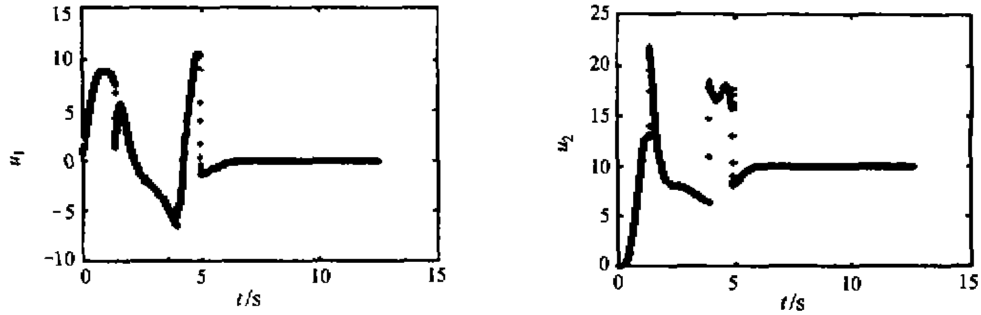

Fig. 3a Control u_1 and u_2 with adaptation

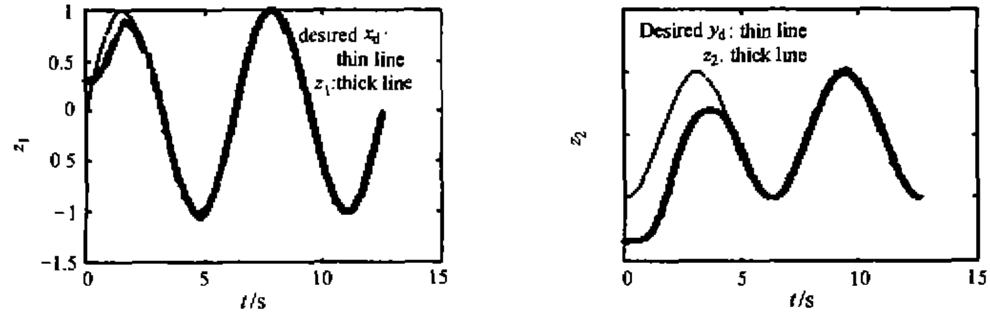

Fig 3b Time curves of output function with adaptation

5 Conclusions

An adaptive variable structure control approach is presented for the robust tracking of two-wheel driven mobile robot via dynamic feedback linearization. The sliding mode control and on-line parameter estimation is strongly coupled so as to have significant advantages over classical 'sliding mode control' both in reducing the control chattering and improving tracking performance in the presence of large parameter uncertainty.

Since adaptive parameter adjustment is not always realizable, it is usually necessary to realize parameter linearization. The approach presented in this paper is effective to solve a class of nonholonomic control problems. The application of the current approach will be carried out to investigate and perform implementations on a mobile robot developed by the researchers.

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