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Robust Stability for Uncertain Lurie Control Systems with Time-Delay *

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Abstract: This paper deals with the problem of robust absolute stability for uncertain nonlinear Lurie control systems with time-delay. The well-known Bellman-Gronwel inequality and the Lyapunov functional are used to study the robust absolute stability of Lurie control systems with time-delay, and the criteria for both delay-dependent and delay-independent stability are obtained. By using these conditions, one can estimate the robust absolute stability bound and the delay bound of uncertain nonlinear Lurie control systems with time-delay directly.

Key words: time-delay; uncertainty; robust absolute stability; Lurie control system

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具有时滞的不确定鲁里叶控制系统的绝对鲁棒稳定性

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摘要: 讨论了具有时滞的非线性不确定鲁里叶控制系统的鲁棒绝对稳定性问题.应用 Bellman-Gronwell 不等式和 Lyapunov 泛函方法研究了不确定鲁里叶控制系统的鲁棒绝对稳定性并给出了系统时滞相关稳定和时滞无关稳定的充分判据.运用这些条件可以直接估计具有时滞的非线性不确定鲁里叶控制系统的鲁棒绝对稳定界和时滞界.

关键词: 时滞; 不确定性; 鲁棒绝对稳定性; 鲁里叶控制系统

1 Introduction

Lurie control system is a class of the most important nonlinear control systems. Uncertainties in a control system may be due to modeling errors, measure errors and linearization approximations, and such errors always exist. The robustness is the most important property considered in the system design. Many researches have been made on the robust absolute stability of Lurie control systems[1-5]. Grujic and Petkovski (1987) presented robust absolute stability criteria in frequency and algebraic domains for a type of Lurie control system with multiple nonlinearities. Tesi and Vicino (1991) obtained frequency domain criteria of robust absolute stability for Lurie control systems in parameter space. Nian (1995, 1998, and 1999) investigated the robust stability of Lurie control system with interval coefficients and obtained some results. Since time-delay is frequently encountered in various engineering systems, the absolute stability of time-delay Lurie control systems has also been explored over the past decades. Konishi and Kokame (1999) discussed the robust absolute stability of uncertain Lurie control systems with time-delay by using LMI. More recently, Nian (1999) gave a simple delay-dependent stability criterion for time-delay Lurie control systems. However, there are few results on robust absolute stability of Lurie control systems with time-delay.

In this paper, the robust absolute stability of Lurie time-delay control systems is studied and both delay-dependent and delay-independent criteria are presented. By using these presented conditions, one can immediately

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estimate the robust stability bound and time-delay bound of Lurie control systems with time-delay.

In the sequel, $\| x \|$ denotes the Euclidean norm of vector x and $\| A \|$ denotes the induced Euclidean norm of matrix A. $\mu(A) = \frac{1}{2}\lambda_{\max}(A^{\mathrm{T}} + A)$ is the matrix measure of A.

2 Delay-independent robust absolute stability

Consider the following uncertain Lurie control system with time-delay

$$\begin{cases} \dot{x}(t) = (A + \Delta A(x(t),t))x(t) + \\ \qquad (B + \Delta B(x(t),t))x(t-\tau) + bu(t), \\ \sigma(t) = c^{\mathrm{T}}x(t), \\ u(t) = f(\sigma(t)), f(\cdot) \in K[0,k], \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $A, B \in \mathbb{R}^{n\times n}$ are real constant matrices satisfying $\mathrm{Re}\,\lambda(A) < 0$. $b, c \in \mathbb{R}^n$ are real constant vectors. $\Delta A(x(t),t), \Delta B(x(t),t) \in \mathbb{C}^{n\times n}[0,\infty)$ are uncertain nonlinear perturbations, which are bounded by the following inequalities:

$$\| \Delta A(x(t),t) \| \leqslant \beta_0, \ \| \Delta B(x(t),t) \| \leqslant \beta_1. \tag{2}$$

$K[0,k]$ is a sector domain described by

$K[0,k] = \{f(\cdot) \mid f(0) = 0;\ 0 < \sigma f(\sigma) \leqslant k\sigma^2, \sigma \neq 0\}$

$f(\cdot) \in K[0,k]$ is a real continuous function.

Theorem 1 System (1) is robust absolute stable if the following inequality holds.

$$\mu(A) + \beta_0 + \beta_1 + \| B \| + k \| b \| \| c \| < 0. \tag{3}$$

Proof The solution $x(t)$ of system (1) for $t > 0$ can be expressed as

$$x(t) = \exp(At)x(0) + \int_0^t \exp(A(t-s))[\Delta A x(s) + (B + \Delta B)x(s-\tau) + bf(\sigma(s))]\mathrm{d}s. \tag{4}$$

Evaluating the norm of both sides of (4) yields

$$\| x(t) \| \leqslant \| \exp(At) \| \| x(0) \| + \int_0^t \| \exp(A(t-s)) \| [\beta_0 \| x(s) \| + (\| B \| + \beta_1) \| x(s-\tau) \| + k \| b \| \| c \| \| x(s) \|]\mathrm{d}s, \tag{5}$$

where

$$\| x_0 \| = \sup_{\theta\in[-\tau,0]} \| x(\theta) \|.$$

Let

$$y(t) = \sup_{\theta\in[-\tau,0]} \| x(t+\theta) \|.$$

Thus, $y(0) = \| x_0 \|$. Using

$$\| \exp(At) \| \leqslant \exp(\mu(A)t),$$

we have

$$\| x(t) \| \leqslant \| \exp(\mu(A)t) \| y(0) + \int_0^t \| \exp(\mu(A)(t-s)) \| \cdot (\beta_0 + B_1 + \| B \| + k \| b \| \| c \|)y(s)\mathrm{d}s. \tag{6}$$

Setting

$$v(s) = \exp(\mu(A)(t-s))y(s),$$

thus

$$v(0) = \exp(\mu(At))y(0),$$

we have

$$v(t) \leqslant v(0) + \int_0^t (\beta_0 + \beta_1 + \| B \| + k \| b \| \| c \|)v(s)\mathrm{d}s. \tag{7}$$

Applying the Bellman-Gronwel inequality to (7) yields

$$\| x(t) \| \leqslant y(t) \leqslant y(0)\exp[(\mu(A) + \beta_0 + \beta_1 + \| B \| + k \| b \| \| c \|)t]. \tag{8}$$

It is evident that system (1) is robust absolute stable if condition (3) holds.

This completes the proof of Theorem 1.

Since matrix A is asymptotically stable, for a given positive definite matrix Q, there exists a unique positive definite matrix P satisfying the following matrix equation

$$A^{\mathrm{T}}P + PA = -Q. \tag{9}$$

Theorem 2 System (1) is robust absolute stable if the following inequality holds:

$$\beta_0 + \beta_1 \leqslant \frac{\lambda_{\min}(Q)}{2\| P \|} - \| B \| - k \| b \| \| c \|. \tag{10}$$

Proof Denote $\gamma = (\| B \| + \beta_1) \| P \|$ and define the Lyapunov functional as

$$V(x(t)) = x^{\mathrm{T}}(t)Px(t) + \gamma\int_{t-\tau}^t x^{\mathrm{T}}(s)x(s)\mathrm{d}s.$$

The derivative of $V(x)$ along (1) is given by

$$\left.\frac{\mathrm{d}V(x)}{\mathrm{d}t}\right|_{(1)} \leqslant -x^{\mathrm{T}}(t)Qx(t) + x^{\mathrm{T}}(t)[\Delta A^{\mathrm{T}}P + P\Delta A]x(t) + \frac{1}{\gamma}x^{\mathrm{T}}(t)(B + \Delta B)^{\mathrm{T}}P^2(B + \Delta B)x(t) + \gamma x^{\mathrm{T}}(t-\tau)x(t-\tau) + \gamma[x^{\mathrm{T}}(t)x(t) - x^{\mathrm{T}}(t-\tau)x(t-\tau)] +$$

$$b^{\mathrm{T}}Px(t)f(\sigma) + x^{\mathrm{T}}(t)Pbf(\sigma) \leqslant$$
$$-[\lambda_{\min}(Q) - \gamma_I - \frac{1}{\gamma}\|B\|+\beta_1)^2\|P\|$$
$$2\beta_0\|P\| - 2k\|b\|\|c\|\|p\|]\|x(t)\|^2 =$$
$$-[\lambda_{\min}(Q) - 2\|B\|\|P\| - 2k\|b\|\|c\|\|P\| -$$
$$2(\beta_0 + \beta_1)\|P\|]\|x(t)\|^2.$$

If condition (10) holds, there exists a real number $\mu > 0$, such that

$$\left.\frac{\mathrm{d}V(x)}{\mathrm{d}t}\right|_{(1)} \leqslant -\mu\|x(t)\|^2.$$

This deduces that system (1) is robust absolute stable. The proof of this theorem is completed.

3 Delay-dependent robust absolute stability

Consider the uncertain Lurie control system (1). Suppose matrix $A + B$ is asymptotically stable. For a given positive definite matrix Q, there exists a unique positive definite matrix P satisfying the following matrix equation

$$(A + B)^{\mathrm{T}}P + P(A + B) = -Q. \qquad (11)$$

Theorem 3 System (1) is robust absolute stable if the following inequality holds.

$$\tau < \frac{\lambda_{\min}(Q) - 2(\beta_0 + \beta_1 + k\|b\|\|c\|)\|P\|}{2\alpha(\|B\|+\beta_1)(\beta_0+\beta_1+\|A\|+\|B\|+k\|b\|\|c\|)\|P\|}, \qquad (12)$$

where $\alpha = \sqrt{\dfrac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \geqslant 1$.

Proof System (1) can be rewritten as

$$x(t) = (A + B + \Delta A + \Delta B)x(t) + bf(\sigma(t)) -$$
$$(B + \Delta B)\int_{t-\tau}^{t}[(A + \Delta A)x(s) +$$
$$(B + \Delta B)x(s - \tau) + bf(\sigma(s))]\mathrm{d}s. \qquad (13)$$

Let $V(x) = x^{\mathrm{T}}Px$. The derivative of $V(x)$ along (13) is given by

$$\left.\frac{\mathrm{d}V(x)}{\mathrm{d}t}\right|_{(13)} =$$
$$x^{\mathrm{T}}(t)[-Q + (\Delta A + \Delta B)^{\mathrm{T}}P +$$
$$P(\Delta A + \Delta B)]x(t) + 2x^{\mathrm{T}}(t)Pbf(\sigma(t)) -$$
$$2\int_{t-\tau}^{t}x^{\mathrm{T}}(t)P(B + \Delta B)(A + \Delta A)x(s)\mathrm{d}s -$$
$$2\int_{t-\tau}^{t}x^{\mathrm{T}}(t)P(B + \Delta B)(B + \Delta B)x(s - \tau)\mathrm{d}s -$$
$$2\int_{t-\tau}^{t}x^{\mathrm{T}}(t)P(B + \Delta B)bf(\sigma(s))\mathrm{d}s \leqslant$$
$$-x^{\mathrm{T}}(t)Qx(t) + x^{\mathrm{T}}(t)[(\Delta A + \Delta B)^{\mathrm{T}}P +$$
$$P(\Delta A + \Delta B)]x(t) + 2x^{\mathrm{T}}(t)Pbf(\sigma(t)) +$$
$$\tau\gamma_1x^{\mathrm{T}}(t)P(B + \Delta B)^{\mathrm{T}}(B + \Delta B)Px(t) +$$
$$\frac{1}{\gamma_1}x^{\mathrm{T}}(\xi_1)(A + \Delta A)^{\mathrm{T}}(A + \Delta A)x(\xi_1) +$$
$$\tau\gamma_2x^{\mathrm{T}}(t)P(B + \Delta B)^{\mathrm{T}}(B + \Delta B)Px(t) +$$
$$\tau\frac{1}{\gamma_2}x^{\mathrm{T}}(\xi_2)(B + \Delta B)]^{\mathrm{T}}(B + \Delta B)x(\xi_2) +$$
$$\tau\gamma_3x^{\mathrm{T}}(t)(B + \Delta B)^{\mathrm{T}}P^2(B + \Delta B)x(t) +$$
$$\tau\frac{1}{\gamma_3}b^{\mathrm{T}}bf^2(\sigma(\xi_3)),$$

where

$$t - \tau < \xi_1,\ \xi_3 < t,\ t - 2\tau < \xi_2 < t - \tau.$$

Suppose

$$V(t + \theta) \leqslant q^2V(t)(q > 1, \theta \in [-\tau, 0]).$$

By using Razumikhin's technique, we get

$$\|x(t+\theta)\| \leqslant q\alpha\|x(t)\|,\ \left(\alpha = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \geqslant 1\right),$$
$$\left.\frac{\mathrm{d}V(x)}{\mathrm{d}t}\right|_{(13)} \leqslant$$
$$-\{\lambda_{\min}(Q)-[2(\beta_0+\beta_1)\|P\|+2k\|b\|\|c\|\|P\|]-$$
$$\tau[(\gamma_1 + \gamma_2 + \gamma_3)\|P\|^2(\|B\| + \beta_1)^2 +$$
$$q^2\alpha^2(\frac{1}{\gamma_1}(\|A\| + \beta_2)^2 + \frac{1}{\gamma_2}(\|B\| + \beta_1)^2 +$$
$$\frac{1}{\gamma_3}k^2\|b\|^2\|c\|^2)]\}\|x(t)\|^2.$$

Let

$$\gamma_1 = \frac{q\alpha(\|A\| + \beta_0)}{(\|B\| + \beta_1)\|P\|},\ \gamma_2 = \frac{q\alpha}{\|P\|},$$
$$\gamma_3 = \frac{q\alpha k\|b\|\|c\|}{(\|B\| + \beta_1)\|P\|},$$

we have

$$\left.\frac{\mathrm{d}V(x)}{\mathrm{d}t}\right|_{(13)} \leqslant$$
$$-[\lambda_{\min}(Q)-2(\beta_0+\beta_1 + k\|b\|\|c\|)\|P\| -$$
$$2\tau q\alpha(\|B\| + \beta_1)(\beta_0 + \beta_1 + \|A\| + \|B\| +$$
$$k\|b\|\|c\|)\|P\|]\|x(t)\|^2 =$$
$$-w\|x(t)\|^2,$$

where

$$w = \lambda_{\min}(Q)-2(\beta_0+\beta_1 + k\|b\|\|c\|)\|P\| -$$
$$2\tau q\alpha(\|B\| + \beta_1)(\beta_0 + \beta_1 + \|A\| +$$
$$\|B\| + k\|b\|\|c\|)\|P\|].$$

If condition (12) holds, there exists a real number $q > 1$, such that $w > 0$. This completes the proof of this theorem.

4 Conclusion

The robust absolute stability of Lurie control system with time-delay has been studied. By using Lyapunov method, both delay-independent and delay-dependent criteria have been given. The presented results are useful to estimate the robust stability bound and delay bound of uncertain Lurie control system with time-delay.

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