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Delay-Dependent Criterion for Absolute Stability of Lurie Type Control Systems with Time Delay

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Abstract: This paper provides a new absolute stability criterion for Lurie type control systems with time delay. Based on an improved upper bound for the inner product of two vectors, a new delay-dependent absolute stability criterion is derived, which is shown by an example less conservative than existing criteria.

Key words: Lurie type control systems; time delay; absolute stability; delay-dependent criterion

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具有时滞的 Lurie 型控制系统绝对稳定的时滞相关准则

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摘要:研究了具有时滞的 Lurie 型控制系统的绝对稳定性,利用矩阵不等式的方法给出了系统绝对稳定的时滞相关准则,与现有的结果相比,具有较小的保守性.

关键词: Lurie 型控制系统; 时滞; 绝对稳定; 时滞相关准则

1 Introduction

Many researchers have investigated the stability of feedback systems whose forward path is a linear time-invariant system and whose feedback path is a memoryless nonlinearity[1-3]. These feedback systems are called Lurie systems. Lurie type control systems with time-delay is an important system, because time-delay is commonly encountered in various engineering systems and the existence of time-delay makes the stability analysis much more complicated. Popov and Halanay[4], Somolines[5] studied the stability of this kind of systems and derived some stability criteria, but the existing criteria are all delay-independent which do not include the information on delay. Generally, abandonment of information on the delay causes conservativeness of the criteria especially when the delay is comparatively small. Recently, a delay-dependent criterion for Lurie type control systems with time delay in finite sector based on the Razumikhin theorem has been derived[6].

In this paper, a new delay-dependent criterion for Lurie type control systems with time delay in infinite sector is given based on an improved upper bound for the inner product of two vectors.

The underlying idea to provide delay-dependent criterion has resorted to the following inequality: given $a, b \in \mathbb{R}^n$,

$$-2a^{\mathrm{T}}b \leqslant \eta_1 = \inf_{N>0}\{a^{\mathrm{T}}Na + b^{\mathrm{T}}N^{-1}b\}, \quad (1)$$

where a^{T} indicates the transpose of a. In this case, the upper bound of $(-2a^{\mathrm{T}}b)$ is always greater than or equal to zero. Therefore the upper bound η_1 is not a good estimate.

To improve the upper bound of $(-2a^{\mathrm{T}}b)$, [7] gives another free matrix M so that

$$-2a^{\mathrm{T}}b \leqslant \eta_2 = \inf_{N>0,M}\{(a+Mb)^{\mathrm{T}}N(a+Mb) + b^{\mathrm{T}}N^{-1}b + 2b^{\mathrm{T}}Mb\}, \quad (2)$$

where clearly $\eta_2 \leqslant \eta_1$. Then we have the following

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Lemma.

Lemma 1 Assume that $a(\alpha) \in \mathbb{R}^{n_x}$ and $b(\alpha) \in \mathbb{R}^{n_y}$ are given for $\alpha \in \Omega$. Then, for any positive definite matrix $N \in \mathbb{R}^{n_x \times n_x}$ and any matrix $M \in \mathbb{R}^{n_y \times n_y}$, the following inequality holds:

$$-2\int_{\Omega} b^{\mathrm{T}}(\alpha)a(\alpha)\mathrm{d}\alpha \leqslant \int_{\Omega}\begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} N & NM \\ M^{\mathrm{T}}N & (2,2)\end{bmatrix}\begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix}\mathrm{d}\alpha, \tag{3}$$

where $(2,2)$ denotes $(M^{\mathrm{T}}N + I)N^{-1}(NM + I)$.

Based on this lemma, the following section will show our main results.

2 Main results

Consider Lurie type direct control system with time delay

$$\begin{cases} \dot{x} = Ax + Bx(t-h) + b\phi(y), \\ y = c^{\mathrm{T}}x, \end{cases} \tag{4}$$

where $x(t) \in \mathbb{R}^n, A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times n}, b \in \mathbb{R}^n, c \in \mathbb{R}^n, h \geqslant 0, \phi(y) \in K[0,\infty)$.

$K[0,\infty) = \{\phi(y) \mid \phi(0) = 0, 0 < y\phi(y) < \infty (y \neq 0)\}$.

In order to show under what condition the origin is globally asymptotically stable for the system (4), we consider the following Lyapunov function

$$V = V_1 + V_2 + V_3, \tag{5}$$

where

$$V_1 = x^{\mathrm{T}}Px + \beta\int_0^{y(t)}\phi(\alpha)\mathrm{d}\alpha,$$

$$V_2 = \int_{-h}^{0}\int_{t+\eta}^{t}\dot{x}^{\mathrm{T}}(\alpha)B^{\mathrm{T}}NB\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\eta,$$

$$V_3 = \int_{t-h}^{t}x^{\mathrm{T}}(\alpha)Qx(\alpha)\mathrm{d}\alpha,$$

$\beta > 0, P > 0, Q > 0, N > 0$. Then we have the following theorem.

Theorem 1 Assume that an uncertain time-invariant delay lies in $[0,\bar{h}]$, i.e., $h \in [0,\bar{h}]$. Then if there exist $P > 0, Q > 0, U > 0$ and W such that

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^{\mathrm{T}} & X_{22} & X_{23} \\ X_{13}^{\mathrm{T}} & X_{23}^{\mathrm{T}} & X_{33}\end{bmatrix} < 0, \tag{6}$$

then the system is absolutely stable. where

$$X_{11} = (A+B)^{\mathrm{T}}P + P(A+B) + \bar{h}^2(W^{\mathrm{T}} + P)U^{-1}(W + P) + A^{\mathrm{T}}B^{\mathrm{T}}UBA + Q + W^{\mathrm{T}}B + B^{\mathrm{T}}W,$$

$$X_{12} = -W^{\mathrm{T}}B + A^{\mathrm{T}}B^{\mathrm{T}}UBB,$$

$$X_{13} = A^{\mathrm{T}}B^{\mathrm{T}}UBb + Pb + \frac{1}{2}\beta A^{\mathrm{T}}c + \frac{1}{2}\mu c,$$

$$X_{22} = -Q + B^{\mathrm{T}}B^{\mathrm{T}}UBB,$$

$$X_{23} = B^{\mathrm{T}}B^{\mathrm{T}}UBb + \frac{1}{2}\beta B^{\mathrm{T}}c,$$

$$X_{33} = b^{\mathrm{T}}B^{\mathrm{T}}UBb + \beta c^{\mathrm{T}}b.$$

Proof We first note that $V(x(t-\alpha), \alpha \in [0,\bar{h}])$ is radially unbound with respect to $x(t)$. Now consider its derivative

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3.$$

Since it holds that

$$x(t) - x(t-h) = \int_{t-h}^{t}\dot{x}(\sigma)\mathrm{d}\sigma. \tag{7}$$

Then (4) can be written as

$$\dot{x}(t) = (A+B)x(t) - B\int_{t-h}^{t}\dot{x}(\alpha)\mathrm{d}\alpha + b\phi(y), \tag{8}$$

and thus the derivative of V_1 satisfies the relation

$$\dot{V}_1 = 2x^{\mathrm{T}}(t)P(A+B)x(t) - 2x^{\mathrm{T}}(t)PB\int_{t-h}^{t}\dot{x}(\alpha)\mathrm{d}\alpha + 2x^{\mathrm{T}}(t)Pb\phi(y) + \beta\dot{y}(t)\phi(y),$$

and using Lemma 1 will supply

$$\dot{V}_1 \leqslant x^{\mathrm{T}}(t)\{(A+B)^{\mathrm{T}}P + P(A+B) + hP(M^{\mathrm{T}}N + I)N^{-1}(NM+I)P\}x(t) + 2x^{\mathrm{T}}(t)PM^{\mathrm{T}}NB\int_{t-h}^{t}\dot{x}(\alpha)\mathrm{d}\alpha + \int_{t-h}^{t}\dot{x}^{\mathrm{T}}(\alpha)B^{\mathrm{T}}NB\dot{x}(\alpha)\mathrm{d}\alpha + 2x^{\mathrm{T}}(t)Pb\phi(y) + \beta c^{\mathrm{T}}[Ax + Bx(t-h) + b\phi(y)]\phi(y).$$

Since $\dot{V}_2$ and $\dot{V}_3$ yield the relation

$$\dot{V}_2 = h\dot{x}^{\mathrm{T}}(t)B^{\mathrm{T}}NB\dot{x}(t) - \int_{t-h}^{t}\dot{x}^{\mathrm{T}}(\alpha)B^{\mathrm{T}}NB\dot{x}(\alpha)\mathrm{d}\alpha,$$

$$\dot{V}_3 = x^{\mathrm{T}}(t)Qx(t) - x^{\mathrm{T}}(t-h)Qx(t-h).$$

Choosing $W = NMP$ and $U = \bar{h}N$ will yield

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \leqslant x^{\mathrm{T}}(t)\{(A+B)^{\mathrm{T}}P + P(A+B) + \bar{h}^2(W^{\mathrm{T}} + P)U^{-1}(W+P)\}x(t) + 2x^{\mathrm{T}}(t)Pb\phi(y) + 2x^{\mathrm{T}}(t)W^{\mathrm{T}}B\int_{t-h}^{t}\dot{x}(\alpha)\mathrm{d}\alpha + \dot{x}^{\mathrm{T}}(t)B^{\mathrm{T}}UB\dot{x}(t) + \beta c^{\mathrm{T}}[Ax + Bx(t-h) + b\phi(y)]\phi(y) + x^{\mathrm{T}}(t)Qx(t) - x^{\mathrm{T}}(t-h)Qx(t-h) + \mu y\phi(y) - \mu y\phi(y) = \xi^{\mathrm{T}}X\xi - \mu y\phi(y).$$

Here $\xi^{\mathrm{T}} = [x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-h)\ \phi(y)]^{\mathrm{T}}$, with some efforts, we can show that (6) guarantees the negativeness of $\dot{V}$, which immediately implies the absolute stability of the system.

Consider Lurie type indirect control system with time delay

$$\begin{cases} \dot{x} = Ax + Bx(t-h) + b\phi(y), \\ \dot{y} = c^{\mathrm{T}}x - \rho\phi(y). \end{cases} \quad (9)$$

Using the same Lyapunov function as (5). Then calculating the derivative, one obtains

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 = \xi^{\mathrm{T}}\bar{X}\xi,$$

where

$$\xi^{\mathrm{T}} = [x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-h) \quad \phi(y)]^{\mathrm{T}},$$

$$\bar{X} = \begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{X}_{13} \\ \bar{X}_{12}^{\mathrm{T}} & \bar{X}_{22} & \bar{X}_{23} \\ \bar{X}_{13}^{\mathrm{T}} & \bar{X}_{23}^{\mathrm{T}} & \bar{X}_{33} \end{bmatrix},$$

$$\begin{aligned} \bar{X}_{11} = {} & (A+B)^{\mathrm{T}}P + P(A+B) + \\ & \bar{h}^2(W^{\mathrm{T}} + P)U^{-1}(W+P) + \\ & A^{\mathrm{T}}B^{\mathrm{T}}UBA + Q + W^{\mathrm{T}}B + B^{\mathrm{T}}W, \end{aligned}$$

$$\bar{X}_{12} = -W^{\mathrm{T}}B + A^{\mathrm{T}}B^{\mathrm{T}}UBB,$$

$$\bar{X}_{13} = Pb + A^{\mathrm{T}}B^{\mathrm{T}}UBb + \frac{1}{2}\beta c,$$

$$\bar{X}_{22} = -Q + B^{\mathrm{T}}B^{\mathrm{T}}UBB,$$

$$\bar{X}_{23} = B^{\mathrm{T}}B^{\mathrm{T}}UBb,$$

$$\bar{X}_{33} = b^{\mathrm{T}}B^{\mathrm{T}}UBb - \beta\rho.$$

Then we have the following theorem.

Theorem 2 Assume that an uncertain time-invariant delay lies in $[0,\bar{h}]$, i.e., $h \in [0,\bar{h}]$. Then if there exist $P > 0, Q > 0, U > 0$ and W such that $\bar{X} < 0$, then system (9) is absolutely stable.

3 Example

We shall illustrate the results by using an example. Consider the system[6]

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} -0.2 & -0.5 \\ 0.5 & -0.2 \end{bmatrix} \begin{bmatrix} x_1(t-h) \\ x_2(t-h) \end{bmatrix} + \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix} \phi(y(t)),$$

$y(t) = 0.6x_1(t) + 0.8x_2(t)$, $\phi(\cdot) \in K[0,0.5]$.

Let $M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $N = P = Q = I$, $\beta = 0.4$, $\mu = 1$. 2, we have $\bar{h} \leqslant 1.268$, when we choose $M = M_1 = \begin{bmatrix} -1 & 0.3 \\ -0.2 & -1.1 \end{bmatrix}$, then obtained $\bar{h} \leqslant 1.825$. And let $\beta = 0.5, \mu = 1, M = M_1$, we have $\bar{h} \leqslant 2.055$, the largest bound is 0.3053 and $\phi(\cdot) \in K[0,0.5]$ via the result of [6]. Based on our result, the bound can be further improved to 2.055 and $\phi(\cdot)$ satisfying the infinite sector condition i.e. $\phi(\cdot) \in K[0,\infty)$.

4 Conclusion

By using a new vector inequality, this paper obtained a new delay-dependent absolute stability criterion for Lurie type control systems with time delay. An example shows that this criterion performs much better than several existing criteria.

References

[1] Wu Yongxian and Zhao Suxia. Absolute stability of control systems with several nonlinear stationary elements in the case of an infinite sector [J]. Automatika i Telemekhanika, 1991, 52(1):34-42

[2] Vidyasagar M. Nonlinear Systems Analysis [M]. London: Prentice-hall, 1993

[3] Park P. A revisited Popov criterion for nonlinear Lurie systems with sector-restrictions [J]. Int. J. Control, 1997, 68(2):461-469

[4] Popov V M and Halanay A. About stability of nonlinear controlled systems with delay [J]. Automation and Remote Control, 1962, 23(7):849-851

[5] Somolines A. Stability of Lurie type functional equations [J]. J. Diff. Eqs., 1997, 26(1):191-199

[6] Nian Xiaohong. Delay dependent conditions for absolute stability of Lurie type control systems [J]. Acta Automatica Sinica, 1999, 25(4):564-556

[7] Park P. A delay-dependent stability criterion for systems with uncertain time-invariant delays [J]. IEEE Trans. Automat. Contr., 1999, 44(4):876-877

[8] Hale J. Theory of Functional Differential Equation [M]. New York: Springer-Verlag, 1997

[9] Kolmanonskii V B. Stability of Functional Differential Equations [M]. London: Academic Press, INC, 1986

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