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The Design of Decentralized Robust Tracking Controllers for Large-Scale Uncertain Systems with Time-Delay *

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Abstract: This paper is to study the decentralized robust tracking control problem for large-scale uncertain systems with time-delay. The uncertainty is value-bounded and can not satisfy its matching conditions. A sufficient condition for the existence of decentralized robust tracking controller is derived. This condition is expressed as the solvability problem of linear matrix inequalities (LMIs). Based on that, a convex optimization problem with LMI constraints is formulated to design a decentralized state feedback control with smaller gain parameters which enables the controlled system to approach the reference input asymptotically.

Key words: large-scale systems with time-delay; LMI; decentralized robust control; output tracking

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不确定性时滞大系统的分散鲁棒跟踪控制器设计

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摘要: 研究不确定性关联时滞大系统的分散鲁棒输出跟踪控制问题.系统中不确定项具有数值界,可不满足匹配条件.基于不确定项的表达形式,给出了存在分散鲁棒跟踪控制器的线性矩阵不等式(LMI)条件.在此基础上,通过建立求解受 LMIs 约束的凸优化问题,提出了具有较小反馈增益 LMI 设计方法,使受控系统渐近跟踪给定的参考输入.LMI 方法求解简单,便于计算.

关键词: 时滞关联大系统; 线性矩阵不等式; 分散鲁棒控制; 输出跟踪

1 Introduction

In recent years, much attention[1] has been paid to decentralized control for interconnected large-scale systems. Since the models often contain uncertainties, expected performance can not be obtained if the controller is designed only based on nominal model. Moreover, time-delay occurs in the interconnection due to the transferring information and is insensitive to the measurement. Therefore, the decentralized robust control for the large-scale systems with time-delay is of great theoretical and practical value. The robust tracking problem for large-scale systems with matching conditions[2,3] and for time-delay system[4,5] has been solved. But the former does not consider the effect of time-delay, and the latter only considers the centralized control.

LMI method has received much attention for its high solvability and becomes an effective method for robust analysis and synthesis[6-8]. In this paper, the problem of decentralized tracking control for a class of interconnected systems with norm-bounded uncertainty is studied using LMI method. The sufficient condition for decentralized robust tracking control based on LMIs is derived and a convex optimization method for designing controller with smaller gain is proposed.

2 Problem description and lemmas

Consider a class of time-varying uncertain time delay large-scale systems with N subsystems. The subsystems can be described as follows:

$$\dot{x}_i(t) = [A_{ii} + \Delta A_{ii}]x_i(t) + [B_i + \Delta B_i]u_i(t) +$$

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$$\sum_{j=1}^{N} A_{ij}x_j(t-\tau_{ij}) + \eta_i, \qquad (1)$$
$$y_i(t) = C_i x_i(t),\ i = 1,2,\cdots,N,$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^{l_i}$ are the state, control and output vectors respectively. A_{ij}, B_i and C_i are with appropriate dimensions representing the system, input and output matrices respectively. (A_{ii}, B_i) is controllable. A_{ij} is the interconnections. $\tau_{ij} \geqslant 0$ denotes the delay in the interconnections. η_i is the constant but unknown disturbance associated with the ith subsystem. The uncertainty ΔA_{ii} and ΔB_i are structured and value-bounded[9,10]:

$$|\Delta A_{ii}| < D_{ii},\ |\Delta B_i| < E_i,\ i,j = 1,2,\cdots,N,$$

where D_{ii} and E_i are known constant matrix with nonnegative entries. The meaning of the $|\Delta| < \bar{\Delta}$ is: $|e_{ij}| \leqslant \bar{e}_{ij}, i,j = 1,2,\cdots,N$, e_{ij} and $\bar{e}_{ij}$ denote the ij-entries of Δ and $\bar{\Delta}$ respectively.

Assumption 1 System (1) satisfies the following condition

$$\operatorname{rank}\begin{bmatrix} A_{ii} & B_i \\ C_i & 0 \end{bmatrix} = n_i + l_i.$$

The objective of this paper is to design a local state feedback control law for each subsystem guaranteeing that the output of each subsystem approaches their reference output asymptotically.

In the following discussion, define a two-valued function $\delta(\cdot)$ as

$$\delta(E) = \begin{cases} 0, & E = 0, \\ 1, & E \neq 0. \end{cases}$$

Here are several important lemmas.

Lemma 1 Suppose $A, B \in \mathbb{R}^{n\times n}$, $A \geqslant B$, then we have $C^{\mathrm{T}}AC \geqslant C^{\mathrm{T}}BC$, $\forall\, C \in \mathbb{R}^{n\times k}$.

Lemma 2 Suppose X and Y are vectors or matrices with suitable dimensions, then for any positive $\alpha > 0$, we have

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leqslant \alpha X^{\mathrm{T}}X + \alpha^{-1}Y^{\mathrm{T}}Y.$$

Lemma 3[11] Suppose Y and Q are vectors or matrices with suitable dimensions, where $Q > 0, \alpha, \beta$ are given positive numbers, then

$$Y^{\mathrm{T}}Y < \alpha I, \text{ if and only if } \begin{bmatrix} -\alpha I & Y^{\mathrm{T}} \\ Y & -I \end{bmatrix} < 0;$$

$$Q^{-1} < \beta I, \text{ if and only if } \begin{bmatrix} Q & I \\ I & \beta I \end{bmatrix} > 0.$$

Proof It can be proven by Schur complement[8] or matrices' transformations.

Lemma 4[10] If $\Delta A \in \mathbb{R}^{n\times m}$ satisfies $|\Delta A| < D$, then

$$\Omega(D) \geqslant \Delta A \Delta A^{\mathrm{T}},\ \Gamma(D) \geqslant \Delta A^{\mathrm{T}}\Delta A,$$

where

$$\Omega(D) = \begin{cases} \|DD^{\mathrm{T}}\| I, & \|DD^{\mathrm{T}}\| I < n\operatorname{diag}(DD^{\mathrm{T}}), \\ n\operatorname{diag}(DD^{\mathrm{T}}), & \text{else} \end{cases}$$

$$\Gamma(D) = \begin{cases} \|D^{\mathrm{T}}D\| I, & \|DD^{\mathrm{T}}\| I < m\operatorname{diag}(D^{\mathrm{T}}D), \\ m\operatorname{diag}(D^{\mathrm{T}}D), & \text{else} \end{cases}$$

where $\operatorname{diag}(R) = \operatorname{diag}(r_{11}, r_{22}, \cdots, r_{nn})$, $R = (r_{ij})$ is n-dimensional real symmetrical non-negative matrix.

Remark 1 $\|M\|$ is defined as the maximum singular value of M, $\|a\|$ is the 2-norm of a, I represents different unit matrices at different position.

3 Decentralized output tracking controller design

In this section, the sufficient condition for the existence of controller for large-scale uncertain systems with time-delay is derived. Then, a convex optimization problem with LMI constraints is formulated to design a decentralized state feedback control with smaller gain parameters.

Construct the augmented system equation as

$$\dot{x}_i(t) = [A_{ii} + \Delta A_{ii}]x_i(t) + [B_i + \Delta B_i]u_i(t) + \sum_{j=1}^{N} A_{ij}x_j(t-\tau_{ij}) + \eta_i, \qquad (2)$$
$$\dot{q}_i(t) = C_i x_i(t) - y_{ri},\ i = 1,2,\cdots,N,$$

where $q_i(t) \in \mathbb{R}^{l_i}$ and $y_{ri} \in \mathbb{R}^{l_i}$ are the state of the augmented system and reference input vectors respectively.

Rewriting (2) into an augmented state equation, we get

$$\dot{z}_i(t) = [A_{zii} + \Delta A_{zii}]z_i(t) + [B_{zi} + \Delta B_{zi}]u_i(t) + \sum_{j=1}^{N} A_{zij}^{d} z_j(t-\tau_{ij}) + \xi_i, \qquad (3)$$

where

$$z_i(t) = \begin{bmatrix} x_i(t) \\ q_i(t) \end{bmatrix},\ A_{zii} = \begin{bmatrix} A_{ii} & 0 \\ C_i & 0 \end{bmatrix},$$

$$\Delta A_{zii} = \begin{bmatrix} \Delta A_{ii} & 0 \\ 0 & 0 \end{bmatrix},\ A_{zij}^{d} = \begin{bmatrix} A_{ij} & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_{zi} = \begin{bmatrix} B_i \\ 0 \end{bmatrix},\ \Delta B_{zi} = \begin{bmatrix} \Delta B_i \\ 0 \end{bmatrix},\ \xi_i = \begin{bmatrix} \eta_i \\ -y_{ri} \end{bmatrix}.$$

The augmented sysem obviously satisfies the following conditions:

$$|\Delta A_{zii}| < D_{zii},\ |\Delta B_{zi}| < E_{zi},\ i,j = 1,2,\cdots,N, \tag{4}$$

where

$$D_{zii} = \begin{bmatrix} D_{ii} & 0 \\ 0 & 0 \end{bmatrix},\ E_{zi} = \begin{bmatrix} E_i \\ 0 \end{bmatrix}.$$

In terms of Assumption 1, the pair (A_{zii}, B_{zi}) of augmented system (3) is controlled. The following theorem is the main results in this section.

Theorem 1 For large-scale system (1), if there exist positive definite matrices $X_i, Z_i \in \mathbb{R}^{(n_i+l_i)\times(n_i+l_i)}$, matrix $Y_i \in \mathbb{R}^{(m_i+l_i)\times(n_i+l_i)}$, and positive number α_i, β_i, which satisfy the LMIs (5) ~ (7), then the decentralized state feedback law (8) will serve as a robust tracking controller and make the closed-loop system internal stable.

$$\begin{bmatrix} \bar{A}_{zii} & A_{zi1}^d X_1 & A_{zi2}^d X_2 & \cdots & A_{ziN}^d X_N \\ X_1(A_{zi1}^d)^{\mathrm{T}} & -\delta(A_{zi1}^d)Z_1 & & & \\ X_2(A_{zi2}^d)^{\mathrm{T}} & & -\delta(A_{zi2}^d)Z_2 & & \\ \vdots & & & \ddots & \\ X_N(A_{ziN}^d)^{\mathrm{T}} & & & & -\delta(A_{ziN}^d)Z_N \end{bmatrix} < 0, \tag{5}$$

$$\begin{bmatrix} -\alpha_i I & X_i\Gamma(D_{zii})^{\frac{1}{2}} \\ \Gamma(D_{zii})^{\frac{1}{2}}X_i & -I \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} -\beta_i I & Y_i\Gamma(E_{zi})^{\frac{1}{2}} \\ \Gamma(E_{zi})^{\frac{1}{2}}Y_i^{\mathrm{T}} & -I \end{bmatrix} < 0, \tag{7}$$

where

$$\bar{A}_{zii} = X_iA_{zii}^{\mathrm{T}} + A_{zii}X_i + Y_i^{\mathrm{T}}B_{zi}^{\mathrm{T}} + B_{zi}Y_i + 2I + \alpha_i I + \beta_i I + \sum_{j=1}^{N}\delta(A_{zji}^d)Z_i,$$

$$u_i = K_iz_i,\ K_i = Y_iX_i^{-1}. \tag{8}$$

Proof The proof is divided into two parts:

1) Internal stability. When the augmented system (3) is controlled by (8) and ξ_i is first neglected, the closed-loop system equation becomes

$$\dot{z}_i = [A_{zii} + \Delta A_{zii}]z_i(t) + [B_{zi} + \Delta B_{zi}]K_iz_i(t) + \sum_{j=1}^{N}A_{zij}^d\ z_j(t-\tau_{ij}). \tag{9}$$

Consider the following Lyapunov function:

$$V(z) = \sum_{i=1}^{N}[z_i^{\mathrm{T}}P_iz_i + \sum_{j=1}^{N}\int_{t-\tau_{ij}}^{t}\delta(A_{zij}^d)z_j^{\mathrm{T}}H_jz_j\mathrm{d}t],$$

where P_i, H_j are positive definite matrices.

The time deviation of $V(z)$ along with the state trajectory of the system of (9) satisfies

$$\begin{aligned}\dot{V}(z) = {}& \\ &\sum_{i=1}^{N}\{\dot{z}_i^{\mathrm{T}}P_iz_i + z_i^{\mathrm{T}}P_i\dot{z}_i + \sum_{j=1}^{N}\delta(A_{zij}^d)z_j^{\mathrm{T}}H_jz_j - \\ &\sum_{j=1}^{N}\delta(A_{zij}^d)z_j^{\mathrm{T}}(t-\tau_{ij})H_jz_j(t-\tau_{ij})\} = \\ &\sum_{i=1}^{N}\{z_i^{\mathrm{T}}(A_{zii}^{\mathrm{T}}P_i + P_iA_{zii} + \Delta A_{zii}^{\mathrm{T}}P_i + P_i\Delta A_{zii} + K_i^{\mathrm{T}}B_{zi}^{\mathrm{T}}P_i + \\ &P_iB_{zi}K_i + K_i^{\mathrm{T}}\Delta B_{zi}^{\mathrm{T}}P_i + P_i\Delta B_{zi}K_i)z_i + \sum_{j=1}^{N}2z_i^{\mathrm{T}}P_iA_{zij}^dz_j(t-\tau_{ij}) + \\ &\sum_{j=1}^{N}\delta(A_{zij}^d)z_j^{\mathrm{T}}H_jz_j - \sum_{j=1}^{N}\delta(A_{zij}^d)z_j^{\mathrm{T}}(t-\tau_{ij})H_jz_j(t-\tau_{ij})\}.\end{aligned} \tag{10}$$

From Lemmas 1,2 and 4, we have

$$\begin{aligned}&P_iP_i + \Gamma(D_{zii}) \geqslant \\ &P_iP_i + \Delta A_{zii}^{\mathrm{T}}\Delta A_{zii} \geqslant P_i\Delta A_{zii} + \Delta A_{zii}^{\mathrm{T}}P_i,\end{aligned} \tag{11}$$

$$\begin{aligned}&P_iP_i + K_i^{\mathrm{T}}\Gamma(E_{zi})K_i \geqslant \\ &P_iP_i + K_i^{\mathrm{T}}\Delta B_{zi}^{\mathrm{T}}\Delta B_{zi}K_i \geqslant P_i\Delta B_{zi}K_i + K_i^{\mathrm{T}}\Delta B_{zi}^{\mathrm{T}}P_i.\end{aligned} \tag{12}$$

Put equations (11) and (12) into (10), we have

$$\begin{aligned}\dot{V}(z) \leqslant {}& \\ &\sum_{i=1}^{N}\{z_i^{\mathrm{T}}[A_{zii}^{\mathrm{T}}P_i + P_iA_{zii} + 2P_iP_i + \Gamma(D_{zii}) + K_i^{\mathrm{T}}B_{zi}^{\mathrm{T}}P_i + \\ &P_iB_{zi}K_i + K_i^{\mathrm{T}}\Gamma(E_{zi})K_i]z_i + \sum_{j=1}^{N}2z_i^{\mathrm{T}}P_iA_{zij}^dz_j(t-\tau_{ij}) + \\ &\sum_{j=1}^{N}\delta(A_{zij}^d)z_j^{\mathrm{T}}H_jz_j - \sum_{j=1}^{N}\delta(A_{zij}^d)z_j^{\mathrm{T}}(t-\tau_{ij})H_jz_j(t-\tau_{ij})\}.\end{aligned} \tag{13}$$

From Lemma 4, $\Gamma(D_{zii})$ and $\Gamma(E_{zi})$ are positive defined or semi-positive defined matrices, which can be divided as $\Gamma(D_{zij}) = \Gamma(D_{zij})^{1/2}\Gamma(D_{zij})^{1/2}$ and $\Gamma(E_{zi}) = \Gamma(E_{zi})^{1/2}\Gamma(E_{zi})^{1/2}$, then there exist $\alpha_i > 0, \beta_i > 0$, which satisfy

$$\Gamma(D_{zij}) < \alpha_iP_iP_i,\ K_i^{\mathrm{T}}\Gamma(E_{zi})K_i < \beta_iP_iP_i. \tag{14}$$

From (13) and (14), we have

$$\begin{aligned}\dot{V}(z) \leqslant {}& \\ &\sum_{i=1}^{N}\{z_i^{\mathrm{T}}(A_{zii}^{\mathrm{T}}P_i + P_iA_{zii} + K_i^{\mathrm{T}}B_{zi}^{\mathrm{T}}P_i + P_iB_{zi}K_i + \\ &2P_iP_i + \alpha_iP_iP_i + \beta_iP_iP_i)z_i + \\ &\sum_{j=1}^{N}2z_i^{\mathrm{T}}P_iA_{zij}^dz_j(t-\tau_{ij}) + \sum_{j=1}^{N}\delta(A_{zij}^d)z_j^{\mathrm{T}}H_jz_j -\end{aligned}$$

$$\sum_{j=1}^{N}\delta(A_{zij}^{d})z_j^{\mathrm{T}}(t-\tau_{ij})H_jz_j(t-\tau_{ij})\} =$$

$$\sum_{i=1}^{N} M^{\mathrm{T}}\begin{bmatrix} \tilde{A}_{zii} & P_iA_{zi1}^{d} & P_iA_{zi2}^{d} & \cdots & P_iA_{ziN}^{d} \\ (A_{zi1}^{d})^{\mathrm{T}}P_i & -\delta(A_{zi1}^{d})H_1 & & & \\ (A_{zi2}^{d})^{\mathrm{T}}P_i & & -\delta(A_{zi2}^{d})H_2 & & \\ \vdots & & & \ddots & \\ (A_{ziN}^{d})^{\mathrm{T}}P_i & & & & -\delta(A_{ziN}^{d})H_N \end{bmatrix}M,$$

where

$$M = \begin{bmatrix} z_i(t) \\ z_1(t-\tau_{i1}) \\ z_2(t-\tau_{i2}) \\ \vdots \\ z_N(t-\tau_{iN}) \end{bmatrix},$$

$$\tilde{A}_{zii} = A_{zii}^{\mathrm{T}}P_i + P_iA_{zii} + K_i^{\mathrm{T}}B_{zi}^{\mathrm{T}}P_i + P_iB_{zi}K_i + 2P_iP_i + \alpha_iP_iP_i + \beta_iP_iP_i + \sum_{i=1}^{N}\delta(A_{zji}^{d})H_i.$$

Based on Lyapunov stability theorem, if the following LMI

$$\begin{bmatrix} \tilde{A}_{zii} & P_iA_{zi1}^{d} & P_iA_{zi2}^{d} & \cdots & P_iA_{ziN}^{d} \\ (A_{zi1}^{d})^{\mathrm{T}}P_i & -\delta(A_{zi1}^{d})H_1 & & & \\ (A_{zi2}^{d})^{\mathrm{T}}P_i & & -\delta(A_{zi2}^{d})H_2 & & \\ \vdots & & & \ddots & \\ (A_{ziN}^{d})^{\mathrm{T}}P_i & & & & -\delta(A_{ziN}^{d})H_N \end{bmatrix}<0 \tag{15}$$

is satisfied, then large-scale system (3) can be state feedback stabilized.

Next, (15) is pre- and post-multiplied by block-diag $\{P_i^{-1}, P_1^{-1}, P_2^{-1}, \cdots, P_N^{-1}\}$ respectively, then we have

$$\begin{bmatrix} \hat{A}_{zii} & A_{zi1}^{d}P_1^{-1} & A_{zi2}^{d}P_2^{-1} & \cdots & A_{ziN}^{d}P_N^{-1} \\ P_i^{-1}(A_{zi1}^{d})^{\mathrm{T}} & -\delta(A_{zi1}^{d})P_1^{-1}H_1P_1^{-1} & & & \\ P_2^{-1}(A_{zi2}^{d})^{\mathrm{T}} & & -\delta(A_{zi2}^{d})P_2^{-1}H_2P_2^{-1} & & \\ \vdots & & & \ddots & \\ P_N^{-1}(A_{ziN}^{d})^{\mathrm{T}} & & & & -\delta(A_{ziN}^{d})P_N^{-1}H_NP_N^{-1} \end{bmatrix} < 0, \tag{16}$$

where

$$\hat{A}_{zii} = P_i^{-1}A_{zii}^{\mathrm{T}} + A_{zii}P_i^{-1} + P_i^{-1}K_i^{\mathrm{T}}B_{zi}^{\mathrm{T}} + B_{zi}K_iP_i^{-1} + 2I + \alpha_iI + \beta_iI + \sum_{i=1}^{N}\delta(A_{zji}^{d})P_i^{-1}H_iP_i^{-1}.$$

Let

$$X_i = P_i^{-1},\ Y_i = K_iP_i^{-1},\ Z_i = P_i^{-1}H_iP_i^{-1},$$

then $X_i > 0, Z_i > 0$ are satisfied. Thus (16) is equivalent to (5) and (14) is equivalent to

$$X_i\Gamma(D_{zii})X_i < \alpha_iI,\ Y_i^{\mathrm{T}}\Gamma(E_{zi})^{\mathrm{T}}Y_i < \beta_iI.$$

From Lemma 3, if and only if (6) and (7) are satisfied, then (14) is satisfied. And the internal stability is then assured.

2) Asymptotic tracking. First define the following matrices.

$A_z = \text{block-diag}[A_{z11}, \cdots, A_{zNN}]$,

$\Delta A_z = \text{block-diag}[\Delta A_{z11}, \cdots, \Delta A_{zNN}]$,

$A_d = \text{block-diag}[A_{z1}, \cdots, A_{zN}]$,

$A_{zi} = [A_{zi1}, \cdots, A_{ziN}],\ i = 1,2,\cdots,N$,

$z(t-\tau) = [z_1^{\mathrm{T}}(t-\tau_1), \cdots, z_N^{\mathrm{T}}(t-\tau_N)]^{\mathrm{T}}$,

$z_i(t-\tau_i) = [z_1(t-\tau_{i1}), \cdots, z_N(t-\tau_{iN})]^{\mathrm{T}}$,

$B_z = \text{block-diag}[B_{z1}, \cdots, B_{zN}]$,

$\Delta B_z = \text{block-diag}[\Delta B_{z1}, \cdots, \Delta B_{zN}]$,

$\xi = [\xi_1^{\mathrm{T}}, \cdots \xi_N^{\mathrm{T}}]^{\mathrm{T}}$,

$K = \text{block-diag}[K_1, \cdots, K_N]^{\mathrm{T}}$.

Then when the feedback law (8) for each subsystem is chosen, large-scale system (3) will be

$$\dot{z}(t) = [A_z + \Delta A_z + (B_z + \Delta B_z)K]z(t) + A_dz(t-\tau) + \xi = A_cz(t) + A_dz(t-\tau) + \xi. \tag{17}$$

Differentiating both sides of (17), we have

$$\ddot{z}(t) = A_c\dot{z}(t) + A_d\dot{z}(t-\tau).$$

From the first part, i.e. internal stability, $\dot{z}(t)$ will approach zero no matter how the initial condition is, that is $\dot{q}(t) = y_i - y_{ri} \to 0$, as $t \to \infty$ for all $i = 1,2,\cdots,N$. Therefore, it is obvious that the property of asymptotic tracking is achieved. Combining (1) and (2), the proof is completed.

Theorem 1 gives the sufficient condition, which, however, can not guarantee the feedback gain as small as possible. In engineering applications, control laws with smaller feedback gain are always adopted to guarantee performance and better disturbance-rejection[12].

To get smaller feedback gain matrix, we consider

$$Y_i^{\mathrm{T}}Y_i < \theta_i I,\ X_i^{-1} < \gamma_i I, \tag{18}$$

where $\theta_i > 0, \gamma_i > 0$. Then the following equation is derived

$$K_i^{\mathrm{T}}K_i = \theta_i X_i^{-1}Y_i^{\mathrm{T}}Y_i X_i^{-1} \leqslant \theta_i \gamma_i^2 I.$$

Therefore, smaller feedback matrices can be obtained by minimizing θ_i, γ_i.

From Lemma 3, (18) is equivalent to

$$\begin{bmatrix} -\theta_i I & Y_i^{\mathrm{T}} \\ Y_i & -I \end{bmatrix} < 0,\ \begin{bmatrix} X_i & I \\ I & \gamma_i I \end{bmatrix} > 0, \tag{19}$$

therefore, system (1) has decentralized tracking control law with smaller feedback gain which can be solved by the following optimization problem

$$\min\left(\sum_{i=1}^{N}\theta_i + \sum_{i=1}^{N}\gamma_i\right),$$

with LMI constraints (5) ~ (7) and (19). This is a convex optimization problem with LMIs constraint, which can be solved by LMI ToolBox[13].

4 Conclusion

For a class of uncertain interconnected large-scale system with time-delay, according to the Lyapunov stability theorem, its decentralized output tracking control can be solved by LMI. By solving the convex optimization problem with LMI constraints, the design of smaller feedback gain is given.

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