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An LMI Approach to Decentralized H_∞-Controller Design of a Class of Uncertain Large-Scale Interconnected Time-Delay Systems

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Abstract: H_∞-controller design for a class of uncertain large-scale interconnected continuous systems with $N \times N$ unknown but constant delays in the interconnections and time varying but norm-bounded parametric uncertainties is addressed. A sufficient condition for the existence of a memoryless robust H_∞-state feedback control law for uncertain large-scale interconnected time-delay systems is derived with LMI (linear matrix inequality) approach. Finally a numerical example is given to demonstrate the design procedure for the decentralized H_∞-state feedback controller.

Key words: H_∞-controller; uncertainties; large-scale interconnected time-delay systems; LMI approach

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一类不确定关联时滞大系统的分散 H_∞ 控制器设计—LMI 方法

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摘要: 研究了一类具有 $N \times N$ 个任意未知常时滞和具有范数有界时变不确定的线性连续大系统的分散鲁棒 H_∞ 状态反馈控制器设计问题,基于线性矩阵不等式方法得到了一个使该系统存在无记忆 H_∞ 状态反馈控制器的充分条件,最后通过一个数值例子来说明分散 H_∞ 状态反馈控制器的设计.

关键词: H_∞ 控制器; 不确定性; 关联时滞大系统; LMI 方法

1 Introduction

In recent years, large-scale systems with time-delays in the interconnections have been receiving considerable attention, beacuse there are a number of large-scale systems with time-delays in the interconnections in practical situations (power systems, communications systems, and so on). To account for implementation constraints, cost and reliability considerations, a decentralized architecture has been developed. Xu and Lam[1,2] established delay-independent decentralized stabilization conditions for large-scale interconnected linear continuous systems with $N \times N$ delays; and de Souza and Li[3] established delay-dependent decentralized stabilization conditions for the same systems. Cheng et al[4] considered H_∞ disturbance attenuation problem of large-scale systems via Riccati equation approach. Mahmoud and Zribi[5] studied the decentralized observer-based feedback H_∞-control problem for uncertain interconnected systems with delays by algebraic Riccati inequalities. Although the Riccati equation or Riccati inequalities is well known and powerful, it can not be directly solved and needs tuning of parameters and/or positive definite matrices. In this paper, a delay-independent decentralized H_∞-controller for uncertain large-scale interconnected linear continuous systems with $N \times N$ unknown but constant delays is established via LMI approach. LMI approach has two advantages: Firstly, it needs no tuning of parameters and/or positive definite matrices. Secondly, it can be efficiently solved numerically by using interior-point algorithms. An example is given to demonstrate the design procedure of the decentralized H_∞-state feedback controller.

2 System description and preliminaries

Consider an uncertain large-scale linear continuous time-delay system S composed of N interconnected subsystems S_i, $i = 1, 2, \cdots N$. Each S_i is described by the equation

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$$S_i:\begin{cases}\dot{x}_i(t)=(A_i+\Delta A_i(t))x_i(t)+(B_i+\Delta B_i(t))u_i(t)+\\ \qquad \sum_{j=1}^{N}(A_{ij}+\Delta A_{ij}(t))x_j(t-\tau_{ij})+G_iw_i(t),\\ z_i(t)=C_ix_i(t)+T_iu_i(t).\end{cases} \tag{1}$$

Here for the i-th subsystem S_i, $x_i\in\mathbb{R}^{n_i}$, $u_i\in\mathbb{R}^{m_i}$, $z_i\in\mathbb{R}^{r_i}$ and $w_i\in\mathbb{R}^{l_i}$ are the state, the control input, the control output and the disturbance input with $\sum_{i=1}^{N}n_i=n$, $\sum_{i=1}^{N}m_i=m$, $\sum_{i=1}^{N}r_i=r$, and $\sum_{i=1}^{N}l_i=l$, respectively, A_i, B_i, G_i, C_i, T_i and A_{ij} are known real constant matrices with appropriate dimensions, $\Delta A_i(\cdot)$, $\Delta B_i(\cdot)$ and $\Delta A_{ij}(\cdot)$ are real-valued continuous matrix functions representing time-varying but norm-bounded parametric uncertainties in the system model with appropiate dimensions, $0\leqslant\tau_{ij}\leqslant\tau<\infty\,(i,j=1,2,\cdots,N)$ are $N\times N$ arbitrary unknown but constant delays

Definition 1 Given a scalar $\gamma>0$, the whole large-scale interconnected time-delay system (1) is said to be decentralized stable with disturbance attenuation γ if there exist local memoryless state feedback matrices $K_i\in\mathbb{R}^{m_i\times n_i}(i=1,2,\cdots,N)$ such that the resulting closed-loop system satisfies the following conditions:

1) The closed-loop system is asymptotically stable whenever $w(t)=0$;

2) Subject to the assumption of zero initial conditions, the following inequality holds

$$\|z(t)\|_2<\gamma\|w(t)\|_2$$

for all $w(t)\neq 0$ and for all admissible uncertainties, where

$$z(t)=[z_1^{\mathrm{T}}(t),z_2^{\mathrm{T}}(t),\cdots,z_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$
$$w(t)=[w_1^{\mathrm{T}}(t),w_2^{\mathrm{T}}(t),\cdots,w_N^{\mathrm{T}}(t)]^{\mathrm{T}}.$$

The problem addressed in this paper is that of designing memoryless local state feedback law

$$u_i(t)=K_ix_i(t), \tag{2}$$

so that the whole closed-loop system is asymptotically stable and to reduce the effect of the disturbance input on the controlled output to a prescribed level γ for all admissible uncertainties. In this paper, we assume that the admissible uncertainties can be described by

$$[\Delta A_i(t)\ \Delta B_i(t)]=D_iF_i(t)[E_{ai}\quad E_{bi}],$$
$$\Delta A_{ij}(t)=M_{ij}F_{ij}(t)N_{ij},$$

where D_i, E_{ai}, E_{bi}, M_{ij} and N_{ij} are known constant real matrices of appropriate dimensions, $F_i(t)$ and $F_{ij}(t)$ are unknown real-valued time-varying matrices with Lebesgue measurable elements satisfying the following bounds:

$$F_i^{\mathrm{T}}(t)F_i(t)\leqslant I,\ F_{ij}^{\mathrm{T}}(t)F_{ij}(t)\leqslant I\ \forall t.$$

Substituting (2) into (1), we obtain the closed-loop system as follows:

$$\hat{S}_i:\begin{cases}\dot{x}_i(t)=(A_i+\Delta A_i(t)+B_iK_i+\Delta B_i(t)K_i)x_i(t)+\\ \qquad \sum_{j=1}^{N}(A_{ij}+\Delta A_{ij}(t))x_j(t-\tau_{ij})+G_iw_i(t),\\ z_i(t)=(C_i+T_iK_i)x_i(t),\ i=1,2,\cdots,N.\end{cases} \tag{3}$$

Furthermore, we define two index sets:

$$\begin{cases}J_i(A)=\{j\mid A_{ij}\neq 0,j=1,2,\cdots,N\},\\ \bar{J}_i(A)=\{j\mid A_{ji}\neq 0,j=1,2,\cdots,N\},\end{cases} \tag{4}$$

and let $\widetilde{N}_A(i)=k(\bar{J}_i(A)),i=1,2,\cdots,N$, where $K(J)$ is the number of the elements that belong to the set J.

The following matrix inequalities will be essential to the proof in this paper, see [1,7].

Lemma 1 For a given constant matrix $M\in\mathbb{R}^{n\times m}$

$$2u^{\mathrm{T}}Mv\leqslant u^{\mathrm{T}}MG^{-1}M^{\mathrm{T}}u+v^{\mathrm{T}}Gv,\ u\in\mathbb{R}^n,\ v\in\mathbb{R}^m \tag{5}$$

holds for any positive definite symmetric constant matrix $G\in\mathbb{R}^{m\times m}$.

Lemma 2 Suppose that X and Y are matrices with appropriate dimensions, and then the following inequality is true.

$$X^{\mathrm{T}}Y+Y^{\mathrm{T}}X\leqslant X^{\mathrm{T}}X+Y^{\mathrm{T}}Y. \tag{6}$$

Lemma 3 Suppose that A, D, E and P are real matrices of appropriate dimensions with $\|F\|\leqslant 1$, then for any matrix $P>0$ and scalar $\epsilon>0$ satisfying $P-\epsilon DD^{\mathrm{T}}>0$, we have

$$(A+DFE)^{\mathrm{T}}P^{-1}(A+DFE)\leqslant A^{\mathrm{T}}(P-\epsilon DD^{\mathrm{T}})^{-1}A+\epsilon^{-1}E^{\mathrm{T}}E. \tag{7}$$

3 H_∞-controller design

In the following, for uncertain large-scale interconnected time-delay system (1), we will present one method for designing H_∞-state feedback controller with linear matrix inequalities that makes the resulting closed-loop system asymptotically stable with disturbance attenuation γ.

Theorem 1 If there are positive definite symmetric matrices $X_i \in \mathbb{R}^{n_i\times n_i}$, $H_j \in \mathbb{R}^{n_j\times n_j}$ and matrices $Y_i \in \mathbb{R}^{m_i\times n_i}$, scalars $\varepsilon_{ij} > 0$ making the following Limes feasible

$$\begin{bmatrix}
\Pi_i & G_i & X_iC_i^{\mathrm T} & Y_i^{\mathrm T}T_i^{\mathrm T} & X_iE_{ai}^{\mathrm T} & Y_i^{\mathrm T}E_{bi}^{\mathrm T} & M_{i1}\varepsilon_{i1}^{-1} & \cdots & M_{iN}\varepsilon_{iN}^{-1} & A_{i1}X_1 & \cdots & A_{iN}X_N & 0 & \cdots & 0 \\
G_i^{\mathrm T} & -\gamma_i^2 I & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
C_iX_i & 0 & -\frac{1}{2}I & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
T_iY_i & 0 & 0 & -\frac{1}{2}I & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
E_{ai}X_i & 0 & 0 & 0 & -I & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
E_{bi}Y_i & 0 & 0 & 0 & 0 & -I & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\varepsilon_{i1}^{-1}M_{i1}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{i1}^{-1}I & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
\varepsilon_{iN}^{-1}M_{iN}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -\varepsilon_{iN}^{-1}I & 0 & \cdots & 0 & 0 & \cdots & 0 \\
X_1A_{i1}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -H_1 & \cdots & 0 & \Gamma_1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
X_NA_{iN}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -H_N & 0 & \cdots & \Gamma_N \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \Gamma_1^{\mathrm T} & \cdots & 0 & -\varepsilon_{i1}^{-1}I & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \Gamma_N^{\mathrm T} & 0 & \cdots & -\varepsilon_{iN}^{-1}I
\end{bmatrix} < 0, \tag{8}$$

where

$$\Pi_i = A_iX_i + X_iA_i^{\mathrm T} + B_iY_i + Y_i^{\mathrm T}B_i^{\mathrm T} + \widetilde{N}_A(i) + 2D_iD_i^{\mathrm T},$$

$$\Gamma_j = X_jN_{ij}^{\mathrm T},\ i,j = 1,2,\cdots,N.$$

Then the closed-loop system (3) asymptotically stable with disturbance attenuation γ, and decentralized local controller gain matrice $K_i = Y_iX_i^{-1}$.

Proof Choose the Lyapunov functional as

$$\begin{cases} V(t) = \sum_{i=1}^{N} V_i(t) = \sum_{i=1}^{N}[x_i^{\mathrm T}(t)P_ix_i(t) + V_{ti}(t)], \\ V_{ti}(t) = \sum_{j\in J_i(A)}\int_{t-\tau_{ij}}^{t} x_j^{\mathrm T}(s)P_jH_jP_jx_j(s)\mathrm{d}s. \end{cases}$$

Then time derivation along the trajectory of the closed-loop system (3) satisfies

$$\begin{aligned}
\dot V(t) = {} & \sum_{i=1}^{N}\{2x_i^{\mathrm T}(t)P_i[(A_i + \Delta A_i(t) + B_iK_i + \Delta B_i(t)K_i)x_i(t) + \\
& \sum_{j\in J_i(A)}(A_{ij} + \Delta A_{ij}(t))x_j(t-\tau_{ij}) + G_iw_i(t)] + \dot V_{ti}(t)\} = \\
& \sum_{i=1}^{N}[x_i^{\mathrm T}(t)(P_iA_i + A_i^{\mathrm T}P_i + P_iB_iK_i + K_i^{\mathrm T}B_i^{\mathrm T}P_i)x_i(t) + \\
& 2x_i^{\mathrm T}(t)P_i\Delta A_i(t)x_i(t) + 2x_i^{\mathrm T}(t)P_i\Delta B_i(t)K_ix_i(t) + \\
& x_i^{\mathrm T}(t)P_iG_iw_i(t) + w_i^{\mathrm T}(t)G_i^{\mathrm T}P_ix_i(t) + \\
& 2x_i^{\mathrm T}(t)P_i\sum_{j\in J_i(A)}(A_{ij} + \Delta A_{ij}(t))x_j(t-\tau_{ij}) + \dot V_{ti}(t)].
\end{aligned}$$

Using Lemma 1 and Lemma 2, we have

$$\begin{aligned}
& 2x_i^{\mathrm T}(t)P_i\Delta A_i(t)x_i(t) = \\
& 2x_i^{\mathrm T}(t)P_iD_iF_i(t)E_{ai}x_i(t) \leqslant \\
& x_i^{\mathrm T}(t)P_iD_iD_i^{\mathrm T}P_ix_i(t) + x_i^{\mathrm T}(t)E_{ai}^{\mathrm T}E_{ai}x_i(t), \\
& 2x_i^{\mathrm T}(t)P_i\Delta B_i(t)K_ix_i(t) = \\
& 2x_i^{\mathrm T}(t)P_iD_iF_i(t)E_{bi}K_ix_i(t) \leqslant \\
& x_i^{\mathrm T}(t)P_iD_iD_i^{\mathrm T}P_ix_i(t) + x_i^{\mathrm T}(t)K_i^{\mathrm T}E_{bi}^{\mathrm T}E_{bi}K_ix_i(t), \\
& 2x_i^{\mathrm T}(t)P_i\sum_{j\in J_i(A)}(A_{ij} + \Delta A_{ij}(t))x_j(t-\tau_{ij}) \leqslant \\
& x_i^{\mathrm T}(t)P_i\sum_{j\in J_i(A)}(A_{ij} + \Delta A_{ij}(t))P_j^{-1}H_j^{-1}P_j^{-1}(A_{ij} + \\
& \Delta A_{ij}(t))^{\mathrm T}P_ix_i(t) + \sum_{j\in J_i(A)}x_j^{\mathrm T}(t - \\
& \tau_{ij})P_jH_jP_jx_j(t-\tau_{ij}).
\end{aligned}$$

Note that

$$\dot V_{ti}(t) = \sum_{j\in J_i(A)}x_j^{\mathrm T}(t)P_jH_jP_jx_j(t) - \sum_{j\in J_i(A)}x_j^{\mathrm T}(t-\tau_{ij})P_jH_jP_jx_j(t-\tau_{ij}).$$

Therefore

$$\dot V(t) \leqslant \sum_{i=1}^{N}\{x_i^T(t)(P_iA_i + A_i^TP_i + P_iB_iK_i + K_i^TB_i^TP_i + 2P_iD_iD_i^TP_i + E_{ai}^TE_{ai} + K_i^TE_{bi}^TE_{bi}K_i)x_i(t) + x_i^T(t)P_iG_iw_i(t) + w_i^T(t)G_i^TP_ix_i(t) + x_i^T(t)P_i\sum_{j\in J_i(A)}(A_{ij}+\Delta A_{ij}(t))P_j^{-1}H_j^{-1}P_j^{-1}(A_{ij}+\Delta A_{ij}(t))^TP_ix_i(t) + \sum_{j\in J_i(A)}x_j^T(t)P_jH_jP_jx_j(t)\}.$$

Using Lemma 3, we get

$$\sum_{j\in J_i(A)}(A_{ij}+\Delta A_{ij}(t))P_j^{-1}H_j^{-1}P_j^{-1}(A_{ij}+\Delta A_{ij}(t))^T = \sum_{j\in J_i(A)}(A_{ij}+M_{ij}F_{ij}(t)N_{ij})P_j^{-1}H_j^{-1}P_j^{-1}(A_{ij}+M_{ij}F_{ij}(t)N_{ij})^T \leqslant \sum_{j\in J_i(A)}A_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^TN_{ij})^{-1}A_{ij}^T + \sum_{j\in J_i(A)}\varepsilon_{ij}^{-1}M_{ij}M_{ij}^T.$$

Furthermore, we have

$$\dot V(t) \leqslant \sum_{i=1}^{N}\{x_i^T(t)[P_iA_i + A_i^TP_i + P_iB_iK_i + K_i^TB_i^TP_i + 2P_iD_iD_i^TP_i + E_{ai}^TE_{ai} + K_i^TE_{bi}^TE_{bi}K_i + P_i\sum_{j\in J_i(A)}A_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^TN_{ij})^{-1}A_{ij}^TP_i + \sum_{j\in J_i(A)}\varepsilon_{ij}^{-1}P_iM_{ij}M_{ij}^TP_i]x_i(t) + x_i^T(t)P_iG_iw_i(t) + w_i^T(t)G_i^TP_ix_i(t) + \sum_{j\in J_i(A)}x_j^T(t)P_jH_jP_jx_j(t)\} =$$

$$\sum_{i=1}^{N}\{x_i^T(t)[P_iA_i + A_i^TP_i + P_iB_iK_i + K_i^TB_i^TP_i + 2P_iD_iD_i^TP_i + E_{ai}^TE_{ai} + K_i^TE_{bi}^TE_{bi}K_i + P_i\sum_{j\in J_i(A)}A_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^TN_{ij})^{-1}A_{ij}^TP_i + \sum_{j\in J_i(A)}\varepsilon_{ij}^{-1}P_iM_{ij}M_{ij}^TP_i + \tilde N_A(i)P_iH_iP_i]x_i(t) + x_i^T(t)P_iG_iw_i(t) + w_i^T(t)G_i^TP_ix_i(t)\}.$$

First, we prove asymptotic stability of closed-loop system (3), let $w(t) = 0$, that is $w_i(t) = 0, i = 1,2,\cdots,N$, obviously $\dot V(t) < 0$ if

$$P_iA_i + A_i^TP_i + P_iB_iK_i + K_i^TB_i^TP_i + 2P_iD_iD_i^TP_i + E_{ai}^TE_{ai} + K_i^TE_{bi}^TE_{bi}K_i + P_i\sum_{j\in J_i(A)}A_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^TN_{ij})^{-1}A_{ij}^TP_i + \sum_{j\in J_i(A)}\varepsilon_{ij}^{-1}P_iM_{ij}M_{ij}^TP_i + \tilde N_A(i)P_iH_iP_i < 0. \quad (9)$$

Next, to establish that $\|z(t)\|_2 < \gamma\|w(t)\|_2$ whenever $w(t)\neq 0$ which implies that the desired robust H_∞ performance is achieved, we introduce

$$J = \int_0^\infty(z^T(t)z(t) - \gamma^2w^T(t)w(t))dt = \sum_{i=1}^{N}\int_0^\infty[z_i^T(t)z_i(t) - \gamma^2w_i^T(t)w_i(t)]dt.$$

Note that for zero initial conditions, $V(0) = 0$, we get

$$J = \sum_{i=1}^{N}\int_0^\infty[z_i^T(t)z_i(t) - \gamma^2w_i^T(t)w_i(t) + \dot V_i(t)]dt - \sum_{i=1}^{\infty}V_i(\infty) \leqslant \sum_{i=1}^{N}\int_0^\infty[z_i^T(t)z_i(t) - \gamma^2w_i^T(t)w_i(t) + \dot V_i(t)]dt \leqslant$$

$$\sum_{i=1}^{N}\int_0^\infty\{x_i^T(t)[P_iA_i + A_i^TP_i + \tilde N_A(i)P_iH_iP_i + P_iB_iK_i + K_i^TB_i^TP_i + 2P_iD_iD_i^TP_i + E_{ai}^TE_{ai} + K_i^TE_{bi}^TE_{bi}K_i + \sum_{j\in J_i(A)}P_iA_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^TN_{ij})^{-1}A_{ij}^TP_i + C_i^TC_i + \sum_{j\in J_i(A)}\varepsilon_{ij}^{-1}P_iM_{ij}M_{ij}^TP_i + C_i^TT_iK_i + K_i^TT_i^TC_i + K_i^TT_i^TT_iK_i]x_i(t) + x_i^T(t)P_iG_iw_i(t) + w_i^T(t)G_i^TP_ix_i(t) - \gamma^2w_i^T(t)w_i(t)\}dt.$$

Using Lemma2, we have

$$J \leqslant \sum_{i=1}^{N}\int_0^\infty\{x_i^T(t)[P_iA_i + A_i^TP_i + \tilde N_A(i)P_iH_iP_i + P_iB_iK_i + K_i^TB_i^TP_i + 2P_iD_iD_i^TP_i + E_{ai}^TE_{ai} + K_i^TE_{bi}^TE_{bi}K_i + \sum_{j\in J_i(A)}P_iA_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^TN_{ij})^{-1}A_{ij}^TP_i + \sum_{j\in J_i(A)}\varepsilon_{ij}^{-1}P_iM_{ij}M_{ij}^TP_i + 2C_i^TC_i + 2K_i^TT_i^TT_iK_i]x_i(t) + x_i^T(t)P_iG_iw_i(t) + w_i^T(t)G_i^TP_ix_i(t) - \gamma^2w_i^T(t)w_i(t)\}dt = \sum_{i=1}^{N}\int_0^\infty\xi_i^T(t)\Psi_i\xi_i(t)dt.$$

where

$$\xi_i = [x_i^T(t) \quad w_i^T(t)]^T,$$

$$\Psi_i = \begin{bmatrix}\Omega_i & P_iG_i\\ G_i^TP_i & -\gamma^2I\end{bmatrix},$$

$$\Omega_i = P_iA_i + A_i^TP_i + \tilde N_A(i)P_iH_iP_i + P_iB_iK_i + K_i^TB_i^TP_i + 2C_i^TC_i + 2K_i^TT_i^TT_iK_i + 2P_iD_iD_i^TP_i + E_{ai}^TE_{ai} +$$

$$K_i^{\mathrm T}E_{bi}^{\mathrm T}E_{bi}K_i + \sum_{j\in J_i(A)} \varepsilon_{ij}^{-1}P_iM_{ij}M_{ij}^{\mathrm T}P_i + \sum_{j\in J_i(A)} P_iA_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^{\mathrm T}N_{ij})^{-1}A_{ij}^{\mathrm T}P_i.$$

Therefore, we obtain $J < 0$ if

$$\Psi_i = \begin{bmatrix} \Omega_i & P_iG_i \\ G_i^{\mathrm T}P_i & -\gamma^2 I \end{bmatrix} < 0. \tag{10}$$

Inequalities (10) pre-multiplying and post-multiplying by Diag $[P_i^{-1} \quad I]$, and introducing the new variables, $X_i = P_i^{-1}, Y_i = K_iX_i$, we get the following

$$\Psi_i = \begin{bmatrix} \Xi_i & G_i \\ G_i^{\mathrm T} & -\gamma^2 I \end{bmatrix} < 0, \tag{11}$$

$$\begin{aligned}\Xi_i = {} & A_iX_i + X_iA_i^{\mathrm T} + \widetilde N_A(i)H_i + B_iY_i + Y_i^{\mathrm T}B_i^{\mathrm T} + \\ & 2X_iC_i^{\mathrm T}C_iX_i + 2Y_i^{\mathrm T}T_i^{\mathrm T}T_iY_i + 2D_iD_i^{\mathrm T} + \\ & X_iE_{ai}^{\mathrm T}E_{ai}X_i + Y_i^{\mathrm T}E_{bi}^{\mathrm T}E_{bi}Y_i + \sum_{j\in J_i(A)} \varepsilon_{ij}^{-1}M_{ij}M_{ij}^{\mathrm T} + \\ & \sum_{j\in J_i(A)} A_{ij}(P_jH_jP_j - \varepsilon_{ij}N_{ij}^{\mathrm T}N_{ij})^{-1}A_{ij}^{\mathrm T}.\end{aligned}$$

Then using Schur's complements, we obtain from (11)

$$\begin{bmatrix}
\Theta_i & G_i & X_iC_i^{\mathrm T} & Y_i^{\mathrm T}T_i^{\mathrm T} & X_iE_{ai}^{\mathrm T} & Y_i^{\mathrm T}E_{bi}^{\mathrm T} & M_{i1} & \cdots & M_{iN} & A_{i1} & \cdots & A_{iN} \\
G_i^{\mathrm T} & -\gamma^2 I & & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
C_iX_i & 0 & -\frac{1}{2}I & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
T_iY_i & 0 & 0 & -\frac{1}{2}I & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
E_{ai}X_i & 0 & 0 & 0 & -I & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
E_{bi}Y_i & 0 & 0 & 0 & 0 & -I & 0 & \cdots & 0 & 0 & \cdots & 0 \\
M_{i1}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{i1}I & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\
M_{iN}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -\varepsilon_{iN}I & 0 & \cdots & 0 \\
A_{i1}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & U_1 & \cdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & 0 \\
A_{iN}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & U_N
\end{bmatrix} < 0, \tag{12}$$

Where

$$\Theta_i = A_iX_i + X_iA_i^{\mathrm T} + B_iY_i + Y_i^{\mathrm T}B_i^{\mathrm T} + \widetilde N_A(i)H_i + 2D_iD_i^{\mathrm T},$$

$$U_j = -P_jH_jP_j + \varepsilon_{ij}N_{ij}^{\mathrm T}N_{ij}.$$

Inequalities (12) pre-multiplying and post-multiplying by Diag $[I \ I \ I \ I \ I \ I \ \varepsilon_{i1}^{-1} \ \cdots \ \varepsilon_{iN}^{-1} \ X_1 \ \cdots \ X_N]$, we have

$$\begin{bmatrix}
\Theta_i & G_i & X_iC_i^{\mathrm T} & Y_i^{\mathrm T}T_i^{\mathrm T} & X_iE_{ai}^{\mathrm T} & Y_i^{\mathrm T}E_{bi}^{\mathrm T} & \varepsilon_{i1}^{-1}M_{i1} & \cdots & \varepsilon_{iN}^{-1}M_{iN} & A_{i1}X_1 & \cdots & A_{iN}X_N \\
G_i^{\mathrm T} & -\gamma^2 I & & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
C_iX_i & 0 & -\frac{1}{2}I & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
T_iY_i & 0 & 0 & -\frac{1}{2}I & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
E_{ai}X_i & 0 & 0 & 0 & -I & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
E_{bi}Y_i & 0 & 0 & 0 & 0 & -I & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\varepsilon_{i1}^{-1}M_{i1}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{i1}^{-1}I & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\
\varepsilon_{iN}^{-1}M_{iN}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & -\varepsilon_{iN}^{-1}I & 0 & \cdots & 0 \\
X_1A_{i1}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -H_1 & \cdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & 0 \\
X_NA_{iN}^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -H_N
\end{bmatrix} +$$

$$\begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ X_1 N_{i1}^{\mathrm{T}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & X_N N_{iN}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \varepsilon_{i1} & \cdots & 0 \\ & \ddots & \vdots \\ 0 & \cdots & \varepsilon_{iN} \end{bmatrix} \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ X_1 N_{i1}^{\mathrm{T}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & X_N N_{iN}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} < 0. \tag{13}$$

Using Schur's complements to (13), it can be readily verified that LMIs (13) are equivalent to LMIs (8), and LMIs (13) can guarantee LMIs (9). Therefore the proof is completed.

Theorem 1 presents a decentralized H_∞ controller design procedure for the large-scale interconnected time-delay system (1). In order to reduce the conservativeness of Theorem 1, a method of designing a decentralizing state feedback control law with smaller feedback gain is formulated by a convex optimization problem.

We enforce

$$\begin{cases} \min\left(\sum_{i=1}^{N} s_i + \sum_{i=1}^{N} t_i\right), \\ Y_i^{\mathrm{T}} Y_i < t_i I,\ X_i^{-1} < s_i I \end{cases} \tag{14}$$

to Theorem 1. It makes $K_i^{\mathrm{T}} K_i = X_i^{-1} Y_i^{\mathrm{T}} Y_i X_i^{-1} < t_i s_i^2 I$. According to Schur's complements, inequalities $Y_i^{\mathrm{T}} Y_i < t_i I$, $X_i^{-1} < s_i I$ equal to the following linear matrix inequalities

$$\begin{bmatrix} -t_i I & Y^{\mathrm{T}} \\ Y & -I \end{bmatrix} < 0,\ \begin{bmatrix} -s_i I & I \\ I & -X_i \end{bmatrix} < 0. \tag{15}$$

Corollary 1 To obtain smaller decentralized H_∞-state feedback gains, we may solve the following LMI optimization problem:

$$\begin{cases} \text{Minimize } \sum_{i=1}^{N} (s_i + t_i), \\ \text{Subject to } (8), (15). \end{cases} \tag{16}$$

4 Example

In this section, we give a numerical example to illustrate the design procedure developed in corollary 1. Consider the uncertain large-scale interconnected time-delay system with $N = 3$.

$$A_1 = \begin{bmatrix} -1.97 & -0.31 \\ 0.91 & 0.8 \end{bmatrix},\ A_{12} = \begin{bmatrix} -0.5 & 0.3 \\ 0 & 1.2 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.4 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.32 & 0.34 \\ 0.1 & -0.87 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -0.5 & 0 \\ 0 & 1 \end{bmatrix},\ A_{23} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -3.42 & 0.06 \\ 0.78 & 0.2 \end{bmatrix},\ A_{31} = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$A_{32} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = [0.1\ \ 0],\ C_2 = [0\ \ 0.1],$$

$$C_3 = [0.2\ \ 0.1],$$

$$G_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\ G_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},\ G_3 = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$

$$T_1 = 0.1,\ T_2 = 0.1,\ T_3 = 0.1,$$

$$D_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},\ D_3 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix},$$

$$E_{a1} = [-0.04\ \ 0.03],\ E_{a2} = [-0.06\ \ 0.04],$$

$$E_{a3} = [-0.07\ \ 0.02],$$

$$E_{b1} = 0.04,\ E_{b2} = -0.05,\ E_{b3} = -0.03,$$

$$M_{12} = \begin{bmatrix} 0.02 \\ 0.01 \end{bmatrix},\ M_{13} = \begin{bmatrix} -0.04 \\ 0.04 \end{bmatrix},$$

$$M_{21} = \begin{bmatrix} 0.05 \\ -0.3 \end{bmatrix},\ M_{23} = \begin{bmatrix} 0.4 \\ 0.05 \end{bmatrix},$$

$$M_{31} = \begin{bmatrix} -0.06 \\ 0.5 \end{bmatrix},\ M_{32} = \begin{bmatrix} 0.2 \\ -0.001 \end{bmatrix},$$

$N_{12} = [0.03 \quad 0.1]$, $N_{13} = [0.3 \quad -0.03]$,
$N_{21} = [-0.01 \quad 0.6]$, $N_{23} = [0.4 \quad 0.05]$,
$N_{31} = [0.3 \quad -0.3]$, $N_{32} = [0.3 \quad -0.3]$,
$A_{11} = 0$, $A_{22} = 0$, $A_{33} = 0$.

Obviously $\tilde{N}_A(1) = 2$, $\tilde{N}_A(2) = 2$, $\tilde{N}_A(3) = 2$.

Let $\gamma = 1$ and solve the LMIs (16), then obtain a group of the parameter matrices as follows:

$$X_1 = \begin{bmatrix} 11.0319 & -2.8358 \\ -2.8358 & 0.9329 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.0139 & -0.0935 \\ -0.0935 & 1.0063 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 4.8499 & -0.4707 \\ -0.4707 & 0.5245 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 18.9519 & -5.2175 \\ -5.2175 & 1.4860 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 5.7394 & -0.5158 \\ -0.5158 & 0.7366 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 13.6789 & -2.8915 \\ -2.8915 & 0.9163 \end{bmatrix},$$

$\epsilon_{12} = 1.3473$, $\epsilon_{13} = 0.7850$, $\epsilon_{21} = 2.1435$,
$\epsilon_{23} = 1.9009$, $\epsilon_{31} = 1.9014$, $\epsilon_{32} = 0.3124$,
$s_1 = 2.7082$, $s_2 = 1.5068$, $s_3 = 1.3217$,
$t_1 = 5.2323$, $t_2 = 3.1535$, $t_3 = 2.1103$,
$Y_1 = [0.3620 \quad -1.6054]$,
$Y_2 = [0.0086 \quad -1.2275]$,
$Y_3 = [-0.1730 \quad -1.1366]$,
$K_1 = [-2.1732 \quad -8.3267]$,
$K_2 = [0.1528 \quad -3.8657]$,
$K_3 = [-0.26942 \quad -2.408]$.

5 Conclusion

In this paper, some criteria of H_∞-controller for a class of uncertain large-scale interconnected continuous systems with $N \times N$ unknown but constant delays in the interconnections time-varying and norm-bounded parametric uncertainties is addressed by LMI approach. A numerical example has been provided to illustrate the procedure of designing decentralized H_∞-state feedback controller.

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