Article ID: 1000 - 8152(2001)06 - 0961 - 03

Comment on "Robust Delay-Dependence Stability for Linear Systems with Nonlinear Parameter Perturbations"*

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Abstract: In a recent paper (Xu and Liu, Control Theory & Applications, vol.15, no.4, pp. 501-506, 1998), a sufficient condition is reported for robust delay-dependence stability in terms of a bound on spectral radius of a prescribed nonnegative matrix. It seems to be fairly powerful, but we argue here that their result may be incorrect. A corrected result is presented in this paper also.

Key words: robustness; stability; delay differential systems

Document code: A

对"具有非线性参数振动的线性系统的时滞相关鲁棒稳定性"一文的评论

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摘要: 最近有一论文根据非负矩阵谱半径性质,给出了系统鲁棒稳定的一个充分条件,这一结果看起来十分有效,但事实上是不可靠的.本文指出了这一结果证明过程中的错误,并给出了修正后的结论.

关键词: 鲁棒性; 稳定性; 时滞微分方程

1 Background and remarks

In a recent paper[1] (Xu and Liu, 1998), the authors proposed a sufficient condition in terms of a bound on the spectral radius of a prescribed nonnegative matrix by delay integral inequality. An example was given to demonstrate the validity of their results, which were much more powerful than those obtained in [2 ~ 5]. Unfortunately their results may be incorrect. In this paper we argue that both Theorem 1 and Theorem 2 in [1] may be incorrect, we also highlight other flaws in their paper.

The system considered in [1] is described as

$$\begin{cases} \dot{x}(t) = A_0 x(t) + A_1 x(t - h_1(t)) + \Delta A_0(t, x(t)) + \\ \qquad \Delta A_1(t, x(t - h_2(t))), \\ x(t) = \phi(t),\ t \in [t_0 - \tau, t_0], \end{cases} \tag{1}$$

$$| \Delta A_0(t, x(t)) | \leqslant A_0 | x(t) |,$$
$$| \Delta A_1(t, x(t - h_2(t))) | \leqslant A_1 | x(t - h_2(t)) |. \tag{2}$$

For the notations see [1].

We now point out some ambiguities and flaws in their paper through the following remarks. For convenience, we assume $[x(0)]^+ = 0$, which does not lose any generality.

Remark 1 In their proof of Theorem1, the authors claimed that one does obtain inequality (9) from inequality (8). We first point out that it is impossible. See this example:

Let $x(t) = 1.5\sin(x - \tau)$, by choosing appropriate t, τ, we get

$$[x(t)]^+ = 0.5,\ y(t) = \sup_{\tau}[x(t)]^+ = 1.5,$$

* Foundation item: supported by the National Natural Science Foundation (6997403).
Received: 2000 - 03 - 27; Revised date: 2001 - 03 - 12.

it is obvious that the inequality

$$[x(t)]^+ \leqslant 0.5y(t)$$

holds, i.e. that inequality (8) (where $\Pi = 0.5$) holds, while the following inequality

$$y(t) \leqslant 0.5y(t)$$

does not hold, i.e. that inequality (9) in [1] does not holds. So their statement "let $y(t) = \sup_{t_0-\tau<s\leqslant t}[x(s)]^+$, then $[x(s)]^+ \leqslant y(t)$ and" should be "let $y(t) = \sup_{t_0-2\tau<s\leqslant t}[x(s)]^+$, then $[x(s)]^+ \leqslant y(t)$ if" (see also Remark 5).

Remark 2 Almost all the results in [1] were obtained from the inequality (9), we think that the authors' Theorem 1 contradicts its premise. In fact $\rho(\Pi) \geqslant 1$ if the inequality (9) in [1] holds. Let's use the contradiction to give a brief proof of this assertion.

Assume the inequality (9) holds, it follows immediately that

$$(I - \Pi)y(t) \leqslant 0, \tag{3}$$

since $y(t) \geqslant 0$, for all t, then

$$I - \Pi \leqslant 0;$$

and if

$$\rho(\Pi) < 1,$$

then

$$(I - \Pi)^{-1} \geqslant 0,$$

it leads to

$$-(I - \Pi)(I - \Pi)^{-1} \geqslant 0,$$

i.e.

$$I \leqslant 0. \tag{4}$$

It contradicts $I \geqslant 0$, so $\rho(\Pi) \geqslant 1$. This completes our assertion. Q.E.D.

So we argue that the results obtained in [1] may be incorrect.

Remark 3 As noted in Remark 1, their Theorem 2 may be incorrect either, in addition, the method does not decrease dimensions. As shown by Example 2 in [1], for the two 2×2 composite systems, computations are required to a 4×4 matrix, such a technique helps little to decrease the computational difficulty.

Remark 4 $[x_t]^+_\tau$ and $[x_t]^+_{2\tau}$ in equation (6) in [1] seems to be $[x_s]^+_\tau$ and $[x_s]^+_{2\tau}$ respectively.

Remark 5 The statement "Let $y(t) = \sup_{t_0-\tau\leqslant s\leqslant t}[x(s)]^+$" seems to be $y(t) = \sup_{t_0-2\tau\leqslant s\leqslant t}[x(s)]^+$.

2 A correction result

Our correction of the result in [1] is as follows

Let $T \in \mathbb{R}^{n\times n}$ be nonsingular, which transfers $T^{-1}(A_0 + A_1)T = J$ and J is the Jordan canonical form of $A_0 + A_1$. Consider the variable transformation

$$z(t) = Tx(t). \tag{5}$$

Substituting (5) into (4) in [1], then $z(t)$ satisfies

$$\begin{aligned}
&\dot{z}(t) = \\
&Jz(t) + T\Delta A_0(t, T^{-1}z(t)) + \\
&T\Delta A_1(t, T^{-1}z(t - h_2)) - \\
&TA_1\int_{t-h_1}^{t}[A_0T^{-1}z(s) + A_1T^{-1}z(s - h_1) + \\
&\Delta A_0(s, T^{-1}z(s)) + \Delta A_1(s, T^{-1}z(s - h_2(t)))]\mathrm{d}s, \\
&z(t) = T\phi(t),\ t \in [-\tau, 0].
\end{aligned} \tag{6}$$

Theorem Let $A_0 + A_1$ be stable, system (1) is asymptotically stable if the test matrix

$$P = \mathrm{Re}(J) + |T|(I + \tau|B|)(A_0 + A_1)|T^{-1}| + \tau(|TA_1A_0T^{-1}| + |TA_1^2T^{-1}|) \tag{7}$$

is stable.

The proof of this theorem can be referred to the appendix.

3 Illustrative example

Example (Su and Huang 1992; Xu 1994; Yongru Gu 1998, Xu and Liu, 1998). Consider the following system described by system (1) where

$$A_0 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},\ A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$\Delta A_0(t, x(t)) = \begin{bmatrix} 0.3\cos t & 0 \\ 0 & 0.2\sin t \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix},$$

$$\Delta A_1(t, x(t)) = \begin{bmatrix} 0.2\cos t & 0 \\ 0 & 0.3\sin t \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

By the Theorem the asymptotic stability delay bound is $\tau < 0.2030$, the results given in [2~5] were $\tau < 0.1614$, $\tau < 0.1575$, $\tau < 0.1583$, $\tau < 0.1575$ respectively. This shows our method is valid. The result obtained in [1] was $\tau < 0.6$, but as we have noted above, this result may be incorrect.

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Appendix

Proof Through the variation of parameter formula, one can obtain the solution of equation (6) as

$$
\begin{aligned}
&z(t) = \\
&\exp(Jt)z(0)+\int_0^t exp(J(t-s))\{T\Delta A_0(s, T^{-1}z(s))+ \\
&T\Delta A_1(s, T^{-1}z(s-h_2)) - TA_1\int_{s-h_2}^{s}[A_0T^{-1}z(u) - \\
&\tau A_1T^{-1}z(u-h_1) + \Delta A_0(u, T^{-1}z(u)) - \\
&\Delta A_1(u, T^{-1}z(u-h_2))]du\}ds.
\end{aligned}
\tag{8}
$$

Define $\underset{s\in[-2\tau,-\tau]}{z(s)} = z(-\tau), y(t) = \sup_{s\in[-2\tau,0]} |z(t+s)|$, then $y(t) \geqslant |z(t)|$, one can obtain the following inequality by applying the modular arithmetic to (8)

$$
\begin{aligned}
|z(t)| \leqslant {} & \exp(\mathrm{Re}(J)t)y(0) + \\
& \int_0^t \exp(\mathrm{Re}(J)(t-s))(P - \mathrm{Re}(J))y(s)ds.
\end{aligned}
\tag{9}
$$

The right half of (8) is the solution of the differential equation

$$\dot{y}(t) = Py(t). \tag{10}$$

Let $W(t) \in \mathbb{R}^n$ denote the maximum solution of differential equation (10), by Lemma 3 in [6] one gets $|z(t)| \leqslant W(t)$. If P defined by (7) is stable, then any solution of (10) will be asymptotically stable. It implies $\lim_{t\to\infty} |z(t)| = 0$, so systems (6) and (1) are asymptotically satble. Since the asymptotic stableness of system (1) is equivalent to that of system (6), system (1) is asymptotically stable. Q.E.D.

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