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Controlling Hyper-Chaotic and Chaotic Dynamics in Noisy Environment *

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Abstract: We consider the problem concerning the control of the discrete-time chaotic and hyper-chaotic dynamical systems in noisy environment. Due to the ergodicity property of orbits in the chaotic attractors, including chaos and hyper-chaos, an optimal control is presented to direct the orbit of the discrete-time chaotic dynamical system quickly towards a pre-specified neighbourhood of the target, and feedback correction is added to deal with noise. After entering the neighbourhood in which the local controller is effective, the controller consisting of small perturbations is used to stabilize the orbit in the system. The numerical simulations of controlling two typical chaotic dynamical systems, one chaotic and the other hyper-chaotic, show that the combined method is effective for a wide range of discrete-time chaotic and hyper-chaotic dynamics.

Key words: chaos; hyper-chaos; optimal control; noise; perturbations

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控制噪声环境中的超混沌与混沌系统

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摘要: 考虑了噪声环境中的超混沌与混沌的离散动力系统的控制问题,提出了一种普遍适用的控制方法.基于混沌吸引子里的轨道的遍历性质,利用最优控制方法快速引导系统轨道进入给定的目标轨道的邻域里;同时增加一个反馈校正器以抑制噪声的干扰,使得系统的轨道不至于偏离最优参考轨道太远.当系统轨道进入给定的目标轨道的邻域后,再设计一个简单的小扰动控制器稳定控制系统运行在目标轨道上.仿真表明,本文的综合控制方法快速、有效.

关键词: 混沌; 超混沌; 最优控制; 噪声; 扰动

1 Introduction

If chaotic motion is undesirable, they should be eliminated and converted into desired low-period motion, especially the period-1 (a fixed point) motion. After the paper by Ott, Grebogi, and Yorke[1] presented a method to make small time-dependent perturbations in an accessible system parameter and thereby achieve a desired periodic output, many techniques of controlling chaos have been proposed over the past decade[2]. In practice, controlling a fully unstable system (hyper-chaos) is as interesting and important as the partly stable manifold (chaos). However, the issue of controlling such hyper-chaos has not been particularly and adequately addressed. Yang et al presented a paper about controlling hyper-chaos[3], yet the controller they designed is effective only in sufficiently small neighbourhood of the target and its control set is not bounded. Due to the ergodicity property, Yang's controller can be utilized to direct the orbits in the hyper-chaotic dynamics in the small neighbourhood of the desired target towards the target. However, the state may wander chaotically for a rather long time before entering the neighbourhood in which the local controller is effective. In this paper, under the condition of arbitrary initial state in the hyper-chaotic attractor, an optimal control method based on the Minimum Principle is introduced to direct the trajectory in hyper-chaotic attractor towards a pre-specified sufficiently small neighbourhood of the target as quickly as possible. The

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hyper-chaotic dynamics in noisy environment is also taken into consideration in the paper, and the feedback correction is added to deal with noise. After entering the neighbourhood in which the local controller is effective, the controller consisting of small perturbations is used to robustly stabilize the orbit at the target in the noisy systems.

Because the hyper-chaotic cases are more general for consideration than chaotic cases, comparatively speaking, the combined method presented to control hyperchaos in the paper is as well effective for controlling chaos, which indicates the combined method could be applied to a wide range of chaotic and hyper-chaotic dynamics. We consider only a class of nonlinear discrete-time dynamics where the analytical expression of the map is assumed available. That is, we may assume that there is enough data for the reconstruction of the map of the hyper-chaotic or chaotic attractor with its gradient. The above assumption is similar to the paper Yang presented.

This paper is organized as follows: in Section 2, small perturbations are used to direct orbits of a nonlinear discrete-time chaotic or hyper-chaotic dynamics quickly towards the target, which is posed as a discrete-time optimal control problem. In Section 3, a design method for the construction of required local feedback correction is given. This local feedback correction acts as a supplementary controller to deal with the effect due to the noisy environment. In Section 4, a controller consisting of small perturbations is used to stabilize the orbit of the dynamics in the neighbourhood of the target and to deal with noise. In Section 5, thorough numerical simulations are given to illustrate the efficiency of the proposed combined method in this paper. And some important conclusions are presented in the last section.

2 Direct the trajectory towards the neighbourhood of target

Consider a class of discrete-time dynamical systems. While the control set is assumed to satisfy the boundedness constraints, it is a typical problem of optimal time control to direct the trajectory in hyper-chaotic attractor towards a pre-specified sufficiently small neighbourhood of the target as quickly as possible. However, the trajectory entering a pre-specified neighbourhood of the target makes the terminal state constrained equation be constrained by inequalities, thus it is not easy to obtain numerical resolutions. The above problem could be substantially converted into another equivalent one, which is to get the optimal control sequence to satisfy the desired minimal distance between the terminal state and the target state within given time. Thus the problem in the case of discrete-time dynamical systems becomes that, the objective function $\| x(N) - x^t \|$ is minimized with given N steps. The following Theorem 1 can provide the solution to the above problem, which is just a special improved application of the Minimum Principle[4], and the process of proving Theorem 1 is ignored.

Theorem 1 Consider the discrete-time dynamical systems,

$$\begin{cases} x(k+1) = f(x(k), u(k), k), \ x(0) = x_0, \\ k = 0, 1, 2, \cdots, N-1, \end{cases} \tag{1}$$

where $x(k) = [x_1(k), \cdots, x_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state, $u(k) = [u_1(k), \cdots, u_m(k)]^{\mathrm{T}} \in \mathbb{R}^m$ is the control sequence and satisfies the boundedness constraints $u(k) \in \Omega$, Ω is the constrained set, and the objective function is given as follows

$$J = \varphi[x(N), N] = \| x(N) - x^t \|^2, \tag{2}$$

where x^t is the terminal state, and $f = (f_1, \cdots, f_n)^{\mathrm{T}}: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^n$, $\varphi: \mathbb{R}^n \to \mathbb{R}$, is continuously differentiable with respect to each of the components of x and u, respectively.

If $u^*(k)$ is the optimal control sequence to minimize the objective function and $x^*(k)$ is the corresponding optimal state sequence, then there exists an n-dimensional vector function $\lambda(k)$ to make $u^*(k), x^*(k)$ and $\lambda(k)$ satisfy the following necessary conditions:

1) $x^*(k)$ and $\lambda(k)$ satisfy the following difference equations

$$x^*(k+1) = \frac{\partial H(k)}{\partial \lambda(k+1)}, \ \lambda(k) = \frac{\partial H(k)}{\partial x(k)}, \tag{3}$$

where $H(k)$ is a discrete Hamiltonian function,

$$H(k) = H[x(k), u(k), \lambda(k+1), k] = \lambda^{\mathrm{T}}(k+1) f[x(k), u(k), k]. \tag{4}$$

2) $x^*(k)$ and $\lambda(k)$ satisfy the following terminal conditions

$$\lambda(N) = \frac{\partial \varphi[x(N), N]}{\partial x(N)}, \ x(0) = x_0. \tag{5}$$

3) The optimal control sequence $u^*(k)$ also make the discrete Hamiltonian function satisfy

$$H[x^*(k),u^*(k),k] = \min_{u(k)\in\Omega} H[x^*(k),u(k),\lambda(k+1),k]. \tag{6}$$

3 Feedback correction in noisy environment

Because the trajectory is extremely sensitive with respect to initial conditions and dense in chaotic or hyper-chaotic attractor, the above open-loop controller is lack of robustness. In an analytical view, it is necessary to add a feedback correction to the optimal control sequence $u^*(k)$ calculated off-line so as to deal with noise. For simplicity, we only consider the uniformly distributed random noise added to the vector function $f(x,k)$ in the Eq.(1).

Without loss of generality, assume the augmented control vector $v(k) = (u_1(k),\cdots,u_m(k),0,\cdots,0)^{\mathrm{T}} \in \mathbb{R}^n$, then the controlled systems is expressed as follows

$$x(k+1) = f(x(k),k) + v(k). \tag{7}$$

Assume the obtained optimal control sequence $v^* = \{v^*(0),v^*(1),\cdots,v^*(N-1)\}$, and the corresponding optimal state sequence $x^* = \{x^*(0),x^*(1),\cdots,x^*(N-1)\}$. Then we may regard x^* as the reference trajectory. To prevent a trajectory from escaping from the neighbourhood of the reference trajectory, a feedback correction based on the reference $v^*(k)$ should be added to make sure that the proposed feedback correction will act as a supplementary controller to pull the trajectory towards the corresponding reference trajectory at each step. Define

$$\bar{v}(k) = v^*(k) + \sigma(k)[x(k) - x^*(k)], \quad \sigma(k) \in \mathbb{R}^{n\times m}. \tag{8}$$

Substituting Eq.(8) into Eq.(7) yields

$$x(k+1) = f(x(k)) + \bar{v}(k) = f(x(k)) + v^*(k) + \sigma(k)(x(k) - x^*(k)). \tag{9}$$

For $x^*(k+1) = f(x^*(k)) + v^*(k)$, thus

$$x(k+1) - x^*(k+1) = f(x(k)) - f(x^*(k)) + \sigma(k)(x(k) - x^*(k)) \approx \{[Df]_{x^*(k)} + \sigma(k)\}(x(k) - x^*(k)), \tag{10}$$

where $[Df]_{x^*(k)}$ is Jacobian matrix. Let $A(k) = [Df]_{x^*(k)} + \sigma(k)$, for arbitrary errors, thus

$$\frac{x(k+1) - x^*(k+1)}{x(k) - x^*(k)} = A(k). \tag{11}$$

Therefore, the sufficient conditions for $x(k)$ contracting around the reference trajectory $x^*(k)$ could be obtained as follows

$$\max |\,\mathrm{eig}\{A(k)\}\,| < 1. \tag{12}$$

In practice, it is not difficult to obtain suitable $\sigma(k)$ to satisfy Eq.(12)[5].

4 Stabilize the trajectory to the target

Suppose all the manifolds are unstable near the target (a fixed point), i.e. it is the hyper-chaotic case we are particularly interested in. Redefine the following canonical discrete-time dynamics

$$x(k+1) = f(x(k),u(k),k), \tag{13}$$

where $x(k) = [x_1(k),\cdots,x_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state, $u(k) = [u_1(k),\cdots,u_n(k)]^{\mathrm{T}} \in \mathbb{R}^n$ is the small control vector, f is a vector-valued function of $x(k)$ and $u(k)$, the equilibrium x' is the origin. Note that other cases can be converted into the canonical form of Eq.(13) by suitable coordinate transformation. Let J be the Jacobian matrix of the map with $u = 0$ evaluated at the fixed point, i.e.,

$$J = \left(\frac{\partial f}{\partial x}\right)_{x=x'}. \tag{14}$$

The eigenvalues of the Jacobian matrix J in Eq.(14) are all with modulus greater than unity for the case we study in this paper. And denote the fixed point by x' and define the following matrix

$$M = \left(\frac{\partial x'}{\partial u}\right)_{u=0}. \tag{15}$$

Hence, for $u \to 0$, we can get

$$x(k+1) - x' = J(x(k) - x'). \tag{16}$$

To stabilize the unstable orbit, we propose to require

$$x(k+1) = \delta x(k), \tag{17}$$

where δ is a constant and $-1 < \delta < 1$. This means that the orbit is forced to contract to the fixed point. On the other hand, from the definition of matrix M, we have the following result, for $u \to 0$,

$$x' = Mu + o(u^2). \tag{18}$$

When matrices $(J - I)$ and M are both invertible, eliminate x' and $x(k+1)$ in Eqs.(16), (17) and (18) to yield

$$u = M^{-1}(J - I)^{-1}(J - \delta I)x(k), \tag{19}$$

where I is the $N \times N$ identity matrix.

Finally, without loss of generality, for the case described in Eq. (1), i.e. the control target is the fixed point, the control vector can be re-described as follows

$$u = M^{-1}(J - I)^{-1}(J - \delta I)[x(k) - x^t]. \tag{20}$$

5 Numerical simulations

The above combined method is applied to control the following two typical systems: hyper-chaotic dynamics and chaotic dynamics, which show that the method has an effective application to both the hyper-chaotic dynamics and the chaotic dynamics.

5.1 Hyper-chaotic example

Consider the following discrete-time hyper-chaotic system presented in Yang's paper[3]

$$f:\mathbb{R}^2 \to \mathbb{R}^2, f(x_1, x_2) = [1-2(x_1^2+x_2^2)^2, \ -4x_1x_2]^{\mathrm{T}}. \tag{21}$$

The target is the fixed point at $x^t = (1/2, 0)$. Thus the controlled system is described as follows

$$f(x_1, x_2, u) = [1-2(x_1^2+x_2^2)^2+u_1, \ -4x_1x_2+u_2]^{\mathrm{T}}. \tag{22}$$

For convenience to gain the optimal control sequence, we add control only to x_1, i.e. $u_2 = 0$ while directing the arbitrarily initial trajectory in the hyper-chaotic attractor towards the neighbourhood of the target. Suppose the uniformly distributed random noise level ζ be given by 0.01, and the control have a constraint given by $-\beta \leqslant u \leqslant \beta$, where β is a small constant.

With given initial condition $x(0) = (0.4, 0.4)^{\mathrm{T}}$ and $\beta = 0.03$, for an additional feedback correction to deal with noise we can set $\Omega = [-0.02, 0.02]$ and get the optimal control sequence and the corresponding state sequence shown in Table 1.

Table 1 The optimal control sequence and the state sequence in system (22)

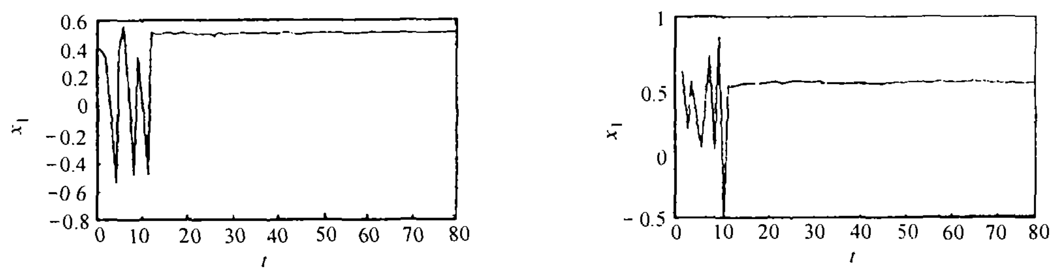
Step	1	2	3	4	5	6	7	8	9	10	11
u_1	-0.02	0.02	-0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	-0.02
x_1	0.34	-0.0304	-0.5370	0.4208	0.5625	0.0943	-0.4805	0.3473	-0.0003	-0.4836	0.5123
x_2	-0.64	0.8704	0.1058	0.2274	-0.3827	0.8610	-0.3247	-0.6241	0.8671	0.0010	0.0020

After the system trajectory enter the neighbourhood of x^t, we can achieve the stabilization of the trajectory to x^t by adjusting control input according to Eq. (20). If $u_1 = u_2 = 0$, then $J = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ and $M = \begin{pmatrix} 1/3 & 0 \\ 0 & 1/3 \end{pmatrix}$. Thus the small perturbation input can be calculated as $u_1 = (2 + \delta)(x_1 - x_1^t)$ and $u_2 = (2 + \delta)(x_2 - x_2^t)$. In this case, Fig. 1(a) shows the numerical simulation of control performance when setting $\delta = 0.75$.

As another illustration with initial condition $x(0) = (0.6, 0.2)^{\mathrm{T}}$ and $\beta = 0.05$, for an additional feedback correction to deal with noise we can set $\Omega = 0.04$ and get the optimal control sequence and the corresponding state sequence shown in Table 2.

Table 2 The optimal control sequence and the state sequence in system (22)

Step	1	2	3	4	5	6	7	8	9	10
u_1	-0.04	0.04	-0.04	-0.04	-0.04	-0.04	0.04	-0.04	-0.04	-0.04
x_1	0.1600	0.5280	0.2137	0.0268	0.3435	0.7170	-0.0015	0.8505	-0.4865	0.4865
x_2	-0.4800	0.3072	-0.6488	0.5546	-0.0594	0.0816	-0.2340	-0.0014	0.0048	0.0092



(a) $x(0)=(0.4,0.4)^{\mathrm{T}}, -0.03 \leqslant u \leqslant 0.03$ (b) $x(0)=(0.6,0.2)^{\mathrm{T}}, -0.05 \leqslant u \leqslant 0.05$

Fig. 1 The state of x_1 in the controlled hyper-chaotic dynamics by the combined control method

In this case, Fig. 1(b) shows the numerical simulation of control performance when setting $\delta = 0.75$ like the first case.

Obviously, the trajectory can be quickly stabilized to the desired target and revolves around the target robustly in steady state. Under similar conditions assumed above, if the initial trajectory has not been pre-directed to the neighbourhood of the target and only using the simple controller presented by Yang, the trajectory will quickly wander out of the attractor to infinity. Comparatively speaking, the controller Yang designed is effective only in sufficiently small neighbourhood of the target and its control set is not bounded. However, the combined controller in this paper is effective for controlling hyper-chaos under arbitrary initial conditions.

5.2 Chaotic example

Another chaotic example, Hénon map, is followed to show that the combined method presented in this paper is also effective for controlling chaotic dynamics. Consider the following Hénon map

$$f:\ \mathbb{R}^2 \to \mathbb{R}^2,\ f(x_1, x_2) = [1 + x_2 - 1.4(x_1)^2, 0.3x_1]^{\mathrm{T}}. \tag{23}$$

The target is the fixed point at $x^t = [0.6314, 0.1894]^{\mathrm{T}}$. Therefore the controlled system is described as follows

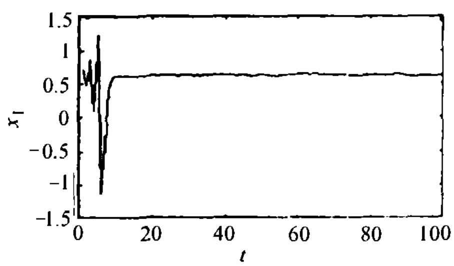
$$f(x_1, x_2, u) = [1 + x_2 - 1.4(x_1)^2 + u_1, 0.3x_1 + u_2]^{\mathrm{T}}. \tag{24}$$

Like the first hyper-chaos example, for convenience to gain the optimal control sequence, we also add control only to x_1, i.e. $u_2 = 0$ while directing the arbitrarily initial trajectory in the chaotic attractor to the neighbourhood of the target. With initial condition $x(0) = [0.6, 0.2]^{\mathrm{T}}$ and $\beta = 0.02$, for an additional feedback correction to deal with noise we can set $\Omega = [-0.01, 0.01]$ and get the optimal control sequence and the corresponding state sequence shown in Table 3.

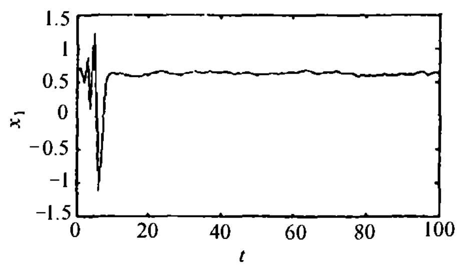
Table 3 The optimal control sequence and the state sequence in system (24)

Step	1	2	3	4	5	6	7	8	9	10
u_1	0.01	0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.01
x_1	0.7060	0.4922	0.8626	0.0958	1.2359	-1.1198	-0.3948	0.4358	0.6056	0.6273
x_2	0.1800	0.2118	0.1477	0.2588	0.0287	0.3708	-0.3359	-0.1184	0.1308	0.1817

After the system trajectory entering the neighbourhood of x^t, we can also achieve the stabilization of the trajectory to x^t by adjusting control input according to Eq.(20). When $u_1 = u_2 = 0$, it is easy to see that $J = \begin{pmatrix} -1.768 & 1 \\ 0.3 & 0 \end{pmatrix}$ and $M = \begin{pmatrix} 0.441 & 0.441 \\ 0.132 & 1.132 \end{pmatrix}$. Therefore the small perturbation input should be given by $u_1 = (1.648 + 0.919\delta)(x_1 - x_1^t) - 0.919(x_2 - x_2^t)$ and $u_2 = -0.300(x_1 - x_1^t) + 1.000\delta(x_2 - x_2^t)$. Set $\delta = 0.9$, Fig.2 shows the numerical simulation results of the above two illustrations respectively while the uniformly distributed random noise level ζ is provided differently.



(a) $x(0) = (0.6, 0.2)^{\mathrm{T}}, -0.02 \leqslant \zeta \leqslant 0.02$



(b) $x(0) = (0.6, 0.2)^{\mathrm{T}}, -0.05 \leqslant \zeta \leqslant 0.05$

Fig. 2 The state of x_1 in the controlled Hénon map by using the combined control method

6 Conclusions

A combined method is presented in this paper to control discrete-time hyper-chaotic dynamics as well as chaotic dynamics quickly. The numerical simulations of controlling two typical chaotic dynamical systems, one is chaotic and the other hyper-chaotic, demonstrate that the combined method presented in the paper is so general and effective that the controlled systems perform strong robustness. In fact, under the condition of limited con-

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