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# Five-Link Biped Robot Hybrid Control via Fuzzy Neural Networks

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**Abstract:** The paper presents a new fuzzy neural networks (FNN) hybrid controller to solve the trajectory tracking problem of biped robots in the single-support phase. The advantages of fuzzy neural network,  $H_{\infty}$  controller and inverse system method are integrated in this paper for control purpose. A new multi-layers fuzzy CMAC is applied to approximate the system information of biped robot. On the one hand, we regard construction errors of FNN as external disturbances, and then use  $H_{\infty}$  controller to attenuate such disturbances. On the other hand, apply the strong approximate capability of FNN to construct the inverse system and offer efficient system information to  $H_{\infty}$  controller. As the result,  $L_2$  gain can be attenuated by the presented fuzzy neural network structure and adaptive algorithm.

Key words: robotic control; hybrid control; FNN; robust control; inverse system method

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# 基于模糊神经网络的5连杆双足机器人混杂控制

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摘要:针对双足机器人单脚支撑期控制问题,提出了一种新型的模糊神经网络混杂控制方法.该种方法结合了模糊神经网络、 $H_{\infty}$ 控制及逆系统方法的优点.应用了一种新的多层模糊 CMAC 神经网络对系统进行逼近,一方面将模糊神经网络的构造误差看作系统的干扰,利用  $H_{\infty}$ 控制对干扰进行抑制.另一方面利用模糊神经网络对系统模型进行逼近,为逆系统的构建和  $H_{\infty}$ 控制率的设计提供了有效的系统信息.并证明了在采用本文提出的模糊神经网络和自适应算法后可以抑制  $L_2$  增益.

关键词: 机器人控制; 混杂控制; 模糊神经网络; 鲁棒控制; 逆系统方法

#### 1 Introduction

The robotic control problems have received considerable attention among researchers<sup>[1,2]</sup>. Many researchers applied the computer torque method<sup>[3]</sup>, fuzzy system<sup>[4]</sup>, neural networks<sup>[1]</sup> to design the control scheme, and obtained several valuable results. J.S. Yang<sup>[3]</sup> and H.K. Lum<sup>[5]</sup> proposed a control algorithm for 5-links biped robots based on inverse dynamic approach, a method which is attractive for its simplicity. In addition, it is useful for nonlinear and decoupled systems. Unfortunately, the robust performance of inverse dynamic control restrict its practical application when the uncertainty and external disturbances of biped systems are considered. S.G. Tzafestas<sup>[6]</sup> applied sliding mode control (SMC) to the control problem, and robust performance is achieved to some extent. However, the slide-mode

control required the bound of disturbance, which is difficult to obtain in real biped robotic systems. Furthermore, the high frequency switch control mode of SMC is not suitable to the smooth trajectory control of biped robot.

In the literature, a new hybrid neural networks controller is presented for the biped robotic control. Inverse system, multi-layer fuzzy CMAC (cerebellar model arcu ation controller)<sup>[7]</sup> and  $H_{\infty}$  control are integrated to construct a whole control system. Firstly, inverse system is employed to cancel the nonlinear and decoupled problem of robotic system. Inverse system is the high level of inverse dynamic; introducing the notion of inverse system which is benefit to the further research work. Secondly,  $H_{\infty}$  controller assures the stability of closed-loop system, and it attenuates the disturbance to a prescribed level.

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Particularly, since the bound of construction errors of FNN are difficult to obtain, we consider the construction errors of FNN as a kind of external disturbance, which can be attenuated by H<sub>∞</sub> controller. Since no bound of disturbance is required, the control scheme is more attractive than that of slide-mode control. On the one hand, we regard construction errors of FNN as external disturbance, and then use H<sub>∞</sub> controller to attenuate such disturbances. On the other hand, we apply the powerful approximate capability of FNN to construct the inverse system and offer efficient system information to H<sub>∞</sub> controller.

# 2 Five-link biped robot model

As is well known, the motion space of a biped robot is often decoupled of two planes that is the sagittal plane and the lateral plane. We assumed that there is no coupled relation between motions of these two planes, and the inter-influence of motions in these two planes seems as disturbances. Only the model of the single-leg support phase in sagittal plane of the 5-link biped robot will be discussed in the paper.

The dynamics of an n-link biped robot may be expressed in the Lagrange form.

$$M_{\theta}(\theta)\ddot{\theta} + C_{\theta}(\theta,\dot{\theta})\dot{\theta} + G_{\theta}(\theta) = D[\tau_{1},\tau_{2},\tau_{3},\tau_{4}]^{\mathrm{T}} + d_{\theta\tau}$$
(1)

with  $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T$  joint variable,  $M_{\theta}(q) =$  $\{r_{ij}\cos(\theta_i - \theta_j)\}\$ inertia matrix,  $C_{\theta}(q,q) = \{r_{ij}\sin(\theta_i)\}$  $-\theta_i \hat{\theta}_i$  Coriolis/Centripetal forces.  $G_{\theta}(\theta) =$  $\operatorname{diag}(-h_i \sin \theta_i)$  gravity. Bounded unknown disturbances are denoted by  $d_{ heta au} \in \mathbb{R}^5$ , and the control input torque is  $\tau_i(i = 1,2,3,4)$ . Other parameters are illustrated in Section 5.

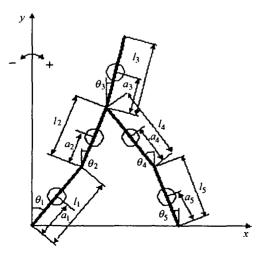


Fig. 1 5-link biped robot

$$D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}.$$

From (1), we obtain:

$$\begin{cases} M(q)\dot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + d_{\tau}, \\ \tau = [0,\tau_1,\tau_2,\tau_3,\tau_4]. \end{cases}$$
 (2)

### 3 Control scheme of inverse system

It is reasonable to use the  $\alpha$ -order integral inverse system to the biped robot is reasonable. Consider the  $\alpha = [2, 2, \dots, 2]^{T}$  integral inverse system of the robotic system, and joint input torque is

$$\tau_1 = \hat{M}(q)\varphi(t) + \hat{C}(q,\dot{q})\dot{q} + \hat{G}(q), \quad (3)$$

$$\varphi = \ddot{q}_d + K_p \dot{e} + K_p e, \qquad (4)$$

where  $e = q_d - q$ ,  $\dot{e} = \dot{q}_d - \dot{q}$ ,  $K_p \in \mathbb{R}^{n \times n}$ , is a position gain matrix and  $K_v \in \mathbb{R}^{n \times n}$  is a velocity gain matrix.  $q_d$ ,  $\dot{q}_d$  and  $\ddot{q}_d$  are the desired signals of q,  $\dot{q}$  and  $\ddot{q}$  respectively. "^" denotes the estimates of homologous terms. Substitute (3) into (2), we will obtain

$$\ddot{q}(t) = \varphi(t) - EM, \qquad (5)$$

where EM denotes the model error.

$$EM = \hat{M}^{-1} [(M - \hat{M}) \dot{q} + (C - \hat{C}) \dot{q} + (G - \hat{G} + d_{\tau}].$$
(6)

When the models are precise, then EM = 0. A pseudolinear system can be obtained as

$$\ddot{q}(t) = \varphi(t). \tag{7}$$

(7) can be rewritten as

$$\ddot{e} + K_{\nu}\dot{e} + K_{\nu}e = 0. \tag{8}$$

Then a biped robot dynamic has been decoupled into nindependence linear systems whose order is two. The complexity of control system design will be reduced by the decouple scheme.

# Fuzzy neural network inverse system $H_{\infty}$ control scheme

The control objectives considered in this paper are to

make the motion of the joint to track a desired path  $q_d$ , with  $\dot{q}_d$  and  $\dot{q}_d$  being continuous. Besides, an  $H_\infty$  performance is to be satisfied by the closed loop system. This  $H_\infty$  performance is represented in terms of a finite  $L_2$  gain relationship. The controller system output is defined as

$$\tau = \tau_l + u_h, \tag{9}$$

$$\tau_{l} = W^{T} \Psi(q, \dot{q}, \dot{q}_{d}) = \hat{M}(q)(\dot{q}_{d} + K_{v}\dot{e} + K_{p}e) + \hat{V}(q, \dot{q})\dot{q} + \hat{G}(q),$$
(10)

where  $\Psi(\,\cdot\,)$  denotes generalization result, W is weight matrix, where  $\tau_l$  is inverse dynamic control and  $u_h$  is  $H_\infty$  control. We assume that there exist  $\Omega_f = \{W \in \mathbb{R}^{pm \times n}: \|W\| \leq m_f\}$ , and ideal parameter is in the compact set  $\Omega_f$ . The ideal parameter can be defined as

$$W^* = \arg \min_{w \in \Omega_f} \{ \sup | M(q)(\dot{q}_d + K_v \dot{e} + K_p e) +$$

$$C(q,\dot{q})\dot{q} + G(q) - W^{\mathrm{T}}\Psi(\cdot) \mid \}$$

and

$$\widetilde{W} = W^* - W.$$

The error of FCMAC (fuzzy cerebellar model arculation controller) can be divided into two parts that is the construct error  $d_f$  and approximate error  $d_\tau$ .

$$d_{f} = M(q)(\ddot{q}_{d} + K_{v}\dot{e} + K_{p}e) + C(q,\dot{q})\dot{q} + G(q) - W^{*T}\Psi(\cdot),$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \hat{M}(q)(\ddot{q}_{d} + K_{v}\dot{e} + K_{p}e) + \hat{V}(q,\dot{q})\dot{q} + \hat{G}(q) + u_{h} - d_{\tau} = M(q)(\ddot{q}_{d} + K_{v}\dot{e} + K_{p}e) + C(q,\dot{q})\dot{q} + G(q) - d_{f} - d_{\tau} - \widetilde{W}^{T}\Psi(\cdot) + u_{h}.$$
(11)
Then

 $M(q)(\ddot{e} + K_n \dot{e} + K_n e) = u_h - d_f - d_r \widetilde{W}^T \Psi(\cdot).$ 

Consider the coordinate transformation

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} I_{n \times n} & 0 \\ \Lambda & I_{n \times n} \end{bmatrix} \begin{bmatrix} e \\ e \end{bmatrix}, \tag{12}$$

where  $\Lambda$  is a  $n \times n$  matrix  $\Lambda^{T} = \Lambda > 0$ .

$$\begin{cases} \dot{x}_{1} = -\Lambda x_{1} + x_{2}, \\ M(q)\dot{x}_{2} = \\ -M(q)(K_{v} - \Lambda)(x_{2} - \Lambda x_{1}) - \\ M(q)K_{p}x_{1} + u - d_{\tau} - d_{f} - \widetilde{W}^{T}\Psi(\cdot) = \\ -M(q)(K_{v} - \Lambda)x_{2} + M(q)[(K_{v} - \Lambda)\Lambda - K_{p}]x_{1} + \\ u - d_{f} - d_{\tau} - \widetilde{W}^{T}\Psi(\cdot). \end{cases}$$
(13)

The disturbance is defined as  $d = -d_f - d_\tau$ .

$$\dot{x} = Ax + B(u - d_{\tau} - d_{f} - \widetilde{W}^{T} \Psi(\cdot)), \qquad (14a)$$

$$A = \begin{bmatrix} -\Lambda & 1 \\ -K_{v} + \Lambda & (K_{v} - \Lambda)\Lambda - K_{p} \end{bmatrix}, B = \begin{bmatrix} 0 \\ M^{-1}(\theta) \end{bmatrix}.$$
(14b)

**Assumption 1** For every construction errors of FNN:  $d_f$ , system disturbance  $d = -d_f - d_r$  and  $\int_0^T \|d\| dt < \infty$ , in another word  $d \in L_2[0, \infty)$ .

**Theorem 1** Considering (14), if Assumption 1 is satisfied, for a prescribed attenuate level  $\rho > 0$ ,  $Q = Q^T > 0$ . There exists a positive matrix  $K = K^T > 0$ , and satisfy:

$$A^{\mathrm{T}}P + PA + \dot{P} - PB\left[2R^{-1} - \frac{I}{\rho^2}\right]B^{\mathrm{T}}P + Q = 0.$$
 (15)

R is a gain matrix, then the control approach is designed as follows:

$$\tau = W^{\mathsf{T}} \Psi(\cdot) + u \tag{16}$$

and

 $\dot{V}(x,t) =$ 

$$\dot{\mathbf{W}} = F^{-1}\boldsymbol{\Psi}(\cdot)PBx_2^{\mathrm{T}},\tag{17}$$

$$u = -R^{-1}B^{\mathrm{T}}Px. \tag{18}$$

Then all the variables of the closed-loop system is bounded, and the  $H_{\infty}$  trajectory performance is achieved:

$$\int_{0}^{T} \|x(t)\|_{Q}^{2} dt \le 2V(0) + \rho^{2} \int_{0}^{T} \|d(t)\|^{2} dt, \ \forall 0 \le T \le \infty.$$
(19)

Proof Define Lyapunov function as follows

$$V(x,t) = \frac{1}{2} x_1^{\mathsf{T}} K x_1 + \frac{1}{2} x_2^{\mathsf{T}} M(\theta) x_2 + \frac{1}{2} \operatorname{tr}(\widetilde{W}^{\mathsf{T}} F^{-1} \widetilde{W}) = \frac{1}{2} x^{\mathsf{T}} P(\theta) x + \frac{1}{2} \operatorname{tr}(\widetilde{W}^{\mathsf{T}} F^{-1} \widetilde{W}).$$
 (20)

Differentiating the above equation yields:

$$x^{T}P\dot{x} + \frac{1}{2}x^{T}\dot{P}x + \operatorname{tr}(\widetilde{W}^{T}F\widetilde{W}) =$$

$$x^{T}PAx - x^{T}PBR^{-1}B^{T}x + \frac{1}{2}x^{T}\dot{P}x + x^{T}PB[-d_{\tau} - d_{\tau} - \widetilde{W}^{T}\Psi(\cdot)] + \operatorname{tr}(\widetilde{W}^{T}F\widetilde{W}) =$$

$$x^{T}PAx - x^{T}PBR^{-1}B^{T}Px + \frac{1}{2}x^{T}\dot{P}x + x^{T}PB[d - d_{\tau} - x^{T}PBR^{-1}B^{T}Px + \frac{1}{2}x^{T}\dot{P}x + x^{T}PB[d - x^{T}PBR^{-1}B^{T}Px + \frac{1}{2}x^{T}\dot{P}x + x^{T}PB[d - x^{T}PBR^{-1}B^{T}Px + \frac{1}{2}x^{T}\dot{P}x + x^{T}PB[d - x^{T}PBR^{-1}B^{T}Px + x^{T}PBR^{-1}B^{T}Px + x^{T}PB[d - x^{T}PBR^{-1}B^{T}Px + x^{T}Px + x^{$$

$$\widetilde{W}^{T}\Psi(\cdot)] + \operatorname{tr}(\widetilde{W}^{T}F(F^{-1}\Psi(\cdot)PBx_{2}^{T})) =$$

$$\frac{1}{2}x^{T}(A^{T}P + PA)x - x^{T}PBR^{-1}B^{T}Px + \frac{1}{2}x^{T}Px + x^{T}PBd =$$

$$\frac{1}{2}x^{\mathsf{T}}\left(A^{\mathsf{T}}P + PA + \dot{P} - PB\left[2R^{-1} - \frac{I}{\rho^{2}}\right]B^{\mathsf{T}}P\right)x - \frac{1}{2}\left(\frac{1}{\rho}B^{\mathsf{T}}Px - \rho d\right)^{\mathsf{T}}\left(\frac{1}{\rho}B^{\mathsf{T}}Px - \rho d\right)^{\mathsf{T}} + \frac{1}{2}\rho^{2}d^{\mathsf{T}}d.$$
(21)

From (15), we obtain:

$$\dot{V}(x,t) \le -\frac{1}{2}x^{\mathrm{T}}Qx + \frac{1}{2}\rho^2 d^{\mathrm{T}}d.$$
 (22)

Integrating the above equation from t = 0 to t = T yields:

$$V(x,T) - V(x,0) \le -\frac{1}{2} \int_{0}^{T} x^{T}(t) Qx(t) dt + \frac{\rho^{2}}{2} \int_{0}^{T} d^{T}(t) d(t) dt.$$
(23)  
Since  $V(x,T) \ge 0$ , we obtain
$$\int_{0}^{T} ||x(t)||_{Q}^{2} dt \le x^{T}(0) Px(0) + \widetilde{W}^{T}(0) F^{-1} \widetilde{W}(0) + \rho^{2} \int_{0}^{T} ||d(t)||^{2} dt.$$
(24)

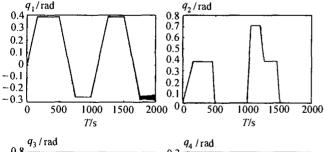
So the  $H_{\infty}$  tracking performance is achieved.

From Assumption 1, since  $d \in L_2[0,\infty)$ , there is a constant  $B_d > 0$ , and  $\int_0^\infty \|d(t)\|^2 dt \le B_d$ , so the following inequality is achieved:

$$V(x,T) \leq V(x,0) + \frac{1}{2}\rho^2 B_d$$

and

$$||x_1(t)|| \le \sqrt{(2V(0) + \rho^2 B_d)/\lambda_{\min}(K)},$$



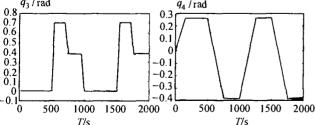


Fig. 2 Track trajectory

#### 6 Conclusion

In the paper, a fuzzy neural network inverse system

$$\|x_2(t)\| \leq \sqrt{(2V(0) + \rho^2 B_d)/\lambda_{\min}(K_M)},$$

where  $\lambda_{\min}(\cdot)$  denote the minimal eigenvalue of matrix,  $K_M < M(\theta)$ , proof completed.

#### 5 Simulation result

Based on the 985 project of Tsinghua University: Design of anthropomorphic robot, we have done some simulation research works for 5-link biped robot. Several model parameters are computed as [2]. Apply the gait pattern of reference [3], the controller parameter are designed as follows:  $K_p = \text{diag}[25, 25, 25, 25, 25], K_v =$  $diag[10,10,10,10,10], \Lambda = diag[5,5,5,5,5], c_1 =$  $c_2 = 1, \gamma = 0.1, F = I$ . Fuzzy neural network used three layers to quantify input states, and choose five fuzzy sets [NB, NS, ZO, PS, PB]. The input vector of FNN is  $[q, \dot{q}, \ddot{q}_d]$ , and the structure of FNN is 12 - 180-5. Firstly, a clustering method<sup>[8]</sup> is utilized to adjust the main parameters of fuzzy neural networks. The offline training number is 10000, and offline training sample pair is  $[u^{*(i)}, u^{(i)}]$ , modification of weight is  $\Delta M_i$  $= \frac{\beta[u^{*(i)}, u^{(i)}]v^{(i)}(s)}{N}, \text{ where } \beta = 0.1 \text{ is learning}$ 

parameter,  $N_e$  is 180 (the number of neurons in hidden layer), v(s) is the generalization result of hidden layer, and after 20000 offline training, the fuzzy neural network can be used in the online control circumstance.

Simulation results are shown in Fig. 2 and Fig. 3.

 $q_1/\text{rad}$ 

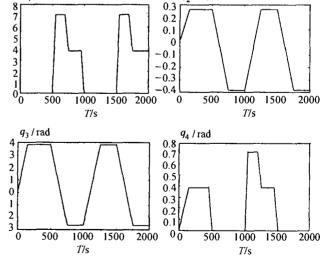


Fig. 3 Reference trajectory

 $H_{\infty}$  controller is proposed for the trajectory tracking problem of bipeds. Firstly, the introduction of inverse

system is beneficial to attenuate the nonlinear and decoupled level of robotic system, then, the new multi-layer fuzzy CMAC is employed to obtain the system information, and supply the basement for  $H_{\infty}$  controller. Thirdly, the  $H_{\infty}$  controller assures the stability of closed-loop system and attenuates the effect of disturbance. This kind of hybrid controller will be the trend of robotic control.

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trol, i.e. to control hyper-chaotic or chaotic dynamics only by using small perturbations, the combined method presented in this paper provides a new idea to direct chaotic orbits to fixed point by using small control action as quickly as possible. Moreover, the combined method could be easily applied to many general practical systems.

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