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General Quasi-infinite Horizon Nonlinear Model Predictive Control *

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Abstract: This paper generalizes the quasi-infinite horizon nonlinear MPC scheme in a more useful form. Conditions for the closed-loop stability of constrained nonlinear systems and for the existence of an optimal solution are presented. Based on feedback linearization, implementation issues of the control scheme including the determination of larger terminal regions are discussed. Computation time for on-line optimization can be reduced significantly.

Key words: nonlinear model predictive control; stability; constrained optimal control; feedback linearization

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广义准无限时域非线性预测控制

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摘要: 将准无限时域非线性预测控制方法推广到更一般的情况,并给出了闭环约束系统的稳定性条件及最优解的存在条件.基于反馈线性化技术讨论了广义准无限时域非线性预测控制的实现及较大终端域的获取.该方法能显著减少在线优化所需的时间.

关键词: 非线性预测控制; 稳定性; 约束最优控制; 反馈线性化

1 Introduction

In the last decade, great advances in nonlinear model predictive control (NMPC) have been achieved. Various NMPC schemes have been developed and successfully applied in industry[1~3], some of which address theoretical problems such as nominal stability (see [2~4]). In [5] and [6], a quasi-infinite horizon nonlinear model predictive control (QIH-NMPC) scheme with guaranteed stability and computational advantages is discussed, where a quadratic objective functional is used. For many practical applications, control performance may be specified in a much general form. This paper extends the results in [5] and [6] to more useful cases.

2 Formulation of the control scheme

Consider a class of systems described by the following general nonlinear ODEs

$$\dot{x}(t) = f(x(t), u(t)),\ x(0) = x_0 \tag{1}$$

subject to input and state constraints

$$u(t) \in U,\ x(t) \in X,\ t \geqslant 0. \tag{2}$$

Some fundamental assumptions are stated as follows:

A1) $f: \mathbb{R}^n \times \mathbb{R}^m \ni (x, u) \to f(x, u) \in \mathbb{R}^n$ is continuous and satisfies $f(0,0) = 0$. In addition, it is locally Lipschitz continuous in x.

A2) $U \subset \mathbb{R}^m$ is compact, $X \subseteq \mathbb{R}^n$ is connected and the point $(0,0)$ is contained in the interior of $X \times U$.

A3) System (1) has a unique continuous solution for any initial condition $x(0) \in X$ and any piecewise continuous input function $u(\cdot): [0, T_p] \to U$.

In the framework of QIH-NMPC[5,6], a finite horizon objective functional in a general form is introduced

$$J(x(t), u(\cdot)) = \int_t^{t+T_p} F(x(\tau), u(\tau))\mathrm{d}\tau + E(x(t+T_P)) \tag{3}$$

for the open-loop optimal control problem with initial state $x(t)$ at time t, where $E(x)$ is positive definite and continuously differentiable in X and satisfies

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$$\frac{\partial E}{\partial x} f(x, k(x)) + F(x, k(x)) \leqslant 0. \tag{4}$$

Moreover, it is assumed that

A4) $F: \mathbb{R}^n \times \mathbb{R}^m \ni (x, u) \rightarrow F(x, u) \in \mathbb{R}$ is continuous, satisfies $F(0,0) = 0$ and $F(x, u) > 0$ for $(x, u) \neq (0,0)$.

A5) $u = k(x)$ is any state feedback law that locally asymptotically stabilizes the nonlinear system (1) and satisfies $k(0) = 0$.

Thus, the constrained open-loop optimal control problem can be formulated as follows:

Problem 1 Find

$$\min_{\bar{u}(\cdot)} J(x(t), \bar{u}(\cdot)) \tag{5}$$

with (3) subject to

$$\dot{\bar{x}} = f(\bar{x}(\tau), \bar{u}(\tau)); \ \bar{x}(t; x(t), t) = x(t), \tag{6a}$$

$$\bar{u}(\tau) \in U, \ \tau \in [t, t + T_P), \tag{6b}$$

$$\bar{x}(\tau; x(t), t) \in X, \ \tau \in [t, t + T_P], \tag{6c}$$

$$\bar{x}(t + T_P; x(t), t) \in \Omega, \tag{6d}$$

where $\bar{x}(\cdot; x(t), t)$ represents the predicted trajectory of (1) starting from the actual state $x(t)$ and driven by a given open-loop input function $\bar{u}(t)$ (it is replaced by $\bar{x}(\cdot)$ for simplicity); Ω is the so-called terminal region defined by

$$\Omega := \{x \in X \mid E(x) \leqslant \alpha, \alpha > 0, k(x) \in U\}. \tag{7}$$

Clearly, Ω has the following properties:

· The point $0 \in \mathbb{R}^n$ is in the interior of Ω, since $E(x) \geqslant 0$ and $(0,0)$ is in the interior of $X \times U$;

· Ω is closed and connected due to $E(x)$ is continuous in X;

· From (4), Ω is invariant for $\dot{x} = f(x, k(x))$.

Remark 1 It follows from the converse theorem[7] that if $u = k(x)$ stabilizes locally asymptotically system (1), there exists, then, a Lyapunov function $E(x)$ satisfying (4). It is not very easy to solve the partial differential inequality (4) to get a function $E(x)$. If the Jacobian linearization of (1) is stabilizable, one can choose $F(x, u) = \| x \|_Q^2 + \| u \|_R^2$ and $E(x) = \| x \|_P^2$, where P is the positive definite solution to a Lyapunov equation[5]. With $Q > 0, R > 0$, functions F and E satisfy (4) and the assumption A4). Thus, in [5] a special case of this paper was discussed. In Section 4, a function $E(x)$ will be determined to satisfy (4) with equality, using the feedback linearization technique. This leads to an NMPC with exactly infinite prediction horizon.

Assume there is an optimal solution to Problem 1 denoted by $\bar{u}^*(\cdot; x(t), t, t + T_P): [t, t + T_P) \rightarrow U$. The feedback control is defined according to the principle of MPC as follows:

$$u^*(\tau) := \bar{u}^*(\tau; x(t), t, t + T_P), \ \tau \in [t, t + \delta), \tag{8}$$

the corresponding closed-loop system is

$$\dot{x}(t) = f(x(t), u^*(t)), \ t \geqslant 0 \tag{9}$$

with $x = 0$ being an equilibrium[5].

3 Nominal stability and conditions for an optimal solution

Let $\sigma = t + T_P$. Corresponding to an optimal solution to Problem 1, the optimal value and open-loop state trajectory are $J^*(x(t), t, \sigma) := J(x(t), \bar{u}^*(\cdot; x(t), t, \sigma))$ and $\bar{x}^*(\cdot; x(t), t, \sigma)$, respectively. Such an explicit notation makes it possible for us to investigate the dependence of J^* on T_P and t separately, and then to show the asymptotic stability of the closed-loop system (9).

Fix σ, say σ_0, the existence of an optimal solution implies that $\bar{x}^*(\tau; x(t), t, \sigma_0) \in X$ for any $\tau \in [t, \sigma_0]$ and $\bar{x}^*(\sigma_0; x(t), t, \sigma_0) \in \Omega$. For any small $\delta > 0$, Problem 1 with $\sigma = \sigma_0 + \delta$ admits a feasible solution:

$$\bar{u}(\tau) = \begin{cases} \bar{u}^*(\tau; x(t), t, \sigma_0), & \tau \in [t, \sigma_0), \\ k(\bar{x}(\tau)), & \tau \in [\sigma_0, \sigma_0 + \delta), \end{cases} \tag{10}$$

which generates a trajectory satisfying the state constraint and the terminal constraint. The corresponding objective value $\bar{J}(x(t), t, \sigma_0 + \delta)$ meets

$$\bar{J}(x(t), t, \sigma_0 + \delta) = J^*(x(t), t, \sigma_0) - E(\bar{x}^*(\sigma_0)) + E(\bar{x}(\sigma_0 + \delta)) + \int_{\sigma_0}^{\sigma_0 + \delta} F(\bar{x}(\tau), k(\bar{x}(\tau))) \mathrm{d}\tau. \tag{11}$$

By (4) and the optimality of J^*, (11) becomes

$$J^*(x(t), t, \sigma_0 + \delta) \leqslant J^*(x(t), t, \sigma_0). \tag{12}$$

Thus, the following result can be stated:

Lemma 1 Let $\sigma = t + T_p$. Suppose that Problem 1 admits an optimal solution, then, the optimal value function $J^*(x(t),t,\sigma)$ is non-increasing in σ.

Proof The proof follows directly from (12).

Remark 2 Lemma 1 implies that for fixed $x(t)$ and t, the value function of Problem 1 is non-increasing in the prediction horizon T_P.

For the period of $[t, t+\delta)$, apply the optimal solution $\bar{u}^*(\cdot;x(t),t,\sigma_0):[t,\sigma_0)\rightarrow U$ of Problem 1 at time t to the nonlinear system (1). For the nominal system without disturbances, the part of the closed-loop trajectory through $x(t)$ is

$$x(\tau)=\bar{x}^*(\tau;x(t),t,\sigma_0),\quad \tau\in[t,t+\delta]. \tag{13}$$

Thus, a feasible solution to Problem 1 at time $t+\delta$ may be chosen as

$$\bar{u}(\tau)=\bar{u}^*(\tau;x(t),t,\sigma_0),\quad \tau\in[t+\delta,\sigma_0). \tag{14}$$

Note that if $\bar{u}^*(\cdot;x(t),t,\sigma_0):[t,\sigma_0)\rightarrow U$ is the optimal solution to Problem 1 at time t, then (14) is the optimal solution at the time $t+\delta$, by Bellman's Optimality Principle. The optimal value corresponding to (14) meets

$$\bar{J}(x(t+\delta),t+\delta,\sigma_0)= J^*(x(t),t,\sigma_0)-\int_t^{t+\delta}F(x(\tau),u^*(\tau))\mathrm{d}\tau. \tag{15}$$

By the optimality of J^*, (15) becomes

$$J^*(x(t+\delta),t+\delta,\sigma_0)\leqslant J^*(x(t),t,\sigma_0)-\int_t^{t+\delta}F(x(\tau),u^*(\tau))\mathrm{d}\tau. \tag{16}$$

Now we are able to state the following result:

Lemma 2 Let $\sigma=t+T_p$. Suppose that Problem 1 admits an optimal solution, then, the value function $J^*(x(t),t,\sigma)$ is non-increasing in t.

Proof Because of the positive definiteness of F, (16) directly implies the result.

Back to Problem 1 with a fixed finite horizon T_P, the stability property of the closed-loop system (9) is stated as follows:

Theorem 1 Suppose that

a) Assumptions A1) ~ A5) are satisfied;

b) The open-loop optimal control problem described by Problem 1 is feasible at time $t=0$.

Then, for a sufficiently small sampling time $\delta>0$, the equilibrium $x=0$ of the closed-loop system (9) is nominally asymptotically stable. Let D be the set of all initial states for which the assumption b) is satisfied, then, D is a region of attraction.

Proof For a sufficiently small $\delta>0$, the assumption b) implies the feasibility of the open-loop optimal control problem at each time $t\geqslant 0$[5]. Define a scale function $V(x):=J^*(x,t,t+T_p)$, it has the following properties[5]:

- $V(0)=0$ and $V(x)>0$ for $x\neq 0$;
- $V(x)$ is continuous at $x=0$.

Moreover, from (12) and (16), we conclude that along the closed-loop trajectory

$$V(x(\tau))-V(x(t))\leqslant-\int_t^{\tau}F(x(s),u^*(s))\mathrm{d}s\leqslant 0 \tag{17}$$

for all $\tau\geqslant t$. Thus, the equilibrium $x=0$ of the system (9) is stable, without having to use the continuous differentiability assumption of $V(x)$[5]. Moreover, (17) implies the existence of $\lim\limits_{t\rightarrow\infty}V(x(t))$. It follows then from $F(x,u)\geqslant 0$ that $F(x(t),u^*(t))\rightarrow 0$ as $t\rightarrow\infty$, which leads to $x(t)\rightarrow 0, u(t)\rightarrow 0$ as $t\rightarrow\infty$. Together with the stability result, we have proven the asymptotic stability of $x=0$. Using the same method in [5], we can show that D is a region of attraction for the closed-loop system (9).

In fact, it can be shown that the feasibility, not necessarily the optimality, of the open-loop optimal control problem is required for the guaranteed stability[2,5]. However, to achieve optimal control performance, one does need the optimality. In the following, we will discuss the existence of an optimal solution to Problem 1, based on some results in [8].

First, we give the following definition of an admissible control input function $u(\cdot)$ to Problem 1.

Definition 1 A piecewise continuous input $u(\cdot)$ defined on $[t,t+T_p)$ is said to be admissible to Problem 1 if there exists a corresponding trajectory of (1) such that

- $u(\tau)\in U, \tau\in[t,t+T_p)$;
- $x(\tau;x(t),t)\in X, \tau[t,t+T_p]$;
- $x(t+T_p;x(t),t)\in\Omega$;
- $\tau\rightarrow F(x(\tau;x(t),t),u(\tau))$ is in $L_1[t,t+T_p]$.

The set of all admissible inputs is denoted by μ. A trajectory corresponding to an admissible input is referred to as an admissible trajectory. Clearly, the existence of a nonempty μ implies that Problem 1 is feasible. In addition, the existence of a non-empty μ is related to the notation of controllability. As a comparison to Definition 1, we give the following definition of constrained controllability, according to [7].

Definition 2 The nonlinear system (1) is said to be constrainedly controllable, if for any two points x_1 and x_2 in X, there exist a finite time T and an input function $u(\cdot):[0,T] \to U$ such that the corresponding trajectory starting from x_1 satisfies $x(t;x_1,0) \in X, t \in [0,T]$ and $x(T;x_1,0) = x_2$.

Next, for a fixed $x \in X$, the values of $F(x,u)$ and $f(x,u)$ for all $u \in \mu$ trace out a set in $\mathbb{R}^{n+1}$, defined by $\mathscr{B}(x) := \{(y^0, y) \mid y^0 \geqslant F(x,u), y = f(x,u), u \in U\}$. Thus, we have

Corallary 1 Suppose that

a) μ is not empty;

b) Assumptions A1) ~ A4) are satisfied and U is convex;

c) for each fixed $x \in X$, the set $\mathscr{B}(x)$ is convex,

then, Problem 1 admits an optimal solution.

Proof Assumption A3) implies that for all admissible inputs $u(\cdot):[t, t+T_p] \to U$, the corresponding trajectories are bounded on $[t, t+T_p]$. It follows that all admissible trajectories lie in some compact set. Moreover, Ω is closed, the proof follows then from the satisfaction of the conditions in Theorem 5.1 in [8].

Remark 3 The convexity of U is true if, for example, $U := \{u \in \mathbb{R}^m \mid u_{\min} \leqslant u \leqslant u_{\max}\}$. The assumption (c) is strong and only sufficient. It is satisfied if the nonlinear system being controlled is affine in u and $F(x,u)$ is a convex function of u on U.

4 Implementation on feedback linearization

If the nonlinear system (1) is affine in u, i.e., $f(x,u) = a(x) + b(x)u$, and exactly feedback linearizable in X, then, with a well-defined coordinate transformation and a nonlinear feedback[9]

$$z = \Phi(x), \quad u = -A(x)^{-1}g(x) + A(x)^{-1}v, \tag{18}$$

the nonlinear system can be transformed into a linear system in a controller normal form: $\dot{z} = Az + Bv$, where v is a new input; $\Phi(x), A(x), g(x)$ and constant matrices A and B have corresponding forms. Without considering constraints for the moment, we can find a linear feedback such that $A_K := A + BK$ is asymptotically stable. Thus, the feedback

$$u = k(x) = -A(x)^{-1}g(x) + A(x)^{-1}K\Phi(x) \tag{19}$$

is asymptotically stabilizing for the nonlinear system (1). For any initial condition $z(t_1) = z_1$, define an infinite horizon objective functional

$$J_e^{\infty}(z_1, u(\cdot)) := \int_{t_1}^{\infty} (\| z(t) \|_Q^2 + \| v(t) \|_R^2) \mathrm{d}t \tag{20}$$

with $Q > 0$ and $R > 0$. The objective value achieved by $v = Kz$ is $J_e^{\infty}(z_1, Kz(\cdot)) = \| z_1 \|_P^2$, where P is the unique positive definite solution to the Lyapunov equation

$$A_K^{\mathrm{T}}P + PA_K = -(Q + K^{\mathrm{T}}RK). \tag{21}$$

Choose an F-function and an E-function in the original coordinates as follows

$$F(x,u) := \| \Phi(x) \|_Q^2 + \| g(x) + A(x)u \|_R^2, \tag{22a}$$

$$E(x) := \| \Phi(x) \|_P^2, \tag{22b}$$

that are equivalent to $F(z,v) = \| z \|_Q^2 + \| v \|_R^2$, $E(z) = \| z \|_P^2$ in the z-coordinates. Clearly, $F(x,u)$ is convex in u, that implies the satisfaction of the condition c) in Corallary 1. Moreover, $E(x)$ satisfies (4) with equality, which leads to an NMPC with exactly infinite prediction horizon.

Remark 4 With the above objective functional, the desired control performance in the original coordinates does not seem to be clear. However, if the affine nonlinear system (1) with given outputs $y_i = h_i(x), i = 1, 2, \cdots, m$ has relative degree $\{r_1, \cdots, r_m\}$ at the origin and $\sum_{i=1}^{m} r_i = n$, we have the fact $z = (y_1, \cdots, y_1^{(r_1-1)}, \cdots, y_m, \cdots, y_m^{(r_m-1)})^{\mathrm{T}}$ and $v_i = y_i^{(r_i)}, i = 1, 2, \cdots, m$. Thus, the weighting matrices Q and R can be chosen to meet the desired control performance in the sense of output control. For example, consider a rigid robot system with n-degrees of freedom. It is described by $M(q)\ddot{q} + c(q, \dot{q}) = u$, where q and $\dot{q}$ represent respectively the position and the velocity of joints; $M(q)$ is the symmetric and positive definite manipulator inertia matrix; $c(q, \dot{q})$ represents the Coriolis, centrifugal and gravitational forces; and u denotes joint torques that can serve as control inputs. Let $x_1 = q, x_2 = \dot{q}$ and $y = x_1$. Define a coordinate transformation $\Phi(x) = x$ and a feedback linearization law $u = M(x_1)v + c(x_1, x_2)$, the robot system is feedback linearizable. Moreover, the new input is $v = \ddot{q}$. Thus, the objective functional defined by (22) admits a clear physical explanation: the first term of (22a) penalties the position and the velocity of joints, while the second penalties the joint acceleration.

As to the constrained case, the following procedure provides the largest possible terminal region Ω for feedback linearizable nonlinear systems:

Step 1 Use the standard feedback linearization technique to construct a coordinate transformation and a feedback law,

Step 2 Solve a linear stabilization problem for the feedback linearized system to get a stabilizing linear state feedback gain K,

Step 3 Solve the Lyapunov equation (21) to get a positive definite and symmetric P,

Step 4 Find the largest possible $\alpha \in (0, \infty)$ in (7) such that $\Omega \subseteq X$ and $k(x) \in U, \forall x \in \Omega$.

5 Example: a robot system

As a numerical example, consider a robot system consisting of a robot arm and a cart[10]. Its motion can be ideally described by

$$(J + mr^2)\ddot{\varphi} + 2mr\dot{r}\dot{\varphi} = T_1, \quad \rho m\ddot{r} - \rho mr\dot{\varphi}^2 = T_2, \tag{23}$$

where m is the mass of the cart; J is the joint moment of inertia; φ and r are positions of the arm and of the cart, respectively; T_1 and T_2 are the torque and the force applied to the arm and the cart, respectively. Constraints are given in [10] as follows:

$$-20\text{Nm} \leqslant T_1 \leqslant 20\text{Nm}, \quad -10\text{N} \leqslant T_2 \leqslant 10\text{N}, \tag{24a}$$

$$0 \leqslant \varphi \leqslant 270°. \tag{24b}$$

Other values for physical parameters are also taken there. Define a state vector and an input vector as $x_1 := \varphi - \varphi_s, x_2 := \dot{\varphi} - \dot{\varphi}_s, x_3 := r - r_s, x_4 := \dot{r} - \dot{r}_s$ and $u_1 := T_1 - T_{1s}, u_2 := T_2 - T_{2s}$, and choose a steady state of the system with positions of $\varphi_s = 135°$ and $r_s = 0.635$m. Thus, assumption A2) is satisfied and U is convex.

In the following, we compare the quasi-infinite horizon nonlinear model predictive controllers based on the feedback linearization (controller A) and the Jacobian linearization discussed in [6] (controller B). Choose performance weighting matrices Q and R as follows (units omitted)

$$Q = \begin{pmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.5 \end{pmatrix}, \quad R = \begin{pmatrix} 0.1 & 0.0 \\ 0.0 & 0.1 \end{pmatrix}. \tag{25}$$

The Jacobian linearization of the robot system (in the x-coordinates) at the steady state gives

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0.0 & 0.0 \\ \dfrac{1}{J + mr_s^2} & 0.0 \\ 0.0 & 0.0 \\ 0.0 & \dfrac{1}{\rho m} \end{pmatrix}. \tag{26}$$

In order to give a fair comparison, the following feedback linearization law is chosen:

$$u_1 = \frac{J + m(x_3 + r_s)^2}{J + mr_s^2} v_1 + 2mx_2x_4(x_3 + r_s), \tag{27a}$$

$$u_2 = v_2 - \rho m x_2^2 (x_3 + r_s). \tag{27b}$$

Thus, the feedback linearized system has the same dynamic and control matrices as in (26). Following the procedure given in Section 4, we obtain a terminal penalty matrix and a terminal region for controller A as follows:

$$P = \begin{pmatrix} 2.1960 & 3.1613 & 0.0 & 0.0 \\ 3.1613 & 4.7463 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0642 & 0.3162 \\ 0.0 & 0.0 & 0.3162 & 0.3365 \end{pmatrix}, \tag{28a}$$

$$\Omega = \{x \in \mathbb{R}^4 \mid x^{\mathrm{T}} P x \leqslant 3.0\}, \tag{28b}$$

where constraints in (24) determine mainly the size of the terminal region. For controller B, the procedure given in [6] yields

$$P = \begin{pmatrix} 44.9619 & 48.8059 & 0.0 & 0.0 \\ 48.8059 & 96.0076 & 0.0 & 0.0 \\ 0.0 & 0.0 & 2.0011 & 0.6200 \\ 0.0 & 0.0 & 0.6200 & 0.4978 \end{pmatrix}, \tag{29a}$$

$$\Omega = \{x \in \mathbb{R}^4 \mid Px \leqslant 2.1\}, \tag{29b}$$

where the nonlinearity of the robot model (23) directly restricts the size of the terminal region. A comparison of (28) to (29) indicates that controller A has a significantly larger terminal region. This implies that a shorter horizon can be chosen to achieve the feasibility of the optimization problem. As a consequence, controller A needs significantly less on-line computation time than controller B. This can be clearly seen in Table 1, where elapsed CPU times for a total simulation time of 10 seconds are listed. For both controllers A and B, the optimization problems are solved in discrete time with a sampling time of $\delta = 0.1$s and the same numerical parameters (optimality tolerance = 10^{-4} and integration step = 0.005s, etc). Moreover, finite horizons in controllers A and B are chosen to be as short as possible such that the corresponding constrained optimization problems are feasible at time $t = 0$. This results in T_p = 1s for controller A and T_p = 2s for controller B. Fig.1 shows time profiles of the closed-loop systems, where it is clear that input and state constraints in (24) are respected.

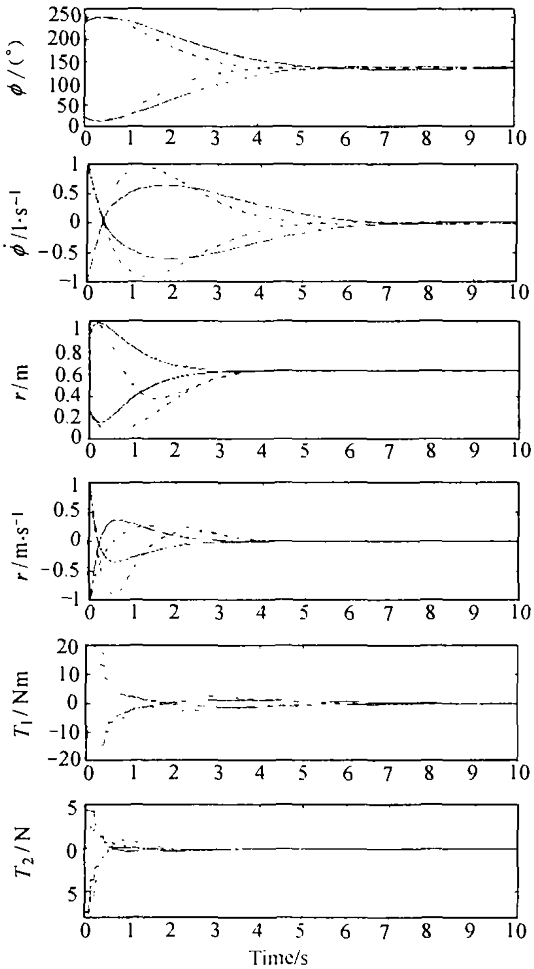

Fig. 1 Simulation results for the robot system with controller A (solid lines) and controller B (dashdot lines)

Table 1 Elapsed CPU time for controllers A and B

initial state				elapsed CPU time/s	
$\varphi(0)$	$\dot{\varphi}(0)$	$r(0)$	$\dot{r}(0)$	A	B
0°	0.0	0.3	0.0	426.93	2593.94
270°	0.0	1.0	0.0	414.94	2514.97
240°	1.0	1.0	1.0	428.89	2728.63
23°	-1.0	0.27	-1.0	405.55	2523.06

The use of the feedback linearization technique does significantly reduce on-line computation time. A real-time implementation of the controller A needs, however, further efforts, due to the existence of state constraints and the use of a continuous-time model for prediction. In the case without state constraints, in order to let controller A real time implementable, we may for example choose a sampling time of $\delta = 0.2$s and an integration step of 0.02s. With these numerical parameters, the elapsed CPU time can be reduced to about 9s for a simulation time of 10 seconds. The price is a small decrease in control performance.

6 Conclusions

This paper generalizes the QIH-NMPC scheme in [5] for a general objective functional, where a general local stabilization controller is used to derive a terminal penalty and an invariant terminal region. Closed-loop stability is guaranteed and sufficient conditions for the existence of an optimal solution to the constrained optimization problem are discussed. Compared to other existing stable NMPC approaches, the proposed one has computational advantages. Especially, for feedback linearizable nonlinear systems, the proposed method leads to an NMPC with exactly infinite prediction horizon. The feedback linearization technique is applied to determine a larger terminal region and thus the control to be determined on-line is only of a short finite horizon. This reduces intensively on-line computation time and may allow a real time implementation even for fast systems.

It should be pointed out that handling state constraints is computationally extremely expensive (see [6]). Thus, effective methods to handle state constraints are highly desired for real time NMPC.

References

[1] Allgower F, Badgwell T A, Qin J S, et al. Nonlinear predictive

control and moving horizon estimation—An introductory overview [A]. In: Frank P M. Advances in Control, Hightlights of European Control Conference'99 [C]. Berlin: Springer-Verlag, 1999, 391 – 449

[2] Mayne D Q, Rawlings J B, Rao C V, et al. Constrained model predictive control: Stability and optimality [J]. Automatica, 2000, 36 (6): 789 – 814

[3] Xi Y G, Geng X J and Chen H. Recent advances in research on predictive control performance [J]. Control Theory and Applications, 2000, 17(4): 469 – 475 (in Chinese)

[4] Chen H and Allgower F. Nonlinear model predictive control schemes with guaranteed stability [A]. In: Berber R and Kravaris C. Nonlinear Model Based Process Control [M]. The Netherlands: Kluwer Academic Publishers, 1998, 465 – 494

[5] Chen H and Allgower F. A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability [J]. Automatica, 1998, 34(10): 1205 – 1217

[6] Chen H and Allgower F. A quasi-infinite horizon predictive control scheme for constrained nonlinear systems [J]. Control Theory and Applications, 1999, 16(3): 313 – 319

[7] Khalil H K. Nonlinear Systems [M]. New York: Macmillan Publishing Company, 1992

[8] Berkovitz L D. Optimal Control Theory [M]. New York: Springer-Verlag, 1974

[9] Isidori A. Nonlinear Control Systems [M]. 3rd edition. Berlin: Springer-Verlag, 1995

[10] Nevistic V and Morari M. Constrained control of feedback linearizable systems [A]. In Proc. 3rd European Control Conference [C], Rome, 1995, 1726 – 1731

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