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Robust Control of a Class of Nonlinear Systems with L_∞-Bounded Disturbance *

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Abstract: In the sense of L_∞ norm, robust stabilization and tracking control problems are first defined for uncertain nonlinear systems. Using the technique of feedback linearization and Lyapunov approach, the robust controllers corresponding to the robust control problems are designed. Finally, a simulation result shows the correctness of the design.

Key words: robust control; nonlinear system; L_∞ norm; feedback linearization

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一类具有 L_∞ 范数有界扰动的非线性系统鲁棒控制

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摘要: 首先针对不确定非线性系统, 基于 L_∞ 范数定义了鲁棒稳定化和鲁棒跟踪控制问题. 然后利用反馈线性化技术和 Lyapunov 方法, 设计了相应的鲁棒控制器. 仿真结果验证了控制器设计的正确性.

关键词: 鲁棒控制; 非线性系统; L_∞ 范数; 反馈线性化

1 Introduction

In recent years, the control problems of nonlinear systems have received much attention. Robust control of nonlinear systems is one of the main topics in the area[1~4].

In the sense of L_2-gain, the nonlinear H_∞ control theory is founded based on the Hamilton-Jaccobi inequalities. By differential game and dissipative theory, the H_∞ control problem of nonlinear systems can be transformed equivalently into the solvability problem of the Hamilton-Jaccobi inequalities or nonlinear matrix inequalities, and the robust controllers can also be constructed via the solution of inequalities.

In the past two decades, an important achievement in the research on nonlinear control systems is the foundation and development of the differential geometrical method[5]. Through feedback linearization, nonlinear systems are transformed into the equivalent forms of linear systems. Based on the state-feedback linearization and I/O linearization, the robust control problems of a class of nonlinear systems are studied in the frame of μ analysis and synthesis[6,7]. Based on the Lyapunov theory, the robust stabilization and robust tracking of nonlinear systems with parameter uncertainty, satisfying the mismatched and matched conditions, are discussed in the literature[8,9]. But they all did not involve the general external disturbance because the necessary assumptions to the results are too rigorous for general disturbance.

This paper deals with the robust control problems of a class of nonlinear systems with L_∞-bounded disturbance. In this paper, considering the fact that a sufficiently small deviation from the ideal control objective is often admissible, the robust stabilization and robust tracking problems are respectively defined in the sense of L_∞-norm.

2 Problem formulation and preliminaries

2.1 Problem formulation

Consider a single-input single-output (SISO) affine

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nonlinear system

$$\dot{x} = f(x) + g(x)(w + u), \tag{1a}$$

$$y = h(x), \tag{1b}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the input, $w \in \mathbb{R}$ is the disturbance, $y \in \mathbb{R}$ is the output. $f(x)$ and $g(x)$ are the known smooth functions with corresponding dimensions. The disturbance w exists in the input channel, and is supposed to satisfy the following assumption:

A1) The disturbance w is L_∞-norm-bounded, i.e.

$$\| w \|_\infty \leqslant l, \tag{2}$$

where l is a known positive value, and $\| \cdot \|_\infty$ stands for L_∞-norm.

The robust stabilization problem is addressed as follows.

Definition 1 Given a scalar $\gamma > 0$, the robust stabilization problem of the nonlinear system (1) is to design a robust control law u such that:

a) When there is no disturbance w, the nonlinear system is asymptotically stable at the expected equilibrium point x_0;

b) When there exists the disturbance w, the nonlinear system asymptotically converges to the set

$$\{x \mid \| x - x_0 \|_\infty \leqslant \gamma\}.$$

Moreover, the robust tracking problem is addressed below.

Definition 2 Given a scalar $\gamma > 0$ and a reference input $y_R(t)$, the robust tracking problem of the nonlinear system (1) is to design a robust control law u such that:

a') When there is no disturbance w, the output y of the nonlinear system converges asymptotically to the prescribed output $y_R(t)$ as time tends to infinity;

b') When there exists the disturbance satisfying the L_∞ norm condition (2), the difference between the output y of the nonlinear system and the reference input $y_R(t)$ converges to the set

$$\{e \mid \| e \|_\infty \leqslant \gamma\}.$$

In order to solve the above problems, an appropriate penalty output to reflect the influence of disturbance is necessary. Customarily, z is used to denote it. The definition of L_∞-performance is introduced as follows.

Definition 3 Consider the nonlinear system (1). For a given positive number γ, let

$$J = \sup_w \| z \|_\infty . \tag{3}$$

Then the nonlinear system has L_∞-performance if $J \leqslant \gamma$ for all w satisfying the assumption A1).

2.2 Exact linearization

Consider the nominal system of the nonlinear system (1), which is described by

$$\dot{x} = f(x) + g(x)u, \tag{4a}$$

$$y = h(x). \tag{4b}$$

The state space exact linearization problem of the nonlinear system (4a) and (4b) is: given a point x_0, find a neighborhood U of x_0, a feedback

$$u = \alpha(x) + \beta(x)v \tag{5}$$

is defined on U, and a transformation $\xi = \phi(x)$ is also defined on U, such that in the coordinates $\xi = \phi(x)$ the corresponding closed loop system

$$\dot{x} = f(x) + g(x)\alpha(x) + g(x)\beta(x)v \tag{6}$$

is of the form

$$\dot{\xi} = A\xi + Bv, \tag{7a}$$

$$y = C\xi, \tag{7b}$$

which is linear and controllable, i.e..

$$\Big[\frac{\partial \phi(x)}{\partial x}f(x) + g(x)\alpha(x)\Big]_{x = \phi^{-1}(\xi)} = A\xi,$$

$$\Big[\frac{\partial \phi(x)}{\partial x}g(x)\beta(x)\Big]_{x = \phi^{-1}(\xi)} = B,$$

$$[h(x)]_{x = \phi^{-1}(\xi)} = C\xi$$

for some suitable matrix $A \in \mathbb{R}^{n\times n}$ and vector $B \in \mathbb{R}^n$ satisfying the condition

$$\operatorname{rank}[B \quad AB \quad \cdots \quad A^{n-1}B] = n.$$

Lemma 1[5] The state space exact linearization problem of the nonlinear system (4a) can be solved if and only if there exists a smooth function $\lambda(x)$ such that the system

$$x = f(x) + g(x)u,$$

$$y = \lambda(x)$$

has relative degree n, i.e.

a) $L_g L_f^k \lambda(x) = 0$ for all x in a neighborhood U of x_0 and all $0 \leqslant k < n - 1$;

b) $L_g L_f^{r-1} \lambda(x_0) \neq 0$.

Where $L_f^k \lambda(x)$ denotes the Lie derivative of $\lambda(x)$ along $f(x)$.

Lemma 2[5] The state-space exact linearization problem of the nonlinear system (4a) can be solved near a point, i.e. there exists an "output" $\lambda(x)$ function for which the system has relative degree n at x_0, if and only if the following conditions are satisfied:

a) The matrix $[g(x_0) \quad ad_f g(x_0) \quad \cdots \quad ad_f^{n-2} g(x_0) \quad ad_f^{n-1} g(x_0)]$ has rank n;

b) The distribution $D = \text{span}[g(x) \quad ad_f g(x) \quad \cdots \quad ad_f^{n-1} g(x)]$ is involutive near x_0.

Lemma 3[5] If nonlinear system (4) has relative degree n at the point x_0, then there exists a coordinate transformation

$$\xi = \phi(x)$$

such that the system can be transformed into the form:

$$\dot{\xi}_i = \xi_{i+1}, \ i = 1, \cdots, n-1, \tag{8a}$$

$$\dot{\xi}_n = a(\xi) + b(\xi) u, \tag{8b}$$

$$y = \xi_1, \tag{8c}$$

where

$$\phi(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{bmatrix}, \tag{9a}$$

$$a(\xi) = L_f^n h(x), \tag{9b}$$

$$b(\xi) = L_g L_f^{n-1} h(x). \tag{9c}$$

Let

$$u = \frac{1}{L_g L_f^{n-1} h(x)} [-L_f^n h(x) + v]. \tag{10}$$

Then the closed-loop system becomes a linear controllable system of the form (7), where the matrices A, B and C are

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \ C^{\mathrm{T}} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{11}$$

Besides the assumptions A1), the following assumption is supposed to hold throughout the paper:

A2) $\| b(\xi) \| \leqslant k$, where k is a positive constant.

3 Main results

3.1 Robust stabilization

According to [5], there exists the following lemma.

Lemma 4[5] If there exists a matrix K such that the following inequality

$$P(A + BK) + (A + BK)^{\mathrm{T}} P < 0 \tag{12}$$

holds, where P is a positive defined matrix, then the controller

$$v = K\xi \tag{13}$$

stabilizes the system (7). Moreover, the corresponding controller

$$u = \alpha(x) + \beta(x) K \phi(x) \tag{14}$$

stabilizes the nominal system (4) of the nonlinear system (1).

Taking the disturbance w into account, in the coordinates $\xi = \phi(x)$, the closed-loop nonlinear system (1) with controller (14) can be exactly linearized as

$$\dot{\xi} = (A + BK)\xi + Bw', \tag{15a}$$

$$y = C\xi, \tag{15b}$$

where $w' = b(\xi) w$. Then the following expression holds:

$$\| w' \| \leqslant kl.$$

Construct a dynamic model of the form

$$\dot{\theta} = (A + BK)\theta, \tag{16a}$$

$$y = C\theta, \tag{16b}$$

and let the initial state $\theta_0 = \phi(x_0)$, then it is obviously found that

$$x = \phi^{-1}(\theta)$$

is equal to the state x of the nonlinear system (1) in the ideal case without disturbance w. Choose

$$z = x - \phi^{-1}(\theta) \tag{17}$$

as the penalty output for the robust stabilization problem because it effectively reflects the deviation of the state x from the ideal case, resulting from the disturbance w.

Based on the Lyapunov theory, the following proposition can be obtained.

Proposition 1 Given positive number $\gamma > 0$, for the nonlinear system (1) satisfying the assumptions A1) and A2), suppose the inverse map $\phi^{-1}(\xi)$ of the coordination transform $\xi = \phi(x)$ satisfies the Lipschizs condition

A3) $\| \phi^{-1}(x) - \phi^{-1}(y) \| \leqslant L \| x - y \|$. Then the controller

$$u_2 = \begin{cases} -kl \dfrac{B^{\mathrm{T}} Pe}{\| B^{\mathrm{T}} Pe \|}, & B^{\mathrm{T}} Pe \neq 0 \text{ and } \| e \|_\infty \geqslant \delta, \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

$$\delta = \frac{\lambda_{\min}^{\frac{1}{2}}}{L \lambda_{\max}^{\frac{1}{2}}} \gamma, \ e = \phi(x) - \theta,$$

such that the systems (1a), (16a) and (17) have the L_∞-performance.

In order to prove Proposition 1, we introduce a property of the Euclid norm $\|\cdot\|$.

Lemma 5 Suppose the vector $x \in \mathbb{R}^n, y \in \mathbb{R}^n$ and the angle between the vector x and y is θ. Then

$$-\|x\|\|y\| \leqslant x^{\mathrm{T}}y = \|x\|\|y\|\cos\theta \leqslant \|x\|\|y\|. \tag{19}$$

Definition 4[4] Given a set

$$\Phi = \{\phi(t,x,w) \mid \forall w \in W, t \in \mathbb{R}^+\},$$

where W is the admissible disturbance set. Φ is invariant for the system (1) if for all $w \in W$ and $x \in \Phi, \phi(t,x,w) \in \Phi$.

Based on Lemma 5, Proposition 1 can be proved as follows.

Proof of Proposition 1 Obviously, to prove the Proposition 1 is equivalent to proving that, with the control law u, the set

$$\{z \mid \|z\|_\infty \leqslant \gamma\}$$

is an invariant set for all w, e and ξ.

By using Lemma 5, we have

$$\|z\|_\infty \leqslant \|z\| \leqslant L\|\phi(x) - \theta\| = L\frac{1}{\lambda_{\min}^{\frac{1}{2}}}\lambda_{\min}^{\frac{1}{2}}\|e\| \leqslant L\frac{1}{\lambda_{\min}^{\frac{1}{2}}}(e^{\mathrm{T}}Pe)^{\frac{1}{2}}.$$

So the sufficient condition that the set $\{z \mid \|z\|_\infty \leqslant \gamma\}$ is an invariant set is that, with the controller (18),

$$e^{\mathrm{T}}Pe \leqslant L^{-2}\lambda_{\min}\gamma^2$$

holds for all w, e and ξ.

Now consider the time derivative of the quadratic function $V(e) = e^{\mathrm{T}}Pe$, i.e.

$$\dot V(e) = e^{\mathrm{T}}[P(A+BK) + (A+BK)^{\mathrm{T}}P]e + 2e^{\mathrm{T}}PB(w' + u_2).$$

By (12), there is

$$e^{\mathrm{T}}[P(A+BK) + (A+BK)^{\mathrm{T}}P]e \leqslant 0.$$

From Lemma 5, it can be obtained that

$$e^{\mathrm{T}}PBw' \leqslant \|e^{\mathrm{T}}PB\|\|w'\| = \|e^{\mathrm{T}}PB\|kl.$$

Taking the form of the controller (18) into account, we can conclude that for all e satisfying $\|e\| \geqslant \delta$,

$$e^{\mathrm{T}}PBu_2 \leqslant -kl\frac{1}{\|B^{\mathrm{T}}Pe\|}e^{\mathrm{T}}PBB^{\mathrm{T}}Pe \leqslant -kl\|e^{\mathrm{T}}PB\| \leqslant 0.$$

Then, for all e satisfying $\|e\| \geqslant \delta$,

$$\dot V(e) \leqslant 0. \tag{20}$$

Because $e^{\mathrm{T}}Pe \leqslant \lambda_{\max}^2\|e\|^2$, the edge points of $\{e \mid e^{\mathrm{T}}Pe \leqslant L^{-2}\lambda_{\min}\gamma^2\}$, i.e. the points satisfying $e^{\mathrm{T}}Pe = L^{-2}\lambda_{\min}\gamma^2$, are included in the set

$$\{e \mid \|e\| \geqslant \frac{\lambda_{\min}^{\frac{1}{2}}}{L\lambda_{\max}^{\frac{1}{2}}}\gamma\}.$$

According to (20), for all e in the set

$$\{e \mid \|e\| \geqslant \frac{\lambda_{\min}^{\frac{1}{2}}}{L\lambda_{\max}^{\frac{1}{2}}}\gamma\},$$

there is $\dot V(e) \leqslant 0$. This means that at the edge points of $\{e \mid e^{\mathrm{T}}Pe \leqslant L^{-2}\lambda_{\min}\gamma^2\}$, the value of $V(e) = e^{\mathrm{T}}Pe$ will decrease. Thus, for the closed-loop system (1a), (16a) and (17) with the controller (18), $e^{\mathrm{T}}Pe \leqslant L^{-2}\lambda_{\min}\gamma^2$ always holds. Then the set $\{z \mid \|z\|_\infty \leqslant \gamma\}$ is an invariant set for the closed-loop system. Therefore, the proposition is proved.

Based on Lemma 3 and Proposition 1, the following conclusion can be drawn.

Theorem 1 For the nonlinear system (1) and a given value $\gamma > 0$, if the system satisfies the assumptions A1), A2) and A3), the robust stabilization problem of the definition 1 can be solved by the controller

$$u = u_1 + u_2 = \alpha(x) + \beta(x)[K\phi(x) + u_2], \tag{21}$$

where

$$\dot\theta = (A+BK)\theta, \tag{22a}$$

$$u_2 = \begin{cases} -kl\dfrac{B^{\mathrm{T}}Pe}{\|B^{\mathrm{T}}Pe\|}, & B^{\mathrm{T}}Pe \neq 0 \text{ and } \|e\|_\infty \geqslant \delta, \\ 0, & \text{otherwise}. \end{cases} \tag{22b}$$

x denotes the state of certain nonlinear system (1), and $\theta(0) = \phi(x_0)$,

$$\delta = \frac{\lambda_{\min}^{\frac{1}{2}}}{L\lambda_{\max}^{\frac{1}{2}}}\gamma,\ e = \phi(x) - \theta.$$

Proof Corresponding to the definition, the proof is divided into two parts:

a) The first is to consider the case of $w = 0$. By the lemma, the part a′) of Definition 2 is obvious;

b) The second is to consider the case of $w \neq 0$ and satisfies the assumption A1). According to the above lemmas, ξ asymptotically converges to the origin, which implies that $\phi^{-1}(\xi)$ converges to the origin. By Proposition 1, x is always kept in the set $\{x \mid \|x - \phi^{-1}(\xi)\| \leqslant \lambda\}$, whose center is $\phi^{-1}(\xi)$. So the part b′) of Definition 2 holds.

Then Theorem 1 is proved.

3.2 Robust tracking

Given a reference input $y_R(t)$, for the I/O linearized system (7) of the nominal system (4), introduce the trajectory error $\eta(t)$ as the difference between the real output and the reference output $y_R(t)$, i.e.

$$\eta(t) = \begin{bmatrix} \eta_1(t) \\ \eta_2(t) \\ \vdots \\ \eta_n(t) \end{bmatrix} = \begin{bmatrix} \xi_1 - y_R(t) \\ \xi_2 - y_R^{(1)}(t) \\ \vdots \\ \xi_n - y_R^{(n-1)}(t) \end{bmatrix},$$

then the system (7) becomes

$$\dot{\eta} = A\eta + Bv, \qquad (23a)$$

$$y = \eta_1. \qquad (23b)$$

So here is the result.

Lemma 6[5] For a given reference input $y_R(t)$, it assumes:

a) There exists the $(n-1)$-th order time derivative of $y_R(t)$;

b) The output $y(t)$ has relative degree n in $\mathbb{R}^n$. Then, the tracking problem can be solved by the controller

$$u_1 = \frac{1}{b(\xi)}(-a(\xi) + y_R^{(n)} - K\eta), \qquad (24)$$

where $K = [k_0\ k_1 \cdots k_{n-1}]$ satisfies that all roots of the equation

$$s^n + k_{n-1}s^{n-1} + \cdots + k_1 s + k_0 = 0 \qquad (25)$$

lie in the left-half complex plane, and $a(\xi)$ and $b(\xi)$ are taken as in Lemma 3.

Through I/O linearization, the nonlinear system with the controller (24) has the following form:

$$\dot{\xi} = A\xi + B[y_R^{(n)} + K(\xi - [y_R\, y_R^{(1)}\, y_R^{(2)} \cdots y_R^{(n-1)}]^{\mathrm{T}})] + Bw, \qquad (26a)$$

$$y = C\xi, \qquad (26b)$$

where $w' = b(\xi)w$. Then the following expression holds:

$$\| w' \| \leqslant kl,$$

where $\xi(0) = [h(x_0)\ h^{(1)}(x_0) \cdots h^{(n-1)}(x_0)]$, and A, B, C are taken as in (11).

Construct a dynamic model as follows:

$$\dot{\theta} = A\theta + B[y_R^{(n)} + K(\theta - [y_R\, y_R^{(1)}\, y_R^{(2)} \cdots y_R^{(n-1)}]^{\mathrm{T}})], \qquad (27a)$$

$$y_n = C\theta, \qquad (27b)$$

where $\theta(0) = [h(x_0)\ h^{(1)}(x_0) \cdots h^{(n-1)}(x_0)]$, and A, B, C are taken as in (11). Clearly, because of the feedback equivalence, the dynamic behavior of model (27) is equivalent to the dynamic behavior of the nominal system, i.e. the ideal case without disturbance w. So the error between the practical output of the system and the output of the artificial nominal system, can be represented by the dynamic model

$$\dot{e} = (A + BK)e + Bw', \qquad (28)$$

where $e(0) = 0$. Choose $z = e$ as the penalty output.

Proposition 2 Given a positive number $\gamma > 0$, the controller

$$u_2 = \begin{cases} -kl\dfrac{B^{\mathrm{T}}Pe}{\| B^{\mathrm{T}}Pe \|}, & B^{\mathrm{T}}Pe \neq 0 \text{ and } \| e \|_\infty \geqslant \delta, \\ 0, & \text{otherwise}. \end{cases} \qquad (29)$$

where

$$\delta = \frac{\lambda_{\min}^{\frac{1}{2}}}{\lambda_{\max}^{\frac{1}{2}}}\gamma$$

guarantees the system

$$\dot{e} = (A + BK)e + Bw' + Bu_2, \qquad (30a)$$

$$z = e, \qquad (30b)$$

where $e(0) = 0$, satisfies the L_∞-performance $\| z \|_\infty \leqslant \gamma$. Here K is taken as in Lemma 6.

Proof The proof is similar to the proof of Proposition 1, and therefore is omitted here.

Here is the results.

Theorem 2 For a given value $\gamma > 0$ and a reference input $y_R(t)$, if the assumptions in Lemma 6 are satisfied, then the robust tracking problem can be solved by the controller

$$u = u_1 + u_2 = \frac{1}{L_g L_f^{n-1}h(x)}(-L_f^n h(x) + y_R^{(n)} - K\eta + u_2), \qquad (31)$$

where

$$\begin{cases} \eta = \xi - [y_R\, y_R^{(1)}\, y_R^{(2)} \cdots y_R^{(n-1)}]^{\mathrm{T}}, \\ \xi = [h(x)\ h^{(1)}(x) \cdots h^{(n-1)}(x)], \\ \dot{\theta} = A\theta + B[y_R^{(n)} + K(\theta - [y_R\, y_R^{(1)}\, y_R^{(2)} \cdots y_R^{(n-1)}]^{\mathrm{T}})], \end{cases} \qquad (32a)$$

$$e = \xi - \theta, \qquad (32b)$$

$$u_2 = \begin{cases} -kl\dfrac{B^{\mathrm{T}}Pe}{\| B^{\mathrm{T}}Pe \|}, & B^{\mathrm{T}}Pe \neq 0 \text{ and } \| e \|_\infty \geqslant \delta, \\ 0, & \text{otherwise}. \end{cases} \qquad (32c)$$

$$\theta(0) = [h(x_0)\ h^{(1)}(x_0)\ \cdots\ h^{(n-1)}(x_0)],$$

$$\delta = \frac{\lambda_{\min}^{\frac{1}{2}}}{\lambda_{\max}^{\frac{1}{2}}}\gamma.$$

x denotes the state of uncertain nonlinear system (1), K is taken as in Proposition 2.

Proof The proof is simple and is omitted.

4 Example

Consider the uncertain nonlinear system (1) with the form

$$f(x) = \begin{bmatrix} x_3^2 \\ x_1 + x_2 + x_3^2 \\ x_1 - x_2 \end{bmatrix},\ g_2(x) = \begin{bmatrix} \exp(x_2) \\ \exp(x_2) \\ 0 \end{bmatrix}. \tag{33}$$

Suppose that the initial state is $[1\quad 2\quad 3]$ and the disturbance is

$$w = \sin(t). \tag{34}$$

The control objective is to stabilize the system at the origin $[0\quad 0\quad 0]$.

For the nominal system (4) of the system (1), Let the coordinate transformation be

$$\xi = \phi(x) = \begin{bmatrix} x_3 \\ x_1 - x_2 \\ -x_1 - x_2 \end{bmatrix}. \tag{35}$$

It is easy to get by calculation

$$L = 0.5, \tag{36}$$

$$a(x) = -x_1 - x_2 - 2x_3^2,\ b(x) = -2\exp(x_2). \tag{37}$$

Then the system (4) is linearized as the system (7). For the system (7), a stabilizing controller is

$$\dot{v} = [-1\quad -3\quad -3]\xi \tag{38}$$

and the positive defined matrix

$$P = \begin{bmatrix} 83.1578 & 60.5567 & 17.9623 \\ 60.5567 & 136.366 & 31.4600 \\ 17.9623 & 31.4600 & 23.8069 \end{bmatrix} \tag{39}$$

satisfies the inequality (12).

By Theorem 1, suppose the permissible L_∞-performance is 0.1, then a robust stabilizing controller is

$$\dot{\theta} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}\theta, \tag{40}$$

$$u(x) = \frac{1}{b(x)}\{-a(x) + [-1\ -3\ -3]\phi(x)\} + u_2(x), \tag{41}$$

where

$$u_2(x) = -2\exp(2)\frac{[17.9623\ 31.4600\ 23.8069][\phi(x)-\theta]}{\|[17.9623\ 31.4600\ 23.8069][\phi(x)-\theta]\|},$$

if $\|\phi(x) - \theta\| \geqslant \frac{\lambda_{\min}^{\frac{1}{2}}}{L\lambda_{\max}^{\frac{1}{2}}}\gamma = 2\times 0.3\times 0.1 = 0.06$ and $[17.9623\quad 31.4600\quad 23.8069][\phi(x) - \theta]] \neq 0$, otherwise $u_2(2) = 0$.

The simulation shows that the stabilizing controller of nominal system is unable to stabilize the uncertain system in Fig. 1, and Fig. 2 shows that the robust controller (41), designed by Theorem 1, is valid. It verifies the correctness of Theorem 1. Theorem 2 can also be proved by simulation in a similar way.

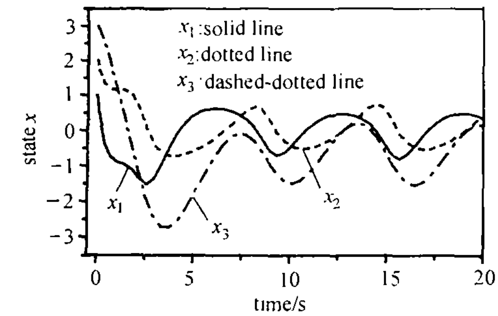

Fig. 1 The state trajectory of uncertain system with the stabilizing controller for nominal system

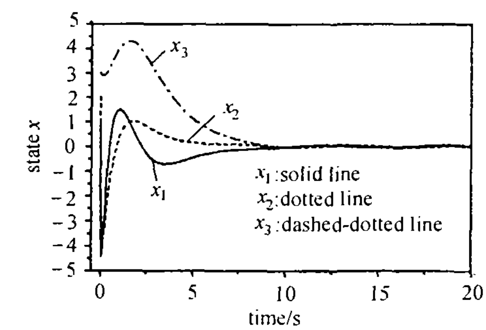

Fig. 2 The state trajectory of uncertain system with the designed robust controller

5 Conclusions

In practice, due to the influence of the uncertain factors such as disturbance, a small deviation to the ideal state is often permissible. Correspondingly, in the stabilization problem, the state is often permitted to swing in a small neighborhood of the expected equilibrium point. Similarly, in the tracking problem, a small tracking error is permitted. Based on these views, the robust stabilization problem and robust tracking problem are defined respectively in the framework of L_∞ norm.

On the basis of the technique of exact linearization, this paper has discussed the robust control of a class of SISO affine nonlinear systems with L_∞-bounded disturbances, and corresponding robust controllers have been obtained. In the final part of the paper, a simulation has been carried by MATLAB and SIMLINK, which illustrates the correctness of the results. Clearly, the results in the paper can further be generalized to the case of MIMO nonlinear systems, and the cases with more uncertainties, but further research is needed.

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