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# Study of Bifurcation and Chaos in the Current-Mode Controlled Buck-Boost DC-DC Converter (Ⅰ)*
## ——Modeling and Simulation

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**Abstract**: This paper studies the bifurcation and chaos in current-mode controlled Buck-Boost converter. The discrete model for Buck-Boost converter in continuous conduction mode (CCM) is derived. The bifurcation and chaos phenomena of Buck-Boost converter have been investigated with input voltage $E$, reference current $I_{ref}$, resistor $R$, inductor $L$ and capacitor $C$ as bifurcation parameters.

**Key words**: power electronics; bifurcation; chaos; model
**Document code**: A

# Buck-Boost DC-DC 变换器中分叉与混沌问题的研究(Ⅰ)
## ——建模与仿真

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**摘要**: 研究了电流控制型 Buck-Boost 变换器中的分叉与混沌问题.首先,对连续模式下的 Buck-Boost 变换器建立了离散数学模型,在此基础上,在输入电压 $E$、参考电流 $I_{ref}$、电阻 $R$、电感 $L$ 和电容 $C$ 等分叉参数作用下,通过数值方法对 Buck-Boost 变换器中的分叉与混沌进行了详细的仿真研究.仿真研究结果表明:Buck-Boost 变换器具有丰富的非线性行为——分叉与混沌,随着各个分叉参数的变化系统会遵循倍周期分叉的规律走向混沌.

**关键词**: 电力电子; 分叉; 混沌; 建模

## 1 Introduction

It is well known that the topologies of DC-DC converters are changed due to the switching operation. Usually, switched-mode DC-DC converters may be thought of as nonlinear and time-varying system. Hence, DC-DC converters exhibit a wide range of bifurcation and chaos behavior under some conditions. The nonlinear phenomena, which appear to behave randomly in a deterministic system even though there is no random input, are particularly interesting and have interested power electronics, circuit and system, mathematics and control. Recently, the research on bifurcation and chaos in DC-DC converters has been greatly developed. In [1], detailed information about recent developments is available. Most published papers are mainly about the bifurcation and chaos study in Buck converter[2~8], Boost converter[9~12] and Cuk converter[13~16]. On the other hand, Buck-Boost converter, one of the important converters that have wide industrial applications, has not yet been reported. Our paper deeply studies the bifurcation and chaos in the current-mode Buck-Boost DC-DC converter. It must be pointed out that power converters usually work at 20kHz to 50kHz in practical engineering. In other published papers, people have used switching frequencies which are much lower than the actual value in practical power electronics to avoid high-frequency problems, such as 500Hz, 2.5kHz, 5kHz, etc. Here, this paper presents the results at 20KHz, which is the highest switching frequency in this area.

This paper is organized as follows. In Section 2, the state equations of the current-mode Buck-Boost DC-DC converter are presented and the discrete model is derived

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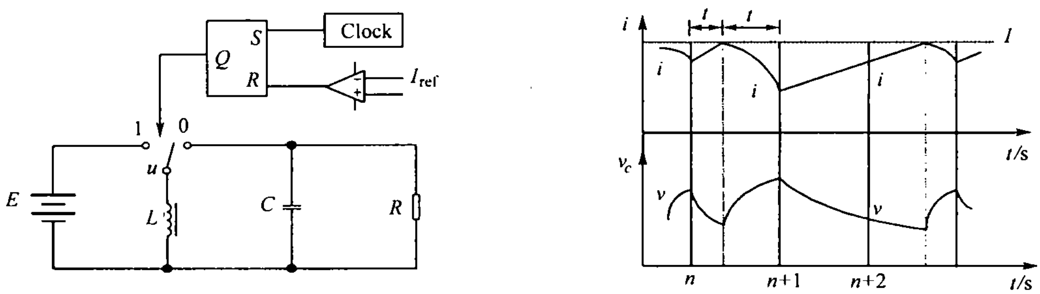
under the condition of continuous conduction mode (CCM). In Section 3, we investigate the bifurcation and chaos phenomena under variation of a range of circuit parameters including input voltage, reference current, load resistance, inductance and capacitance. The simulation results including the strange attractors, bifurcation diagrams and waveforms are presented. A conclusion is given in Section 4

## 2 Iterative model for the Buck-Boost converter

In power electronics circuits, the famous state-space-averaging model is widely adopted by power electronics engineers in their analysis and design. However, an averaged model abandons the switching details and only focuses on the envelope of the dynamical motion. It is only useful to analyze the low-frequency characteristics in the power electronics circuits. Therefore, in order to explore the nonlinear phenomena which may appear across a wide spectrum of frequency, the exact discrete-time maps must be derived. In this paper, we use stroboscopic map, the most widely used type of discrete-time maps for modeling DC-DC converters, to obtain the Poincare section. i.e. the system states, the inductor current and capacitor voltage, are periodically sampled at time instants, $t = nT$.

The current-mode Buck-Boost converter is shown in Fig.1(a). The waveforms of inductor current and capacitor voltage are shown in Fig.1(b). We assume that the converter operates in CCM, where the inductance and switch period $T$ are so chosen that the inductor current never falls to zero. Hence, there are two circuit configurations, according to whether $S$ is closed or opened. It is assumed that $S$ is closed at the beginning of each cycle, i.e., at $t = nT$. The inductor current rises linearly until $i = I_{ref}$. Any clock pulse arriving during this period is ignored. When $i = I_{ref}$, switch $S$ opens and remains open until the arrival of the next cloak pulse, where it is closed again.




(a) (b)

Fig. 1 The circuit diagram and waveforms in current-mode Buck-Boost converter

When switch $S$ is closed, the state equations are (1) and (2):

$$\begin{cases} \dfrac{\mathrm{d}i}{\mathrm{d}t} = \dfrac{E}{L}, & (1) \\ \dfrac{\mathrm{d}v_c}{\mathrm{d}t} = -\dfrac{1}{RC}v_c. & (2) \end{cases}$$

When switch $S$ is opened, the state equations are (3) and (4):

$$\begin{cases} \dfrac{\mathrm{d}i}{\mathrm{d}t} = -\dfrac{1}{L}v_c, & (3) \\ \dfrac{\mathrm{d}v_c}{\mathrm{d}t} = \dfrac{1}{C}i - \dfrac{1}{RC}v_c. & (4) \end{cases}$$

Where we omit the parasitic elements of inductor and capacitor so as to simplify the derivation of discrete model.

Let $(i_n, v_n)$ be the inductor current and capacitor voltage at a clock pulse at which the switch is closed.

From Fig.1(b), we know that the switch $S$ is opened when the inductor current reaches reference current $I_{ref}$. The close-state time $t_n$ can be obtained from (1) by integration, so the closed-state time $t_n$ is calculated by (5):

$$t_n = \frac{L}{E}(I_{ref} - i_n). \tag{5}$$

The capacitor voltage corresponding to instant $t_n$ is calculated by (6):

$$v_c(t_n) = v_n e^{-\frac{t_n}{RC}}. \tag{6}$$

The iterative model for the Buck-Boost converter can be derived as follows according to two situations, i.e., $t_n \geqslant T$ and $t_n < T$.

**Case 1** $t_n \geqslant T$. It means that the switch $S$ in the converter is held on closed-state during a switching period $T$. The values of $i_n$ and $v_n$ at next clock instant, $i_{n+1}$ and $v_{n+1}$, are calculated from (1) and (2) with $i_n$ and $v_n$ as initial values.

$$\begin{cases} i_{n+1} = i_n + \dfrac{E}{L}T, \\ v_{n+1} = v_n \cdot \mathrm{e}^{-\frac{T}{RC}}. \end{cases} \tag{7}$$

**Case 2** $t_n < T$. It means that the switch $S$ in the converter is switched from closed-state to open-state during a switching period $T$. The values of $i_n$ and $v_n$ at next clock instant, $i_{n+1}$ and $v_{n+1}$, are calculated from (3) and (4) with $I_{\mathrm{ref}}$ and $v_n \mathrm{e}^{-\frac{t_n}{RC}}$ as initial values.

According to the relationship of $R$, $L$ and $C$, the iterative model for the Buck-Boost converter is divided into three forms.

① $1 - \dfrac{4R^2C}{L} > 0$, i.e. $R < \dfrac{1}{2}\sqrt{\dfrac{L}{C}}$. In this case, the roots of the characteristic equation corresponding to the original differential equations (3) and (4) are real and distinct. It describes a monotonous rising process without oscillatory motion. Hence, (8) and (9) give the iterative map for the Buck-Boost converter.

$$i_{n+1} = c_1 \mathrm{e}^{r_1(T-t_n)} + c_2 \mathrm{e}^{r_2(T-t_n)}, \tag{8}$$

$$v_{n+1} = -L[c_1 r_1 \mathrm{e}^{r_1(T-t_n)} + c_2 r_2 \mathrm{e}^{r_2(T-t_n)}], \tag{9}$$

where

$$r_1 = -\frac{1}{2RC} + \frac{1}{2RC}\sqrt{1 - \frac{4R^2C}{L}},$$

$$r_2 = -\frac{1}{2RC} - \frac{1}{2RC}\sqrt{1 - \frac{4R^2C}{L}},$$

$$c_1 = I_{\mathrm{ref}} - \frac{v_n \mathrm{e}^{-\frac{t_n}{RC}} + L r_1 I_{\mathrm{ref}}}{L(r_1 - r_2)},$$

$$c_2 = I_{\mathrm{ref}} - \frac{v_n \mathrm{e}^{-\frac{t_n}{RC}} + L r_1 I_{\mathrm{ref}}}{L(r_1 - r_2)}.$$

② $1 - \dfrac{4R^2C}{L} = 0$, i.e. $R = \dfrac{1}{2}\sqrt{\dfrac{L}{C}}$. In this case, the roots of the characteristic equation corresponding to the original differential equations (3) and (4) are real and equal. It also describes a monotonous rising process without oscillatory motion. Hence, (10) and (11) give the iterative map for the Buck-Boost converter.

$$i_{n+1} = [c_1 + c_2(T - t_n)]\mathrm{e}^{r_1(T-t_n)}, \tag{10}$$

$$v_{n+1} = -L\mathrm{e}^{r_1(T-t_n)}[c_2 + r_1 c_1 + r_1 c_2(T - t_n)], \tag{11}$$

where $r_1 = r_2 = -\dfrac{1}{2RC}$, $c_1 = I_{\mathrm{ref}}$, $c_2 = -\dfrac{v_n \cdot \mathrm{e}^{-\frac{t_n}{RC}}}{L} - r_1 I_{\mathrm{ref}}$.

③ $1 - \dfrac{4R^2C}{L} < 0$, i.e. $R > \dfrac{1}{2}\sqrt{\dfrac{L}{C}}$. In this case, the solutions of the characteristic equation corresponding to the original differential equations (3) and (4) are a pair of complex conjugate roots. It leads to a damped oscillatory process. Hence, (12) and (13) give the iterative map for the Buck-Boost converter.

$$i_{n+1} = \mathrm{e}^{\alpha(T-t_n)}[c_1 \cos\beta(T - t_n) + c_2 \sin\beta(T - t_n)], \tag{12}$$

$$v_{n+1} = -L\mathrm{e}^{\alpha(T-t_n)}[(c_1\alpha + c_2\beta)\cos\beta(T - t_n) + (c_2\alpha - c_1\beta)\sin\beta(T - t_n)], \tag{13}$$

where

$$\alpha = -\frac{1}{2RC}, \quad \beta = \frac{1}{2RC}\sqrt{\frac{4R^2C}{L} - 1},$$

$$c_1 = I_{\mathrm{ref}}, \quad c_2 = -\frac{1}{\beta}\left(\frac{v_n \cdot \mathrm{e}^{-\frac{t_n}{RC}}}{L} + I_{\mathrm{ref}}\alpha\right).$$

## 3 Simulation study of bifurcation and chaos in the Buck-Boost converter

Based on the iterative map in Section 2, we can study the bifurcation and chaos in the Buck-Boost converter with numerical method. Several visual aids that we can utilize to visualize chaos and bifurcation phenomena including time-waveform of state variables, phase portrait, Poincare or first-return maps and bifurcation diagrams. Among them, bifurcation diagram is the most powerful tool to investigate the nonlinear phenomena. In a bifurcation diagram, a periodic steady state of the system is represented as a signal point or several points equal to the periodicity of the system for a fixed parameter. For chaos, numerous points are plotted in the diagram because chaos means period infinity and the points never fall at the same position. Therefore, in such a bifurcation diagram, the change of behavior of a system is clearly shown as a parameter is varied.

From the theoretical point of view, any circuit parameter can work as bifurcation parameter. In this section, we will investigate the change of behavior in the Buck-Boost converter when a parameter, such as input voltage $E$, reference current $I_{\mathrm{ref}}$, load resistance $R$, inductance $L$ and capacitance $C$, is varied.

In the simulation, the values of the components are chosen to ensure that the converter operates theoretically in the continuous conduction mode.

### 3.1 $E$ as the bifurcation parameter

In the Buck-Boost converter with input voltage $E$ as bifurcation parameter, input voltage $E$ is varied from 45V to 7V with a step of 0.1V, while other circuit parameters are fixed at the following values. i.e., $I_{ref}$ = 4A, $R$ = 20Ω, $L$ = 0.5mH, $C$ = 4μF, $T$ = 50μs ($f$ = 20kHz). The bifurcation diagram of the converter is shown in Fig.2.

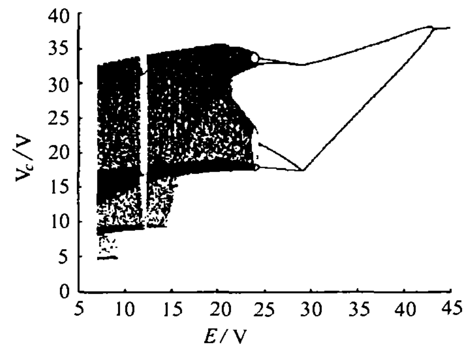



Fig. 2 The bifurcation diagram in the Buck-Boost converter with input voltage as parameter

As shown in Fig.2, the Buck-Boost converter goes through period-1, period-2, period-4, period-8, and eventually exhibits chaos as input voltage $E$ is varied from 45V to 7V. The stable period-1 is observed while the input voltage $E$ is varied from 45V to 43.2V. The first bifurcation occurs at $E$ = 43.3V and the converter enters a stable period-2 region. As the input voltage is continuously decreased to 29.0V, the converter bifurcates to period-4. Further, peroid-4 bifurcates to period-8 at 24.4V and so on. Hence, the converter goes to chaos via period-doubling route.

In Fig.2, it can be interestingly observed that a small periodic window, which also exhibits period-doubling cascade, is embedded in the chaos region. In the periodic window, the converter experiences period-3 to period-6 and so on when the input voltage $E$ is changed from 12.4V to 11.8V.

At $E$ = 20V, the waveforms and phase portrait of the converter are shown in Fig.3(a),(b),(c). In Fig.3, it can be observed that the waveforms appear to behavior randomly and there is a strange attractor in the phase portrait. A strange attractor occurs means that the converter is working in the chaotic state.

Moreover, if input voltage is respectively equal to 50V, 35V, and 25V, which correspond to period-1, period-2, and period-4, the waveforms and phase portraits of the converter are shown in Fig.4, Fig.5, and Fig.6 respectively.

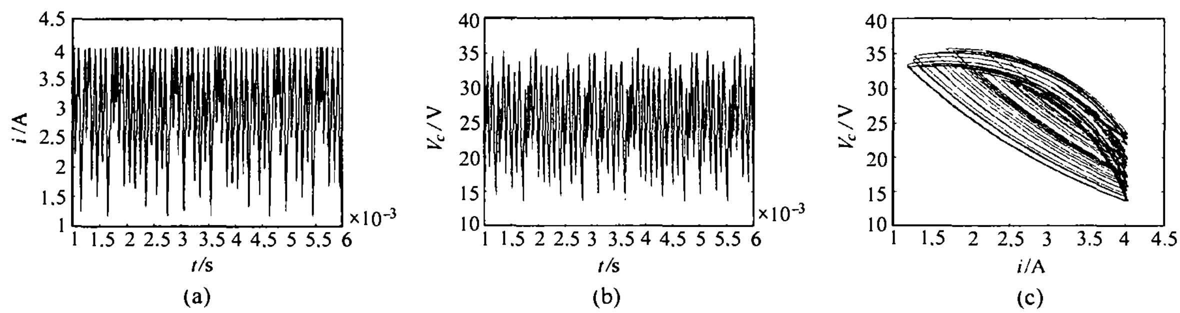



Fig. 3 The waveforms and phase portrait in the Buck-Boost converter with $E$=20V

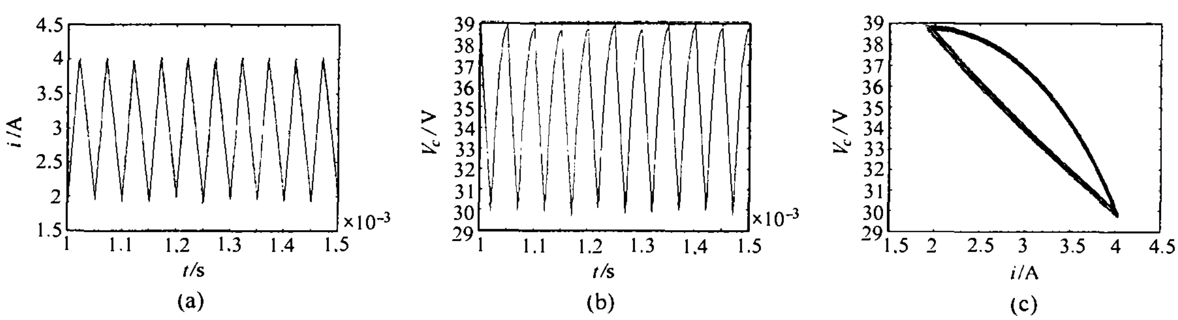



Fig. 4 The waveforms and phase portrait in the Buck-Boost converter with $E$=50V

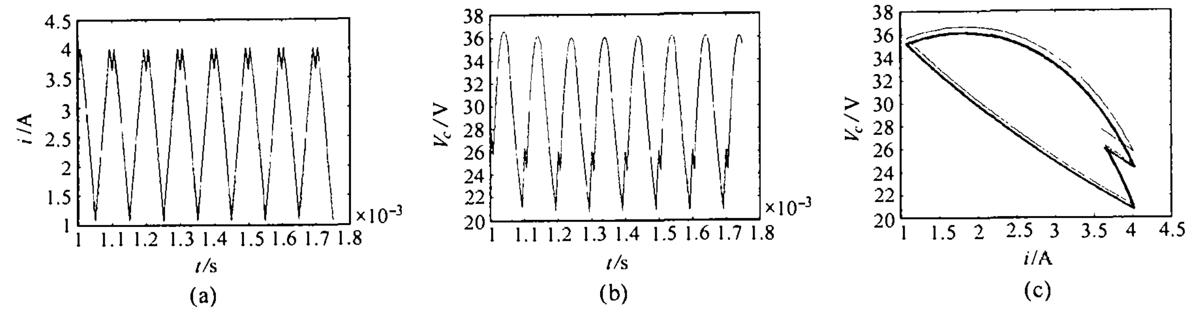



Fig. 5 The waveforms and phase portrait in the Buck-Boost converter with $E$=35V

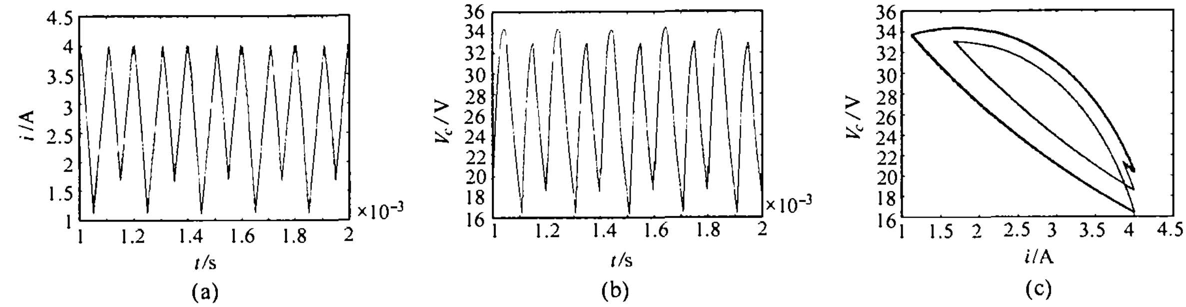



Fig. 6 The waveforms and phase portrait in the Buck-Boost converter with $E$=25V

### 3.2 $I_{ref}$ as the bifurcation parameter

In this section, we use reference current $I_{ref}$ as bifurcation parameter. Reference current $I_{ref}$ is varied from 0.8A to 4.5A with a step of 0.01A, while other circuit parameters are fixed at the following values. i.e., $E$ = 12V, $R$ = 20Ω, $L$ = 0.5mH, $C$ = 4μF, $T$ = 50μs ($f$ = 20kHz). The bifurcation diagram of the converter is shown in Fig.7.

Similarly in Fig. 7, the Buck-Boost converter goes through period-1, period-2, period-4, period-8, and eventually exhibits chaos as reference current $I_{ref}$ is varied from 0.8A to 4.5A.

At $I_{ref}$ = 4.5A, the waveforms and phase portrait of the converter are shown in Fig. 8 (a), (b), (c). In Fig. 8, it can be observed that the waveforms appear to behave randomly and there is a strange attractor in the phase portrait.

For the concision of the paper, here, the waveforms and phase portrait of period-1, period-2, and period-4 are not presented again.

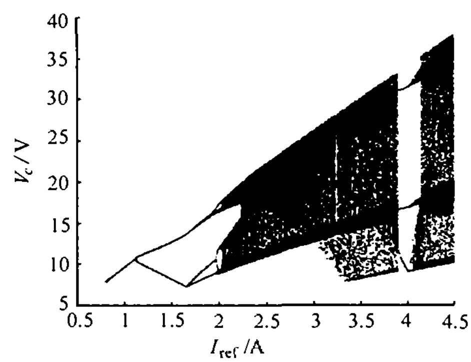



Fig. 7 The bifurcation in the Buck-Boost converter with reference current as parameter

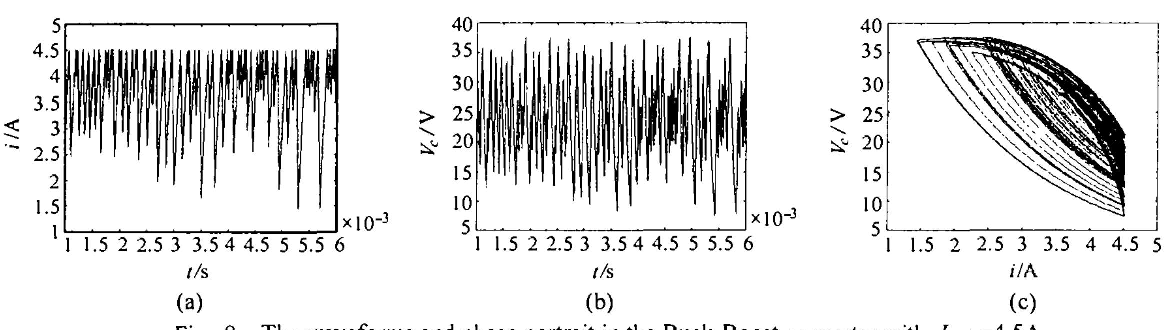



Fig. 8 The waveforms and phase portrait in the Buck-Boost converter with $I_{ref}$ =4.5A

### 3.3 $R$ as the bifurcation parameter

Similarly, load resistor $R$ works as bifurcation parameter, the resistance is varied from 1Ω to 25Ω with a step of 0.05Ω, while other circuit parameters are fixed at the following values. i.e., $E = 12\text{V}$, $I_{ref} = 4\text{A}$, $L = 0.5$ mH, $C = 4\mu\text{F}$, $T = 50\mu\text{s}$ ($f = 20\text{kHz}$). The bifurcation diagram of the converter is shown in Fig. 9.

Similarly in the bifurcation diagram in Fig. 9, the Buck-Boost converter goes through period-1, period-2, period-4, period-8, and eventually exhibits chaos as resistor $R$ is varied from $1\Omega$ to $25\Omega$.

At $R = 15\Omega$, the waveforms and phase portrait of the converter are shown in Fig. 10 (a), (b), (c).

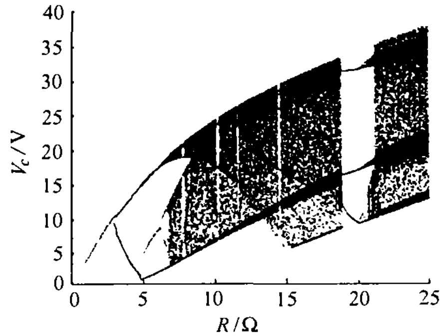



Fig. 9 The bifurcation in the Buck-Boost converter with resistance as parameter

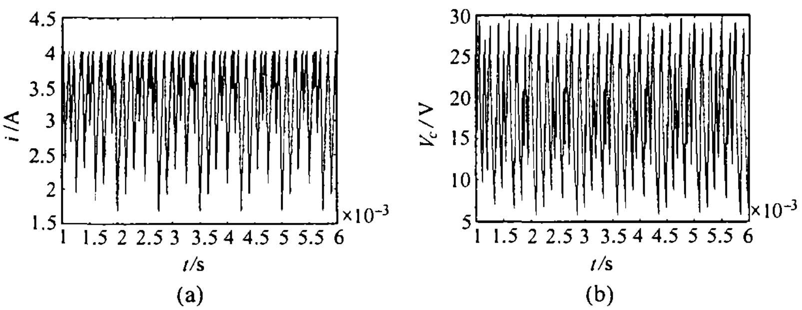



Fig. 10 The waveforms and phase portrait in the Buck-Boost converter with $R = 15\Omega$

### 3.4 $L$ as the bifurcation parameter

Similarly, in this section, inductor $L$ is used as bifurcation parameter. The inductance is varied from 0.07mH to 1.5mH with a step of 0.001mH, while other circuit parameters are fixed at the following values. i.e., $E = 12\text{V}$, $I_{ref} = 1.7\text{A}$, $R = 20\Omega$, $C = 2\mu\text{F}$, $T = 50\mu\text{s}$ ($f = 20\text{kHz}$). The bifurcation diagram of the converter is shown in Fig. 11.

Similarly in the bifurcation diagram in Fig. 11, the Buck-Boost converter goes through period-1, period-2, period-4, period-8, and eventually exhibits chaos as inductance is varied from 0.07mH to 1.5mH.

Fig. 12 (a), (b), (c) show the waveforms and phase portrait of the converter at $L = 1.5\text{mH}$.

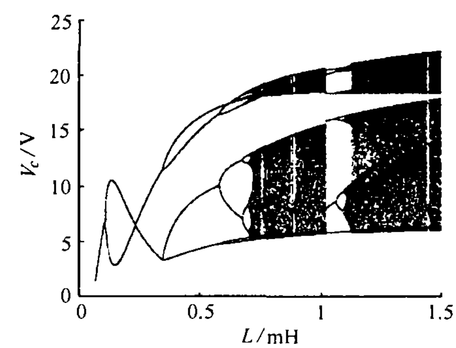



Fig. 11 The bifurcation in the Buck-Boost converter with inductor as parameter

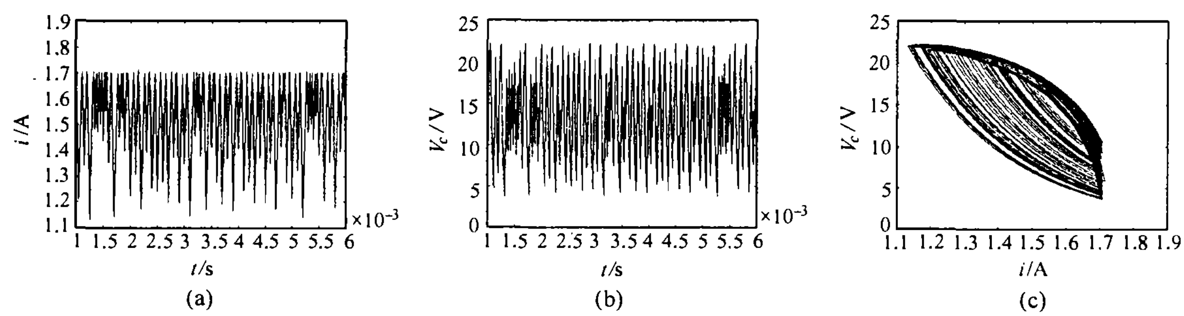



Fig. 12 The waveforms and phase portrait in the Buck-Boost converter with $L = 1.5\text{mH}$

### 3.5 $C$ as the bifurcation parameter

Similarly, we use capacitor $C$ as bifurcation parameter. The capacitance is varied from $38\mu\text{F}$ to $0.3\mu\text{F}$ with a step of $0.01\mu\text{F}$, while other circuit parameters are fixed at the following values. i.e., $E = 12\text{V}$, $I_{ref} = 4\text{A}$, $R = 16\Omega$, $L = 1\text{mH}$, $T = 50\mu\text{s}$ ($f = 20\text{kHz}$). The bifurcation diagram of the converter is shown in Fig. 13. The zoom-in of Fig. 13 (a) is shown in Fig. 13 (b).

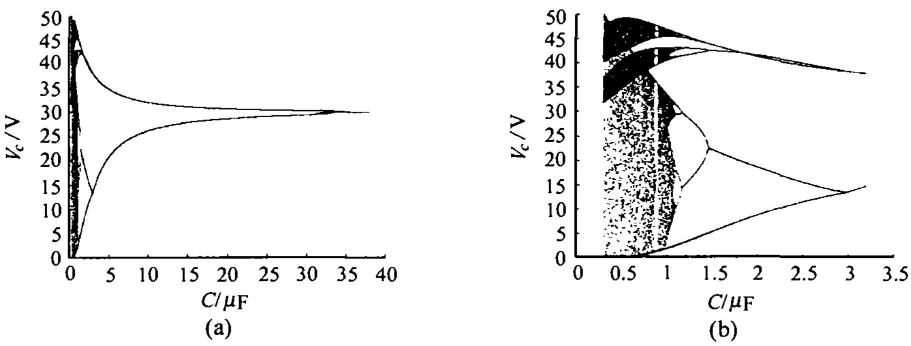



Fig. 13 The bifurcation in the Buck-Boost converter with capacitor as parameter

From the bifurcation diagram shown in Fig. 13 (a) and Fig. 13 (b), we can see that the Buck-Boost converter goes through period-1, period-2, period-4, period-7, and eventually exhibits chaos as capacitance $C$ is varied from 38μF to 0.3μF. The stable period-1 is observed while the capacitance $C$ is varied from 38μF to 33.99μF. At $C$ = 1μF, the waveforms and phase portrait of the converter are shown in Fig. 14 (a), (b), (c).

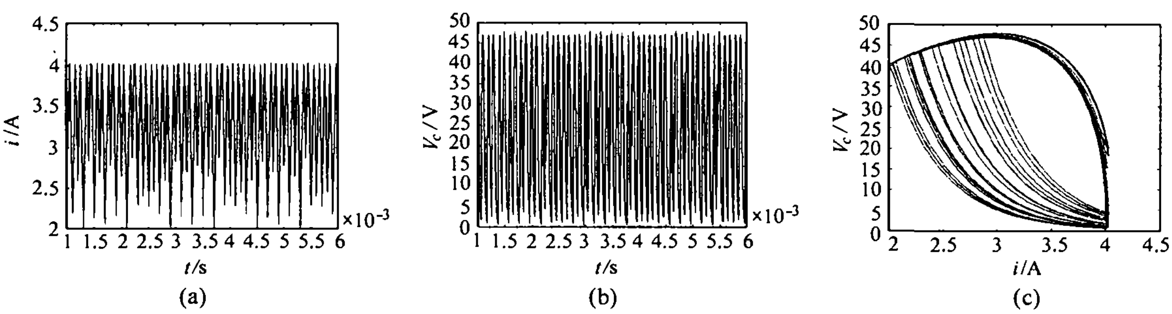



Fig. 14 The waveforms and phase portrait in the Buck-Boost converter with $C$=1μF

## 4 Conclusion

It is well known that the topologies of DC-DC converters are changed due to the switching operation. This results in a nonlinear time-varying system. Hence, DC-DC converters exhibit a wide range of bifurcation and chaos behavior under some conditions. In this paper, we deeply study the bifurcation and chaos phenomena in the current-mode Buck-Boost converter.

This paper derives an iterative map for the Buck-Boost converter under current-mode control. On the basis of the model, the bifurcation phenomena under variation of a range of circuit parameters including input voltage, reference current, load resistance, inductance and capacitance have been investigated.

The simulation results state that the Buck-Boost converter exhibits a wide range of nonlinear behavior. As the bifurcation parameter is varied, the system goes to chaos via period-doubling route. It must be specially pointed out that sometimes period-adding occurs in the period-doubling route. It can also be seen that the solutions of the system equations appear to behave randomly in a deterministic system even though there is no random input.

The research about the domains of bifurcation and chaos in the parameter space is particularly important because the power electronics engineers must choose the parameter values in order to obtain the desirable behavior. Moreover, the engineers will consciously avoid the bifurcation and chaos domains if they thoroughly understand when the nonlinear phenomena occur. We will discuss this topic in our future papers, numerical analysis and experiment.

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