Article ID: 1000 - 8152(2002)03 - 0428 - 03

Discriminance for the Similar Structure of Generalized Composite Systems and Robust Stabilization *

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Abstract: Similar structure of generalized composite systems is defined for the first time and the discriminance is given. So the new concept of similarity is proposed. Robust controllers for the system are designed so that the generalized composite systems are asymptotically stable. Since the controllers also possess similar structures, they are easy to perform in engineering practice.

Key words: similar structure; generalized composite system; robust stabilization

Document code: A

广义组合系统相似结构的判别和鲁棒镇定

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摘要: 首次定义了广义组合系统的相似结构并给出了判别方法,提出了新的相似性概念,对系统设计了鲁棒控制器使得组合系统渐近稳定.由于控制器本身也具有相似结构,因此易于工程实现.

关键词: 相似结构; 广义组合系统; 鲁棒镇定

1 Introduction

Similar structure for a class of generalized composite systems is firstly defined based on the research of generalized system[1] and similarity[2~4], so new conception and new research region of similarity are presented here. The uncertain interconnections of systems discussed in this paper are nonlinear and do not satisfy matching conditions. A simple method is given that can find out the similar structure in generalized composite systems. Robust controllers are designed so that the composite systems are asymptotically stable. We know from the research that the controllers also possess similar structure, so the controller design is largely simplified.

2 Description for the similar structure

Consider the following generalized composite systems

$$E_i\dot{x}_i = A_ix_i + B_i[u_i + H_i(x) + \Delta H_i(x)] + \sum_{j=1,j\neq i}^{N} \Phi_{ij}(x_j),\ i = 1,2,\cdots,N, \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ are states and output of the i-th sub-system, respectively; E_i, A_i are constant matrices of order n, $\operatorname{rank} E_i < n$; B_i are constant matrices of order $n \times m$; $H_i(x) \in V_m^{\omega}(\Omega)$, $\sum_{j=1,j\neq i}^{N} \Phi_{ij}(x_j) \in V_n^{\omega}(\Omega)$ and $\Delta H_i(x)$ are matching interconnections, non-matching interconnections and uncertain matching interconnections, respectively, here Ω_i is a neighborhood of $x_i = 0$, $\Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_N$ is a neighborhood of $x = 0$, $V_n^{\omega}(\Omega)$ is smooth vector field with dimension n defined on Ω, $x = \operatorname{col}(x_1, x_2, \cdots, x_N)$. We assume that $H_i(0) = 0$, $\Phi_{ij}(0) = 0$, $i = 1,2,\cdots,N$, $i \neq j$ and each subsystem is regular. Systems $E_i\dot{x}_i = A_ix_i + B_iu_i$ are simply denoted as (E_i, A_i, B_i), $i = 1,2,\cdots,N$.

Definition Systems (1) are called similar generalized composite systems, or possessing similar structure, if there exist matrices L_i, F_i of orders $m \times n$ and nonsingular matrix T_i, S_i of order n, so that

$$\begin{cases} T_i(E_i + B_iL_i)S_i = E,\ T_i(A_i + B_iF_i)S_i = A, \\ T_iB_i = B,\ i = 1,2,\cdots,N, \end{cases} \tag{2}$$

and (T_i, S_i, L_i, F_i) is called the similar parameters of

* Foundation item: supported by National Natural Science Foundation of China (79970114, 69934010, 69974007).
Received date: 2000 - 04 - 10; Revised date: 2001 - 04 - 25.

i-th sub-system, $i = 1,2,\cdots,N$.

3 Discriminance for the similar structure

Theorom 1 Systems (1) possess similar structure, if all (E_i, A_i, B_i) are single input and controllable.

Proof We can obtain the similar parameters (T_i, S_i, L_i, F_i) satisfying (2) according to theory of linear systems and singular linear system, while

$$A = \begin{bmatrix} 0 & 0 & & \\ \vdots & \ddots & \ddots & \\ 0 & \cdots & \cdots & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

4 Design of robust controllers

During the proof of Theorem 1 (A, B) is controllable, hence there exists $m \times n$ matrix K so that $(A + BK)$ is stable, and for arbitrary positive definite matrix Q, Lyapunov equation $(A + BK)^{\mathrm{T}}P + P(A + BK) = -Q$ has unique positive definite matrix solution P. Since $\Phi_{ij}(x_j) \in V_n^{\omega}(\Omega)$, $\Phi_{ij}(0) = 0$, there exist smooth functional matrices $R_{ij}(x_j)$ according to [5] so that $\Phi_{ij}(x_j) = R_{ij}(x_j) \cdot x_j$, $i, j = 1,2,\cdots,N$, $i \neq j$. Let $W = (\omega_{ij}(x_j))_{N\times N}$, while

$$\omega_{ij}(x_j) = \begin{cases} 1, & i = j, \\ -2\lambda_M((Q^{-\frac{1}{2}})^{\mathrm{T}}PT_iR_{ij}(x_j)S_j^{-1}Q^{-\frac{1}{2}}), & i \neq j, \end{cases}$$

where $\lambda_M(\cdot)$ denotes the maximum singular value of matrices. Let $\|\cdot\|$ be the Euclid norm of vectors.

Theorem 2 Systems (1) could be robust stabilized under the following conditions:

i) (E_i, A_i, B_i) are single input and controllable, $i = 1,2,\cdots,N$;

ii) $\| H_i(x) \| \leqslant \lambda(x)$, $\| \Delta H_i(x) \| \leqslant \rho(x)$, $i = 1,2,\cdots,N$;

iii) $W^{\mathrm{T}}(x) + W(x)$ is positive definite on Ω.

Proof Theorem 1 holds according to condition i), so there are similar parameters (T_i, S_i, L_i, F_i) that satisfying (2). Then the closed-loop systems of systems (1) and controllers

$$u_i = u_i^0(x) + u_i^1(x) + u_i^2(x), \quad i = 1,2,\cdots,N \tag{3}$$

are asymptotically stable on Ω, where

$$u_i^0(x) = -L_i\dot{x}_i + (F_i + K)S_ix_i,$$
$$u_i^1(x) = -\operatorname{sign}[(S_i + x_i)^{\mathrm{T}}PB]\lambda(x),$$
$$u_i^2(x) = -\operatorname{sign}[(S_i + x_i)^{\mathrm{T}}PB]\rho(x).$$

Remark We know from (3) that N controllers have similar structure, difference among them lays on $L_i, F_i, S_i, \lambda(x), \rho(x)$, so the design of controllers are simplified.

5 Simulation

Consider generalized composite systems consists of two subsystems

$$\begin{pmatrix} 0 & 0 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 3.5 \\ -1 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -0.5 \\ 1 \end{pmatrix}\left(u_1 + 0.3\left\|\begin{pmatrix} x_3 \\ x_4 \end{pmatrix}\right\| + \Delta H_1(x)\right) + \begin{pmatrix} 0.2x_3\sin x_3 \\ 0.1x_4\cos x_4 \end{pmatrix},$$

$$\begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ -9 & -2 \end{pmatrix}\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}\left(u_2 + 0.4\left\|\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right\| + \Delta H_2(x)\right) + \begin{pmatrix} 0.05x_1e^{-|x_1|} \\ 0.02x_2e^{-|x_2|} \end{pmatrix},$$

where

$$\| \Delta H_1(x) \| \leqslant 0.7 \| x \|,$$
$$\| \Delta H_2(x) \| \leqslant 0.2 \| x \|,$$
$$R_{12} = \begin{pmatrix} 0.1\sin x_3 & 0 \\ 0 & 0.1\cos x_4 \end{pmatrix},$$
$$R_{21} = \begin{pmatrix} 0.02x_1e^{-|x_1|} & 0 \\ 0 & 0.02x_2e^{-|x_2|} \end{pmatrix}.$$

Chose

$$T_1 = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad L_1 = (-1 \quad -3),$$
$$F_1 = (1 \quad 2), \quad S_1 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix},$$
$$T_2 = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}, \quad L_2 = (-5 \quad 1), \quad F_2 = (3 \quad 1),$$
$$S_2 = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}, \quad K = (-6 \quad -5), \quad Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

the solution of Lyapunov equation is $P = \begin{pmatrix} 1.1167 & 0.0833 \\ 0.0833 & 0.1167 \end{pmatrix}$. $\lambda_M[PT_1R_{12}S_2^{-1}] + \lambda_M[PT_2R_{21}S_1^{-1}] < 1$, so $(W + W^{\mathrm{T}})$ is positive definite. Design

$$u_1^0 = (1 \quad 3)\dot{x}_1 - (5 \quad 23)x_1,$$
$$u_1^1(x) = u_1^2(x) = -\operatorname{sign}(0.0833x_1 + 0.4499x_2)\| x \|,$$

$$u_2^0 = (5 \quad -1)\dot{x}_2 - (18 \quad 4)x_2,$$

$$u_2^1(x) = u_2^2(x) = \\ -\operatorname{sign}(0.5167x_3 + 0.1167x_4)\,\|x\|.$$

We can obtain the following figures of state responses, where initial values are $x_0^{(1)} = (-0.3 \quad 0.2 \quad -0.1 \quad -0.2)^{\mathrm{T}}$ and $x_0^{(2)} = (-0.6 \quad 0.2 \quad -0.1 \quad -0.2)^{\mathrm{T}}$, respectively.

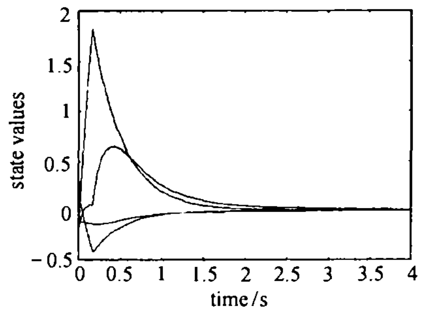

Fig. 1 State response under initial value $x_0^{(1)}$

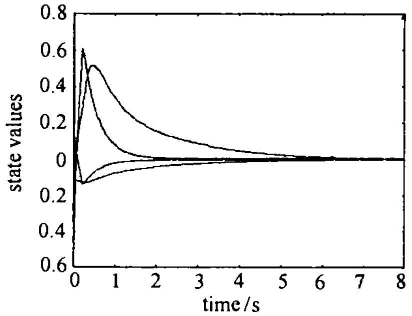

Fig. 2 State response under initial value $x_0^{(2)}$

6 Conclusion

Similar structure of uncertain generalized composite systems are defined in this paper, so the research of similarity is extended into generalized composite systems. Robust controllers possessing similar structure are designed for the controlled systems. Compare with the common control of composite system, we only need solve one Lyapunov equation, so the computation is largely simplified.

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