Article ID: 1000 - 8152(2002)03 - 0450 - 03

The Reaching Law for Variable Structure Control of Discrete Time System Based on Attenuating Control

XIAO Yanhong, ZHOU Jinglin, GE Zhaoyan and PENG Yongjin
(Institute of Electrical and Information Engineering, Hunan University · Changsha, 410082, P. R. China)

Abstract: The design of variable structure control for discrete time system using discrete reaching law is discussed. Based on the attenuating control, the attenuating variable rate and the attenuating power reaching law are proposed in this paper. The theory analysis and simulation demonstrate the effectiveness of the methods, which not only retain the asymptotic stability, but also reduce the chattering of the system.

Key words: discrete time system; variable structure control; attenuating control; reaching law
Document code: A

离散时间系统变结构控制基于衰减控制的趋近律

肖雁鸿 周靖林 葛召炎 彭永进
(湖南大学电气与信息工程学院·长沙,410082)

摘要: 讨论了利用离散趋近律设计离散时间系统的变结构控制问题,在衰减控制的基础上,提出了衰减变速趋近律和衰减幂次趋近律.理论分析和仿真结果表明,本文所提的方法是有效的,既保证了闭环系统的渐进稳定性,又大大减少了系统的抖动现象.

关键词: 离散时间系统; 变结构控制; 衰减控制; 趋近律

1 Introduction

The research of variable structure control for discrete time system is an important branch of control theory[1,2]. Almost the focus of all the research of variable structure control for discrete time system is the reaching condition. The traditional inequality reaching conditions[3] are not sufficient or too harsh which make the solution of variable structure control difficult or complicated.

Gao Weibing put forward the equation reaching condition in 1995[4], that is the discrete form of exponential reaching law:

$$s(k+1) = (1-qT)s(k) - \varepsilon T\mathrm{sgn}(s(k)), \quad (1)$$

where T is the sampling cycle, $\varepsilon > 0, q > 0, qT < 1$.

The main drawback of (1) is that when the system is in switching region, it can not reach the original point, but tends to reach a chattering close to the original point. The stationary chattering region is $2\varepsilon T/(2 - qT)$[5].

Based on the analysis of inner relation between attenuating control and discrete exponential reaching law, two kinds of new discrete reaching law: attenuating variable rate reaching law and attenuating power reaching law are proposed, which not only retain the asymptotic stability of variable structure closed loop system, but also reduce the chattering of the system.

2 Attenuating control and discrete exponential reaching law

Theorem 1 Consider the following SISO discrete time system:

$$x(k+1) = Ax(k) + bu(k), \quad (2)$$

$$s(x(k)) = C^{\mathrm{T}}x(k), \quad (3)$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}, A \in \mathbb{R}^{n\times n}, b \in \mathbb{R}^{n\times 1}, C^{\mathrm{T}} = [C_1, C_2, \cdots, C_{n-1}, 1]$. It is assumed that C satisfies the stable condition of sliding mode and A, b are completely controllable pair.

When the system (2) is controlled by the following rule:

Received date: 2001 - 05 - 28; Revised date: 2002 - 01 - 06.

$$U_r = -(C^{\mathrm{T}}b)^{-1}C^{\mathrm{T}}(A - \rho I)x(k), \ |\rho| < 1, \tag{4}$$

the system state locus will converge to the original point with the attenuating rate coefficient ρ of $s(x(k))$. U_r is called attenuating control and the attenuating coefficient is as follows:

$$\rho = s(k+1)/s(k). \tag{5}$$

The proof of Theorem 1 can be found in reference [3].

Theorem 2 Discrete exponential reaching law is the constant rate reaching law under the attenuating control.

Analyzing the control U for the discrete exponential reaching law, we conclude that U can be decomposed into two parts: $\widetilde{U}_r$ and $\widetilde{U}_d$. $\widetilde{U}_r$ is the attenuating control and $\widetilde{U}_d$ is the discrete variable structure control based on the constant rate reaching law, which only differs in a coefficient. Thus, the discrete exponential reaching law can be equivalent to the combination of attenuating control and constant rate reaching law.

For convenience, assume $0 < \tilde{\rho} = 1 - qT < 1$ and the equation (1) can be rewritten as:

$$s(k+1) = \tilde{\rho}s(k) - \varepsilon T\mathrm{sgn}(s(k)). \tag{6}$$

In no confusion cases, the equation (6) is just called constant rate reaching law based on attenuating control in the following.

Here, $\widetilde{U}_r$ enables the closed loop system to converge to original point asymptotically, but the chattering produced by $\widetilde{U}_d$ depends upon the constant rate. With the constant rate, the chattering can not be weakened and the system locus can not converge to the original point. The following attenuating variable rate reaching law and attenuating power reaching law are proposed to solve this problem.

3 Main results

Combining the attenuating control and discrete variable rate reaching law, we get the following reaching law:

$$s(k+1) = \tilde{\rho}s(k) - \varepsilon T \| x(k) \|_1 \mathrm{sgn}(s(k)), \tag{7}$$

where $\| x(k) \|_1 = \sum_{i=1}^{n} | x_i(k) |$ is the state norm, $0 < \tilde{\rho} < 1, \varepsilon > 0$.

Equation (7) is the variable rate reaching law based on attenuating control (AVR reaching law) proposed in this paper. The corresponding control is:

$$U = -(C^{\mathrm{T}}b)^{-1}[C^{\mathrm{T}}(A - \tilde{\rho}I)x(k) + \varepsilon T \| x(k) \|_1 \mathrm{sgn}(s(k))]. \tag{8}$$

Theorem 3 Let the systems (2), (3) be controlled by the equation (8) based on the attenuating variable rate reaching law (7), the system stationary switching region is $| s(k) | < \varepsilon T \| x(k) \|_1$. When $0 < \varepsilon T < \min\{(1 - c_i)/2, (1 + c_i)/2\}$ and $| c_i | < 1, i = 1, 2, \cdots, n$, the system state locus will converge to the original point asymptotically.

Similarly, adding the attenuating coefficient to the discrete power reaching law, it can be rewritten as:

$$s(k+1) = \tilde{\rho}s(k) - \varepsilon T | s(k) |^{\alpha}\mathrm{sgn}(s(k)), \tag{9}$$

where $0 < \tilde{\rho} < 1, \varepsilon > 0, 0 < \alpha < 1$.

Equation (9) is called the discrete power reaching law based on attenuating control (AP reaching law). The corresponding control is:

$$U = -(C^{\mathrm{T}}b)^{-1}[C^{\mathrm{T}}(A - \tilde{\rho}I)x(k) + \varepsilon T | s(k) |^{\alpha}\mathrm{sgn}(s(k))]. \tag{10}$$

Theorem 4 Let the systems (2), (3) be controlled by the equation (10) based on the attenuating power reaching law (9), the system stationary switching region is $2\exp(\ln(\frac{\varepsilon T}{1+\tilde{\rho}})/(1 - \alpha)$. When, $\varepsilon T < 1$, the system state locus will converge to the switching region asymptotically with faster reaching rate. The chattering can be weakened by increasing the power coefficient α.

Due to limited space, the proof of Theorems 3,4 is omitted.

Note that the stationary switching region for AVR reaching law is variable and attenuating, but the stationary switching region for AP reaching law is constant.

4 Simulation example

Consider the following simulation model[4]:

$$x(k+1) = Ax(k) + bu(k) + D(x(k)), \tag{11}$$

$$s(k) = C^{\mathrm{T}}x(k), \tag{12}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \ b = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C^{\mathrm{T}} = [-0.8 \quad 1], \ x(0) = \begin{bmatrix} 0.25 \\ 0.5 \end{bmatrix}.$$

$D(x(k))$ is the uncertain part of the system, which sat-

isfies the matching condition, that is $D(x(k)) = bdx(k)$.

Assume $\varepsilon = 0.09, q = 0.3, \alpha = 0.6, T = 1, d = 0$. The switching function and the state locus for the PCV reaching law[6] and AVR, AP reaching laws are listed in Fig. 1, Fig. 2 and Fig. 3.

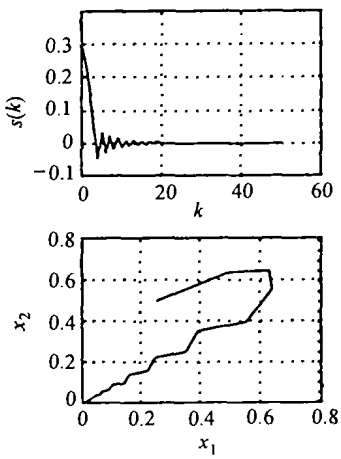

Fig. 1 PCV reaching law

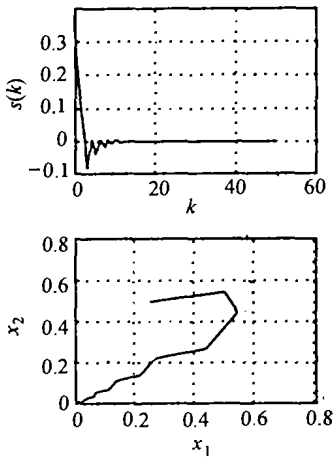

Fig. 2 AVR reaching law

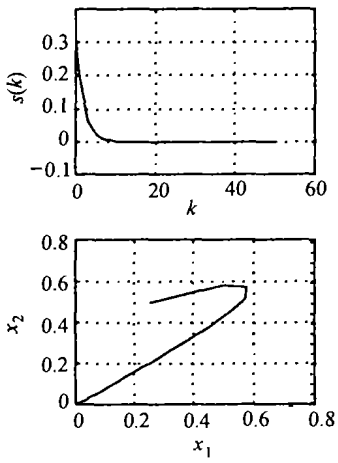

Fig. 3 AP reaching law

The simulation results demonstrate that AVR and AP reaching law not only retain the asymptotical stability of PCV reaching law, but also weaken the chattering. Moreover, the control calculation of AVR and AP reaching law are simpler than that of PCV reaching law.

When $d \neq 0$, the simulation also demonstrates that the variable structure control systems designed by AVR and AP reaching law have good robustness under certain parameter conditions.

5 Conclusion

Based on the attenuating control, two kinds of new discrete reaching laws are proposed to design discrete variable structure control system. Under certain parameter conditions, the methods retain the asymptotical stability of the system and weaken the chattering. The designed system is enabled to have good dynamic and static performance. Furthermore, these methods can be extended to the design of multi-variable discrete variable structure control system.

References

[1] Gao Weibing. Variable Structure Control Theory and Design Method [M]. Beijing: Science Press, 1996

[2] Zhai Changlian and Wu Zhiming. Variable structure control design for uncertain discrete time systems [J]. Acta Automatic Sinica, 2000, 26(2): 184 - 191

[3] Yao Qinghui, Huang Jiqi and Wu Hangshong. Variable Structure Control System [M]. Chongqing: Chongqing University Press, 1997

[4] Gao Weibing. Variable structure control of discrete-time systems [J]. Acta Automatic Sinica, 1995, 21(2): 154 - 161

[5] Bartoszewicz A. Remarks on "Discrete-time variable structure control system" [J]. IEEE Trans. on Industrial Electronics, 1996, 43(1): 235 - 238

[6] Yao Qinghui, Song Lizhong and Wen Hong. Proportional-constant-variable rate control for discrete-time variable structure systems [J]. Control and Decision, 2000, 15(3): 329 - 332

本文作者简介

肖雁鸿 1964年生. 副教授, 在职博士. 主要研究方向: 变结构控制, DEDS, 电力电子技术. Email: xiaoyanhong@21cn.com

周靖林 1976生. 硕士研究生. 主要研究方向: 变结构控制, 电力电子技术等.

葛召炎 1962年生. 高级工程师, 在读博士. 主要研究方向: 变结构控制, 微机控制, 汽车电器技术等.

彭永进 1938年生. 教授, 博士生导师. 主要研究方向: 变结构控制, DEDS, 电力电子技术等.