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Decentralized Stabilization of Large-Scale Interconnected Time-Delay Systems: an LMI Approach *

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Abstract: Decentralized stabilization conditions in the form of linear matrix inequalities (LMIs) for large-scale interconnected linear continuous systems with unknown constant delays are established under a certain interconnection decomposition. An example is given to illustrate the proposed LMI approach and to compare the obtained results with those in the literature.

Key words: large-scale interconnected systems; delays; decentralized stabilization; LMI approach

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关联时滞大系统的分散镇定:线性矩阵不等式方法

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摘要: 针对具有未知常时滞的关联大系统,在一定关联分解情况下,建立了可由线性矩阵不等式表示的分散镇定条件.文中给出了一个例子用来说明所提出的线性矩阵不等式方法并比较文献中已有结果.

关键词: 关联大系统; 时滞; 分散镇定; 线性矩阵不等式方法

1 Introduction

As we know, the main time-domain methods for stability analysis of time-delay systems are Lyapunov function method and Lyapunov functional method[1]. By Lyapunov functional method, Lee and Radovic[2,3] establish some decentralized stabilization conditions for large-scale linear time-delay systems consisting of N interconnected subsystems and including $N \times N$ constant delays. In their methods, more structure information of the interconnections are taken into account by introducing some cardinalities corresponding to the interconnections and by considering some different decomposition cases of the interconnection matrices. Recently, Hu[4] and Trinh and Aldeen[5] also use Lyapunov functional method to study the same problems for large-scale linear continuous systems with N constant delays. However, no numerical schemes for designing the stabilization controllers are proposed in the above references. In this paper, we establish decentralized stabilization conditions expressed in LMIs for large-scale interconnected linear continuous systems with $N \times N$ unknown constant delays under a certain interconnection decomposition. An example with three interconnected subsystems is given to illustrate the proposed LMI approach and to compare the obtained results with those in the literature.

2 System description and preliminaries

Let us consider a large-scale linear continuous time-delay system $\tilde{S}$ consisting of N interconnected subsystems $\tilde{S}_i, i = 1,2,\cdots,N$, as follows:

$$\tilde{S}_i:\ \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j=1}^{N} A_{ij} x_j(t - \tau_{ij}) \tag{1a}$$

with the memoryless local state feedback control law

$$u_i(t) = K_i x_i(t), \tag{1b}$$

where $x_i \in \mathbb{R}^{n_i}$ and $u_i \in \mathbb{R}^{m_i}$ denote the state and input of the subsystem $\tilde{S}_i$ with $\sum_{i=1}^{N} n_i = n$ and $\sum_{i=1}^{N} m_i = m$, A_i, B_i and A_{ij} are constant matrices with appropriate di-

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mensions, $\tau_{ij} \in [0, \tau]$ denotes $N \times N$ arbitrary unknown constant delays, $K_i \in \mathbb{R}^{m_i \times n_i}$ is the constant local controller gain matrix. Then, the closed-loop system $\hat{S}$ corresponding to system $\tilde{S}$ can be written as follows:

$$\hat{S}_i:\ \dot{x}_i(t) = (A_i + B_iK_i)x_i(t) + \sum_{j=1}^{N} A_{ij}x_j(t-\tau_{ij}) \tag{2}$$

for $i = 1,2,\cdots,N$. Assume that for each isolated delay-free subsystem: $\dot{z}_i(t) = A_iz_i(t) + B_iu_i(t)$, (A_i, B_i) is stabilizable and also A_{ij} satisfies the following decomposition[2-6]:

$$A_{ij} = B_iH_{ij} + D_{ij},\ i,j = 1,2,\cdots,N. \tag{3}$$

Here, we define some sets of indices:

$$\begin{cases} J_i(H) = \{j \mid H_{ij} \neq 0,\ j = 1,2,\cdots,N\}, \\ \bar{J}_i(H) = \{j \mid H_{ji} \neq 0,\ j = 1,2,\cdots,N\}, \\ J_i(D) = \{j \mid D_{ij} \neq 0,\ j = 1,2,\cdots,N\}, \\ \bar{J}_i(D) = \{j \mid D_{ji} \neq 0,\ j = 1,2,\cdots,N\}, \end{cases} \tag{4}$$

and let

$$\begin{cases} \tilde{N}_H(i) = k(\bar{J}_i(H)),\ \tilde{N}_D(i) = k(\bar{J}_i(D)), \\ i = 1,2,\cdots,N, \end{cases} \tag{5}$$

where $k(J)$ denotes cardinality of the set J.

Definition 1 The system $\tilde{S}$ is said to be decentrally stabilizable by the memoryless local state feedback control if every solution $x(t)$ of the corresponding closed-loop system $\hat{S}$ starting from an arbitrary initial function $\phi \in C[-\tau,0],\mathbb{R}^n)$ converges asymptotically to zero as $t \to \infty$, where $C([-\tau,0],\mathbb{R}^n)$ denotes space of continuous functions mapping $[-\tau,0]$ into $\mathbb{R}^n$ with given $\tau > 0$.

Lemma 1[7] For a given constant matrix $M \in \mathbb{R}^{n\times m}$,

$$2u^{\mathrm{T}}Mv \leqslant u^{\mathrm{T}}MG^{-1}M^{\mathrm{T}}u + v^{\mathrm{T}}Gv,\ u \in \mathbb{R}^n,\ v \in \mathbb{R}^m \tag{6}$$

holds for any symmetric and positive definite constant matrix $G \in \mathbb{R}^{m\times m}$.

3 Decentralized stabilization condition and local controllers design

In the following, for the system $\tilde{S}$ in the decomposition case (3), we establish the decentralized stabilization conditions and also provide the approach of designing the memoryless local state feedback controllers.

Theorem 1 If the following LMI problem:

$$\begin{bmatrix} A_iY_i + Y_iA_i^{\mathrm{T}} + \frac{1}{2}B_iW_i + \frac{1}{2}W_i^{\mathrm{T}}B_i^{\mathrm{T}} + (\tilde{N}_H(i) + \tilde{N}_D(i))F_i & D_{i1}Y_1 & \cdots & D_{ij}Y_j & \cdots & D_{iN}Y_N \\ Y_1D_{i1}^{\mathrm{T}} & -F_1 & & \cdots & & 0 \\ \vdots & & \ddots & & & \\ Y_jD_{ij}^{\mathrm{T}} & \vdots & & -F_j & & \vdots \\ \vdots & & & & \ddots & \\ Y_ND_{iN}^{\mathrm{T}} & 0 & & \cdots & & -F_N \end{bmatrix} < 0, \tag{7}$$

$Y_i > 0,\ F_i > 0,\ i = 1,2,\cdots,N,\ j \in J_i(D)$ in variables $Y_i \in \mathbb{R}^{n_i\times n_i}$, $F_i \in \mathbb{R}^{n_i\times n_i}$ and $W_i \in \mathbb{R}^{m_i\times n_i}$ is feasible, the system (1a) is decentrally stabilizable by the local control law (1b) with the following gain matrix

$$K_i = \frac{1}{2}(W_i - \sum_{j\in J_i(H)} H_{ij}Y_jF_j^{-1}Y_jH_{ij}^{\mathrm{T}}B_i^{\mathrm{T}})Y_i^{-1}. \tag{8}$$

Proof Let

$$\begin{cases} V(x_t) = \sum_{i=1}^{N}[x_i^{\mathrm{T}}(t)Y_i^{-1}x_i(t) + V_t(i)], \\ V_t(i) = \sum_{j\in J_i(H)}\int_{t-\tau_{ij}}^{t} x_j^{\mathrm{T}}(s)Y_j^{-1}F_jY_j^{-1}x_j(s)\mathrm{d}s + \\ \qquad \sum_{j\in J_i(D)}\int_{t-\tau_{ij}}^{t} x_j^{\mathrm{T}}(s)Y_j^{-1}F_jY_j^{-1}x_j(s)\mathrm{d}s, \end{cases} \tag{9}$$

$x_i(t) \in \mathbb{R}^{n_i}$, $x(t) \in \mathbb{R}^n$ with $n = \sum_{i=1}^{N} n_i$, $F_i = F_i^{\mathrm{T}} > 0 \in \mathbb{R}^{n_i\times n_i}$ and $Y_i = Y_i^{\mathrm{T}} > 0 \in \mathbb{R}^{n_i\times n_i}$. Along the trajectory of the closed-loop system (2) and by Lemma 1, we obtain

$$\begin{aligned} &\dot{V}(x_t) \leqslant \\ &\sum_{i=1}^{N}[x_i^{\mathrm{T}}(t)(Y_i^{-1}A_i + A_i^{\mathrm{T}}Y_i^{-1} + \\ &\frac{1}{2}Y_i^{-1}(B_iW_i + W_i^{\mathrm{T}}B_i^{\mathrm{T}})Y_i^{-1})x_i(t) + \\ &x_i^{\mathrm{T}}(t)\sum_{j\in J_i(D)} Y_i^{-1}D_{ij}Y_jF_j^{-1}Y_jD_{ij}^{\mathrm{T}}Y_i^{-1}x_i(t) + \\ &\sum_{j\in J_i(H)} x_j^{\mathrm{T}}(t-\tau_{ij})Y_j^{-1}F_jY_j^{-1}x_j(t-\tau_{ij}) + \\ &\sum_{j\in J_i(D)} x_j^{\mathrm{T}}(t-\tau_{ij})Y_j^{-1}F_jY_j^{-1}x_j(t-\tau_{ij}) + \dot{V}_t(i)]. \end{aligned} \tag{10a}$$

By $\dot{V}_t(i) =$

$$\sum_{j\in J_i(H)} x_j^{\mathrm{T}}(t)Y_j^{-1}F_jY_j^{-1}x_j(t) - \sum_{j\in \tilde{J}_i(H)} x_j^{\mathrm{T}}(t-\tau_{ij})Y_j^{-1}F_jY_j^{-1}x_j(t-\tau_{ij}) + \sum_{j\in J_i(D)} x_j^{\mathrm{T}}(t)Y_j^{-1}F_jY_j^{-1}x_j(t) - \sum_{j\in \tilde{J}_i(D)} x_j^{\mathrm{T}}(t-\tau_{ij})Y_j^{-1}F_jY_j^{-1}x_j(t-\tau_{ij}), \quad (10b)$$

and the definitions of $J_i(H), \tilde{J}_i(H), J_i(D)$, and $\tilde{J}_i(D)$, we further obtain form (10a) and (10b)

$$\dot{V}(x_t) \leqslant \sum_{i=1}^{N}[x_i^{\mathrm{T}}(t)(Y_i^{-1}A_i + A_i^{\mathrm{T}}Y_i^{-1} + \frac{1}{2}Y_i^{-1}(B_iW_i + W_i^{\mathrm{T}}B_i^{\mathrm{T}})Y_i^{-1})x_i(t) + x_i^{\mathrm{T}}(t)\sum_{j\in J_i(D)} Y_i^{-1}D_{ij}Y_jF_i^{-1}Y_jD_{ij}^{\mathrm{T}}Y_i^{-1}x_i(t) + \sum_{j\in J_i(H)} x_j^{\mathrm{T}}(t)Y_j^{-1}F_jY_j^{-1}x_j(t) + \sum_{j\in J_i(D)} x_j^{\mathrm{T}}(t)Y_j^{-1}F_jY_j^{-1}x_j(t)] =$$

$$\sum_{i=1}^{N}[x_i^{\mathrm{T}}(t)(Y_i^{-1}A_i + A_i^{\mathrm{T}}Y_i^{-1} + \frac{1}{2}Y_i^{-1}(B_iW_i + W_i^{\mathrm{T}}B_i^{\mathrm{T}})Y_i^{-1})x_i(t) + x_i^{\mathrm{T}}(t)\sum_{j\in J_i(D)} Y_i^{-1}D_{ij}Y_jF_j^{-1}Y_jD_{ij}^{\mathrm{T}}Y_i^{-1}x_i(t) + \sum_{j\in J_i(H)} x_i^{\mathrm{T}}(t)Y_j^{-1}F_iY_i^{-1}x_i(t) + \sum_{j\in J_i(D)} x_i^{\mathrm{T}}(t)Y_i^{-1}F_iY_i^{-1}x_i(t)] =$$

$$\sum_{i=1}^{N}[x_i^{\mathrm{T}}(t)Y_i^{-1}(A_iY_i + Y_iA_i^{\mathrm{T}} + \frac{1}{2}(B_iW_i + W_i^{\mathrm{T}}B_i^{\mathrm{T}}) + (\tilde{N}_H(i) + \tilde{N}_D(i))F_i + \sum_{j\in J_i(D)} D_{ij}Y_jF_j^{-1}Y_jD_{ij}^{\mathrm{T}})Y_i^{-1}x_i(t)]. \quad (11)$$

Note that the LMI problem (7) can be converted to nonlinear inequalities by using Schur complements, and thus (7) is equivalent to

$$A_iY_i + Y_iA_i^{\mathrm{T}} + \frac{1}{2}(B_iW_i + B_i^{\mathrm{T}}B_i^{\mathrm{T}}) + (\tilde{N}_H(i) + \tilde{N}_D(i))F_i + \sum_{j\in J_i(D)} D_{ij}Y_jF_j^{-1}Y_jD_{ij}^{\mathrm{T}} < 0. \quad (12)$$

Therefore, if the LMI problem (7) is feasible, then we have $\dot{V}(x_t) \leqslant \mu\sum_{i=1}^{N} \| Y_i^{-1}x_i(t)\|^2$ for all $t \geqslant 0$, where

$$\mu = \max_{i=1,\cdots,N}[\lambda_{\max}(A_iY_i + Y_iA_i^{\mathrm{T}} + \frac{1}{2}(B_iW_i + W_i^{\mathrm{T}}B_i^{\mathrm{T}}) + (\tilde{N}_H(i) + \tilde{N}_D(i))F_i + \sum_{j\in J_i(D)} D_{ij}Y_jF_j^{-1}Y_jD_{ij}^{\mathrm{T}})] < 0, \quad (13)$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue. By Theorem 2.1 in [1], we complete the proof of the theorem.

Remark 1 The LMI problem (7) is an LMIP in the matrix variables Y_i, W_i and F_i. According to [8], this LMIP is equivalent to the following LMIP with fewer matrix variables:

$$\begin{bmatrix} A_iY_i + Y_iA_i^{\mathrm{T}} - \sigma_iB_iB_i^{\mathrm{T}} + (\tilde{N}_H(i) + \tilde{N}_D(i))F_i & D_{i1}Y_1 & \cdots & D_{ij}Y_j & \cdots & D_{iN}Y_N \\ Y_1D_{i1}^{\mathrm{T}} & -F_1 & & \cdots & & 0 \\ \vdots & & \ddots & & & \\ Y_jD_{ij}^{\mathrm{T}} & \vdots & & -F_j & & \vdots \\ \vdots & & & & \ddots & \\ Y_ND_{iN}^{\mathrm{T}} & 0 & & \cdots & & -F_N \end{bmatrix} < 0 \quad (14)$$

with $Y_i > 0 \in \mathbb{R}^{n_i\times n_i}$, $F_i > 0 \in \mathbb{R}^{n_i\times n_i}$ and $\sigma_i > 0 \in \mathbb{R}^1$, where $i = 1,2,\cdots,N$ and $j \in J_i(D)$. Then, the corresponding gain matrix for i becomes

$$K_i = -\frac{1}{2}\sigma_iB_i^{\mathrm{T}}Y_i^{-1} - \frac{1}{2}\sum_{j\in J_i(H)} H_{ij}Y_jF_j^{-1}Y_jH_{ij}^{\mathrm{T}}B_i^{\mathrm{T}}Y_i^{-1}. \quad (15)$$

The feasibility problems (7) and (14) can be solved by using the MATLAB LMI Toolbox which has been used worldwide.

4 An illustrative example

Let us consider the interconnected time-delay system (1a) with $N = 3$,

$$A_1 = \begin{bmatrix} -6 & 2 & 0 \\ 0 & -7 & 0 \\ 4 & -1 & 2 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -4 & -1 \\ 1 & 1 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -2 & 2 \\ 3 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & -1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix},\ B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$H_{12} = [-2 \quad 1 \quad 2],\ D_{12} = \begin{bmatrix} 3 & -2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix},$$

$$H_{21} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 2 \end{bmatrix},\ H_{23} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$

$$H_{31} = [3 \quad -1 \quad 1],\ D_{31} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & -1 & 0 \end{bmatrix},$$

$$H_{32} = [2 \quad 2 \quad 0],\ D_{32} = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix},$$
$$H_{11} = 0,\ H_{13} = 0,\ H_{22} = 0,\ H_{33} = 0,$$
$$D_{11} = 0,\ D_{13} = 0,\ D_{21} = 0,$$
$$D_{22} = 0,\ D_{23} = 0 \text{ and } D_{33} = 0.$$

Therefore, we have $\widetilde{N}_H(1) = 2, \widetilde{N}_H(2) = 2, \widetilde{N}_H(3) = 1$, $\widetilde{N}_D(1) = 1, \widetilde{N}_D(2) = 2$ and $\widetilde{N}_D(3) = 0$.

According to Remark 1, we solve the LMI problem (14) and obtain a group of the parameter matrices as follows:

$$\sigma_1^* = 35.1465,\ \sigma_2^* = 1.6273,\ \sigma_3^* = 100.5535,$$
$$F_1^* = \begin{bmatrix} 42.2221 & -2.3047 & -23.7527 \\ -2.3047 & 44.5701 & -1.1380 \\ -23.7527 & -1.1380 & 17.9957 \end{bmatrix},$$
$$F_2^* = \begin{bmatrix} 0.6606 & 0.3109 & -0.9267 \\ 0.3109 & 9.9203 & -2.3648 \\ -0.9267 & -2.3648 & 1.7292 \end{bmatrix},$$
$$F_3^* = \begin{bmatrix} 89.2736 & -1.9528 \\ -1.9528 & 22.7085 \end{bmatrix},$$
$$Y_1^* = \begin{bmatrix} 16.5076 & 2.4084 & -7.2878 \\ 2.4084 & 16.8537 & -2.0579 \\ -7.2878 & -2.0579 & 4.8960 \end{bmatrix},$$
$$Y_2^* = \begin{bmatrix} 0.7826 & 0.9225 & -1.0235 \\ 0.9225 & 5.0666 & -2.1067 \\ -1.0235 & -2.1067 & 1.8829 \end{bmatrix},$$
$$Y_3^* = \begin{bmatrix} 19.4347 & -11.2449 \\ -11.2449 & 20.6891 \end{bmatrix}.$$

Based on the above parameter matrices and Eq. (15), we obtain the local controller gain matrices as follows

$$\begin{cases} K_1 = [-6.1625 \quad -0.8584 \quad -14.2425], \\ K_2 = \begin{bmatrix} -2.7153 & -2.8237 & -4.6520 \\ 4.4821 & -5.0178 & -5.6574 \end{bmatrix}, \\ K_3 = [-10.7905 \quad -10.3666]. \end{cases}$$

This example has also been studied in [2]. Let us consider the Frobenius norm of K given by $\| K \|_F = \{\sum_{i=1}^{N} \mathrm{tr}(K_i^{\mathrm{T}} K_i)\}^{1/2}$ to measure the size of the decentralized gain, where $K = \mathrm{blockdiag}[K_1, K_2, \cdots, K_N]$. Here, we obtained $\| K \|_F = 24.0775$ while the decentralized gains from [2] and [6] give $\| K \|_F = 26.2819$ and $\| K \|_F = 31.4519$, respectively. Thus, it turns out that a smaller decentralized gain given by our method is sufficient to stabilize the overall system.

5 Conclusion

Decentralized stabilization conditions for large-scale interconnected linear continuous systems with $N \times N$ unknown constant delays have been estabilished for a certain interconnection decomposition. The conditions are expressed in LMIs and hence they are numerically tractable. An illustrative example has been given to compare the proposed LMI approach with those in the literature.

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