文章编号：1000-8152(2002)06-0927-05

具有多状态和控制时滞的控制系统的绝对稳定性分析*

徐炳吉[1]，廖晓昕[1]，刘新芝[1,2]
（1. 华中科技大学 控制科学与工程系，武汉 430074；2. 滑铁卢大学 应用数学系，加拿大滑铁卢）

摘要：基于线性矩阵不等式方法，对具有多个状态和控制的时变时滞的 Lurie 型控制系统的稳定性进行分析，得到了系统绝对稳定的几个充分条件，这些条件用线性矩阵不等式表示，具有较低的保守性. 最后通过一个实例验证了所得条件的保守性较以往结果的保守性小.

关键词：时滞；绝对稳定性；线性矩阵不等式
中图分类号：TP13　　**文献标识码**：A

Absolute stability analysis for control systems with multiple time-varying delays in both state and control input

XU Bing-ji[1], LIAO Xiao-xin[1], LIU Xin-zhi[1,2]
(1. Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China;
2. Deptartment of Applied Mathematics, University of Waterloo, Waterloo, Canada)

Abstract: Based on the linear matrix inequality (LMI) approach, this paper analyzes the absolute stability of Lurie type control systems with multiple time-varying delays in both state and control input. Several sufficient conditions for the absolute stability of such systems are obtained. The derived sufficient conditions are expressed in terms of LMI to find the less conservative criteria. Finally, a numerical example is given to demonstrate that the derived conditions are less conservative than that given in the literature.

Key words: time delay; absolute stability; linear matrix inequality (LMI)

1 引言(Introduction)

对于具有时滞的 Lurie 型控制系统的绝对稳定性问题，许多国内外学者进行了研究，给出了很多检验系统绝对稳定的充分条件[1~6]，这些充分条件大多是用矩阵范数或矩阵测度表示的，然而矩阵范数通常使得这些条件比较保守. 下面用 Lyapunov 泛函方法对于具有多个状态和控制的时变时滞的 Lurie 型控制系统的绝对稳定性进行分析，给出几个新的系统绝对稳定的充分条件，这些条件用 LMI 表示，具有较低的保守性.

2 主要结论(Main results)

考虑如下具有多个状态和控制时变时滞的控制系统

$$
\begin{aligned}
\dot{x}(t) &= A_0x(t)+\sum_{i=1}^{l}A_ix(t-h_i(t))+b_0f_0(\sigma_0(t))+\sum_{i=1}^{m}b_if_i(\sigma_i(t-h_{i+l}(t))),\\
\sigma_i(t) &= c_i^{\mathrm{T}}x(t),\ i=0,1,\cdots,m,\\
x(t) &= \varphi(t),\ t\in[-H,0].
\end{aligned}
\tag{1}
$$

其中向量函数 $x(t)\in\mathbb{R}^n$，实矩阵 $A_i\in\mathbb{R}^{n\times n}$，$i=0,1,\cdots,l$；向量 $b_i,c_i\in\mathbb{R}^n$，$i=0,1,\cdots,m$，时变时滞 $h_i(t)$ 满足条件 $0<h_i(t)\leqslant h_i<\infty$，$\dot{h}_i(t)\leqslant d_i<1$，$i=1,2,\cdots,m+l$；$H=\max\limits_{1\leqslant i\leqslant m+l}\{h_i\}$，$f_i(\cdot)\in F_{[0,k_i]}=\{f_i(\sigma_i)\mid f_i(0)=0, 0<\sigma_if_i(\sigma_i)\leqslant k_i\sigma_i^2, \sigma_i\neq 0, f_i(\cdot)\text{ 连续}\}$，$i=0,1,\cdots,m$；$\varphi(t)$ 为连续的向量初始函数.

文中约定：$U<0$ 表示 U 为负定矩阵，$\|\cdot\|$ 为 $\mathbb{R}^n$ 中的欧氏范数，

$$\|\varphi\|_{C([-H,0])}=\sup_{\theta\in[-H,0]}\|\varphi(\theta)\|.$$

引理[7]　设 $a(s)\in\mathbb{R}^{n_x}$，$b(s)\in\mathbb{R}^{n_y}$，$s\in\Omega$，则对任意正定矩阵 $X\in\mathbb{R}^{n_x\times n_x}$ 及任意矩阵 $M\in\mathbb{R}^{n_y\times n_y}$ 下式成立

* 基金项目：国家自然科学基金(60074008)和高校博士点基金(20010487005)资助项目.
收稿日期：2001-03-19；收修改稿日期：2002-01-09.

$$-2\int_{\Omega} b^{\mathrm{T}}(s)a(s)\mathrm{d}s \leqslant$$

$$\int_{\Omega}\begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} X & XM \\ M^{\mathrm{T}}X & (2,2) \end{bmatrix}\begin{bmatrix} a(s) \\ b(s) \end{bmatrix}\mathrm{d}s.$$

其中 $(2,2) = (M^{\mathrm{T}}X + I)X^{-1}(XM + I)$.

定理 1 若存在常数 $\varepsilon > 0, \alpha_i > 0, i = 1,2,\cdots,l; \beta_j > 0, j = 1,2,\cdots,m$ 及对称矩阵 $P > 0$ 使得

$$\begin{bmatrix} A_0^{\mathrm{T}}P + PA_0 + \Big(\sum_{i=1}^{l}\alpha_i + \sum_{i=1}^{m}\beta_i k_i^2 c_i^{\mathrm{T}} c_i + \varepsilon b_0^{\mathrm{T}} b_0 k_0^2 c_0^{\mathrm{T}} c_0\Big) I_n & P & L^{\mathrm{T}} \\ P & -\varepsilon I_n & 0 \\ L & 0 & -R \end{bmatrix} < 0, \tag{2}$$

则系统(1)绝对稳定.其中

$$L^{\mathrm{T}} = [PA_1 \ \cdots \ PA_l \ \ Pb_1 \ \cdots \ Pb_m],$$

$$R = \mathrm{diag}(\alpha_1(1-d_1)I_n \ \cdots \ \alpha_l(1-d_l)I_n \ \ S),$$

$$S = \mathrm{diag}(\beta_1(1-d_{l+1}) \ \cdots \ \beta_m(1-d_{l+m})).$$

证 构造 Lyapunov 泛函

$$V = x^{\mathrm{T}}(t)Px(t) + \sum_{i=1}^{l}\alpha_i\int_{t-h_i(t)}^{t}x^{\mathrm{T}}(s)x(s)\mathrm{d}s + \sum_{i=1}^{m}\beta_i\int_{t-h_{i+l}(t)}^{t}f_i^2(\sigma_i(s))\mathrm{d}s,$$

则存在常数 $a_1 > 0, a_2 > 0$ 使得

$$a_1\|\varphi(0)\|^2 \leqslant V \leqslant a_2\|\varphi\|^2_{C([-H,0])}.$$

由向量不等式

$$2u^{\mathrm{T}}v \leqslant \varepsilon u^{\mathrm{T}}u + \frac{1}{\varepsilon}v^{\mathrm{T}}v. \tag{3}$$

其中 $u \in \mathbb{R}^n, v \in \mathbb{R}^n, \varepsilon > 0$ 为任意实数,我们有

$$\frac{\mathrm{d}V}{\mathrm{d}t}\Big|_{(1)} \leqslant$$

$$x^{\mathrm{T}}(t)[A_0^{\mathrm{T}}P + PA_0 + (\sum_{i=1}^{l}\alpha_i + \sum_{i=1}^{m}\beta_i k_i^2 c_i^{\mathrm{T}} c_i +$$

$$\varepsilon b_0^{\mathrm{T}} b_0 k_0^2 c_0^{\mathrm{T}} c_0)I_n + \frac{1}{\varepsilon}P^2]x(t) +$$

$$2\sum_{i=1}^{l}x^{\mathrm{T}}(t)PA_i x(t-h_i(t)) +$$

$$2\sum_{i=1}^{m}x^{\mathrm{T}}(t)Pb_i f_i(\sigma_i(t-h_{i+l}(t))) -$$

$$\sum_{i=1}^{l}\alpha_i(1-d_i)x^{\mathrm{T}}(t-h_i(t))x(t-h_i(t)) -$$

$$\sum_{i=1}^{m}\beta_i(1-d_{i+1})f_i^2(\sigma_i(t-h_{i+l}(t))) = y^{\mathrm{T}}(t)\Phi y(t).$$

其中

$$\Phi = \begin{bmatrix} B & L^{\mathrm{T}} \\ L & -R \end{bmatrix},$$

$$B = A_0^{\mathrm{T}}P + PA_0 + \frac{1}{\varepsilon}P^2 + (\sum_{i=1}^{l}\alpha_i + \sum_{i=1}^{m}\beta_i k_i^2 c_i^{\mathrm{T}} c_i + \varepsilon b_0^{\mathrm{T}} b_0 k_0^2 c_0^{\mathrm{T}} c_0)I_n,$$

$$y(t) = [x^{\mathrm{T}}(t) \ \ x^{\mathrm{T}}(t-h_1(t)) \ \cdots \ x^{\mathrm{T}}(t-h_l(t)) \ \ f(t)]^{\mathrm{T}},$$

$$f(t) = [f_1(\sigma_1(t-h_{l+1}(t))) \ \cdots \ f_m(\sigma_m(t-h_{l+m}(t)))].$$

由 Schur 补[8]知, $\Phi < 0$ 与 LMI(2)等价.因此当式(2)成立时,存在常数 $a > 0$ 使 $\frac{\mathrm{d}V}{\mathrm{d}t}\Big|_{(1)} \leqslant -a\|x(t)\|^2$. 由文[2]知系统(1)绝对稳定.

证毕.

定理 2 对于给定的 $\bar{H} > 0$, 若存在常数 $\alpha > 0, \alpha_i > 0, i = 1,2,\cdots,m$,对称矩阵 $P > 0, U > 0, Q_i > 0, i = 1,2,\cdots,l$ 及矩阵 W 使得

$$\begin{bmatrix} B_0 & B_1^{\mathrm{T}} & B_2^{\mathrm{T}} & \cdots & B_l^{\mathrm{T}} \\ B_1 & (d_1-1)\Lambda & & & \\ B_2 & & (d_2-I)\Lambda & & \\ \vdots & & & \ddots & \\ B_l & & & & (d_l-I)\Lambda \end{bmatrix} < 0, \tag{4}$$

则当 $H \leqslant \bar{H}$ 时系统(1)绝对稳定,其中

$$B_0 = \begin{bmatrix} \bar{A} & -W^{\mathrm{T}}A_1 & \cdots & -W^{\mathrm{T}}A_l & Pb_0 & Pb_1 & \cdots & Pb_m \\ -A_1^{\mathrm{T}}W & (d_1-1)Q_1 & & & & & & \\ \vdots & & \ddots & & & & & \\ -A_l^{\mathrm{T}}W & & & (d_l-1)Q_l & & & & \\ b_0^{\mathrm{T}}P & & & & -\alpha & & & \\ b_1^{\mathrm{T}}P & & & & & \alpha_1(d_{l+1}-1) & & \\ \vdots & & & & & & \ddots & \\ b_m^{\mathrm{T}}P & & & & & & & \alpha_m(d_{l+m}-1) \end{bmatrix},$$

$$\bar{A} = A^{\mathrm{T}}P + PA + \sum_{i=1}^{l} Q_i + W^{\mathrm{T}}\sum_{i=1}^{l} A_i + \sum_{i=1}^{l} A_i^{\mathrm{T}}W + \sum_{i=1}^{m} \alpha_i k_i^2 c_i^{\mathrm{T}} c_i I_n + \alpha c_0^{\mathrm{T}} c_0 I_n,\ A = \sum_{i=0}^{l} A_i,$$

$$B_i = \begin{bmatrix} UA_iA_0 & UA_iA_1 & \cdots & UA_iA_l & UA_ib_0 & UA_ib_1 & \cdots & UA_ib_m \\ \theta\bar{H}(W+P) & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix},\ i = 1,2,\cdots,l,$$

$$\Lambda = \mathrm{diag}(U \quad U),\ \theta = \left(\frac{1}{l}\sum_{i=1}^{l}\frac{1}{1-d_i}\right)^{-1/2}.$$

证　令 $\varphi(t) = \varphi(-H), t \in [-2H, -H]$，由 $t \geqslant h_i(t)$ 时，

$$x(t-h_i(t)) = x(t) - \int_{t-h_i(t)}^{t} \dot{x}(s)\mathrm{d}s,\ i = 1,2,\cdots,l,$$

则可由式(1)得到

$$\dot{x}(t) = Ax(t) - \sum_{i=1}^{l} A_i\int_{t-h_i(t)}^{t}\dot{x}(s)\mathrm{d}s + b_0 f_0(\sigma_0(t)) + \sum_{i=1}^{m} b_i f_i(\sigma_i(x(t-h_{i+l}(t)))), \tag{5a}$$

$$x(t) = \varphi(t), t \in [-2H, 0]. \tag{5b}$$

由文[2]知，若系统(5)渐近稳定，则系统(1)渐近稳定.因此讨论系统(1)的绝对稳定性，只需讨论系统(5)的绝对稳定性.

令 $V_1 = x^{\mathrm{T}}(t)Px(t)$，则

$$\left.\frac{\mathrm{d}V_1}{\mathrm{d}t}\right|_{(5)} = x^{\mathrm{T}}(t)[A^{\mathrm{T}}P + PA]x(t) + 2\sum_{i=1}^{m} x^{\mathrm{T}}(t)Pb_i f_i(\sigma_i(t - h_{i+l}(t))) + 2x^{\mathrm{T}}(t)Pb_0 f_0(\sigma_0(t)) - 2\sum_{i=1}^{l} x^{\mathrm{T}}(t)PA_i\int_{t-h_i(t)}^{t}\dot{x}(s)\mathrm{d}s.$$

由引理得

$$-2\sum_{i=1}^{l} x^{\mathrm{T}}(t)PA_i\int_{t-h_i(t)}^{t}\dot{x}(s)\mathrm{d}s \leqslant \sum_{i=1}^{l} h_i x^{\mathrm{T}}(t)P(M^{\mathrm{T}}X + I)X^{-1}(XM + I)Px(t) + 2\sum_{i=1}^{l} x^{\mathrm{T}}(t)PM^{\mathrm{T}}XA_i\int_{t-h_i(t)}^{t}\dot{x}(s)\mathrm{d}s + \sum_{i=1}^{l}\int_{t-h_i(t)}^{t} x^{\mathrm{T}}(s)A_i^{\mathrm{T}}XA_i x(s)\mathrm{d}s.$$

令 $W = XMP, U = HX$，则

$$\left.\frac{\mathrm{d}V_1}{\mathrm{d}t}\right|_{(5)} \leqslant x^{\mathrm{T}}(t)[A^{\mathrm{T}}P + PA + lH^2(W^{\mathrm{T}} + P)U^{-1}(W + P)]x(t) + 2\sum_{i=1}^{l} x^{\mathrm{T}}(t)W^{\mathrm{T}}A_i\int_{t-h_i(t)}^{t}\dot{x}(s)\mathrm{d}s + \frac{1}{H}\sum_{i=1}^{l}\int_{t-h_i(t)}^{t}\dot{x}^{\mathrm{T}}(s)A_i^{\mathrm{T}}UA_i\dot{x}(s)\mathrm{d}s + 2\sum_{i=1}^{m} x^{\mathrm{T}}(t)Pb_i f_i(\sigma_i(t - h_{i+l}(t))) + 2x^{\mathrm{T}}(t)Pb_0 f_0(\sigma_0(t)).$$

令

$$V_2 = \frac{1}{H}\sum_{i=1}^{l}\frac{1}{1-d_i}\int_{0}^{h_i(t)}\int_{t-s}^{t}\dot{x}^{\mathrm{T}}(\tau)A_i^{\mathrm{T}}UA_i\dot{x}(\tau)\mathrm{d}\tau\mathrm{d}s,$$

$$V_3 = \sum_{i=1}^{l}\int_{t-h_i(t)}^{t} x^{\mathrm{T}}(s)Q_i x(s)\mathrm{d}s + \sum_{i=1}^{m}\alpha_i\int_{t-h_{i+l}(t)}^{t} f_i^2(\sigma_i(s))\mathrm{d}s,$$

则

$$\frac{\mathrm{d}V_2}{\mathrm{d}t} \leqslant \sum_{i=1}^{l}\frac{1}{1-d_i}\dot{x}^{\mathrm{T}}(t)A_i^{\mathrm{T}}UA_i\dot{x}(t) - \frac{1}{H}\sum_{i=1}^{l}\int_{t-h_i(t)}^{t}\dot{x}^{\mathrm{T}}(s)A_i^{\mathrm{T}}UA_i\dot{x}(s)\mathrm{d}s,$$

$$\frac{\mathrm{d}V_3}{\mathrm{d}t} \leqslant x^{\mathrm{T}}(t)\sum_{i=1}^{l} Q_i x(t) - \sum_{i=1}^{l}(1-d_i)x^{\mathrm{T}}(t - h_i(t))Q_i x(t-h_i(t)) + \sum_{i=1}^{m}\alpha_i k_i^2 c_i^{\mathrm{T}}c_i x^{\mathrm{T}}(t)x(t) - \sum_{i=1}^{m}\alpha_i(1-\mathrm{d}_{i+l})f_i^2(\sigma_i(t - h_{i+l}(t))) + \alpha c_0^{\mathrm{T}}c_0 x^{\mathrm{T}}(t)x(t) - \alpha f_0^2(\sigma_0(t)).$$

取 Lyapunov 泛函 $V = V_1 + V_2 + V_3$，则存在常数 $\alpha_3 > 0, \alpha_4 > 0$ 使得 $\alpha_3\|\varphi(0)\|^2 \leqslant V \leqslant \alpha_4\|\varphi\|^2_{C([-2H,0])}$，当 $H \leqslant \bar{H}$ 时

$$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(5)} \leqslant x^{\mathrm{T}}(t)[\bar{A} + lH^2(W^{\mathrm{T}} + P)U^{-1}(W + P)]x(t) + 2\sum_{i=1}^{m} x^{\mathrm{T}}(t)Pb_i f_i(\sigma_i(t - h_{i+l}(t))) + 2x^{\mathrm{T}}(t)Pb_0 f_0(\sigma_0(t)) - 2\sum_{i=1}^{l} x^{\mathrm{T}}(t)W^{\mathrm{T}}A_i x(t - h_i(t)) + \sum_{i=1}^{l}\frac{1}{1-d_i}\dot{x}^{\mathrm{T}}(t)A_i^{\mathrm{T}}UA_i\dot{x}(t) - \sum_{i=1}^{l}(1-d_i)x^{\mathrm{T}}(t-h_i(t))Q_i x(t-h_i(t)) -$$

$$\sum_{i=1}^{m}\alpha_i(1-d_{i+l})f_i^2(\sigma_i(t-h_{i+l}(t)))-\alpha f_0^2(\sigma_0(t)) = y^{\mathrm T}(t)\Omega y(t).$$

其中

$$y(t) = [x^{\mathrm T}(t)\quad x^{\mathrm T}(t-h_1(t))\quad\cdots\quad x^{\mathrm T}(t-h_l(t))\quad \bar f(T)]^{\mathrm T},$$

$$\bar f(t) = [f_0(\sigma_0(t))\quad f_1(\sigma_1(t-h_{l+1}(t)))\quad \cdots\quad f_m(\sigma_m(t-h_{l+m}(t)))],$$

$$\Omega = B_0 + \sum_{i=1}^{l}\frac{1}{1-d_i}B_i^{\mathrm T}\Lambda^{-1}B_i.$$

由 Schur 补[8]知，$\Omega<0$ 与 LMI(4)等价. 因此当 $H\leqslant\bar H$ 时，若式(4)成立，则存在常数 $b>0$ 使 $\left.\frac{\mathrm dV}{\mathrm dt}\right|_{(5)}\leqslant -b\|x(t)\|^2$. 由文[2]知系统(1)绝对稳定. 证毕.

定理 3 对于给定的 $\bar H>0$，若存在常数 $\varepsilon>0$，$\lambda_j>0, j=1,2,\cdots,m$；$\alpha_i>0,\beta_i>0,\gamma_i>0,\delta_i>0$，$i=1,2,\cdots,l$，及对称矩阵 $P>0$ 使得

$$\begin{bmatrix} \Sigma & P & \Gamma & \Pi & \Pi & \Pi & \Pi \\ P & -\varepsilon I_n & & & & & \\ \Gamma^{\mathrm T} & & -\Theta_1 & & & & \\ \Pi^{\mathrm T} & & & -\Theta_2 & & & \\ \Pi^{\mathrm T} & & & & -\Theta_3 & & \\ \Pi^{\mathrm T} & & & & & -\Theta_4 & \\ \Pi^{\mathrm T} & & & & & & -\Theta_5 \end{bmatrix} < 0, \tag{6}$$

则当 $H\leqslant\bar H$ 时系统(1)绝对稳定. 其中

$$\Sigma = A^{\mathrm T}P + PA + \bar A + \mu I_n,\quad A = \sum_{i=0}^{l}A_i,$$

$$\bar A = \sum_{i=1}^{l}\alpha_i A_0^{\mathrm T}A_i^{\mathrm T}A_iA_0 + \sum_{i=1}^{l}\sum_{j=1}^{l}\frac{\beta_i}{1-d_j}A_j^{\mathrm T}A_i^{\mathrm T}A_iA_j,$$

$$\Gamma = [P\quad P\quad\cdots\quad P],\quad \Pi = [\bar HP\quad \bar HP\quad\cdots\quad \bar HP],$$

$$\Theta_1 = \mathrm{diag}(\lambda_1I_n\quad \lambda_2I_n\quad\cdots\quad\lambda_mI_n),$$

$$\Theta_2 = \mathrm{diag}((1-d_1)\alpha_1I_n\quad (1-d_2)\alpha_2I_n\quad\cdots\quad(1-d_l)\alpha_lI_n),$$

$$\Theta_3 = \mathrm{diag}\left(\frac{(1-d_1)}{l}\beta_1I_n\quad \frac{(1-d_2)}{l}\beta_2I_n\quad\cdots\quad\frac{(1-d_l)}{l}\beta_lI_n\right),$$

$$\Theta_4 = \mathrm{diag}((1-d_1)\gamma_1I_n\quad (1-d_2)\gamma_2I_n\quad\cdots\quad(1-d_l)\gamma_lI_n),$$

$$\Theta_5 = \mathrm{diag}\left(\frac{(1-d_1)}{m}\delta_1I_n\quad \frac{(1-d_2)}{m}\delta_2I_n\quad\cdots\quad\frac{(1-d_l)}{m}\delta_lI_n\right),$$

$$\mu = \sum_{i=1}^{m}\frac{\lambda_i}{1-d_{i+l}}k_i^2b_i^{\mathrm T}b_ic_i^{\mathrm T}c_i + \sum_{i=1}^{l}\sum_{j=1}^{m}\frac{\delta_i}{1-d_{j+l}}b_j^{\mathrm T}A_i^{\mathrm T}A_ib_j\,k_j^2c_j^{\mathrm T}c_j + \sum_{i=1}^{l}\gamma_ik_0^2b_0^{\mathrm T}A_i^{\mathrm T}A_ib_0c_0^{\mathrm T}c_0 + \varepsilon b_0^{\mathrm T}b_0k_0^2c_0^{\mathrm T}c_0.$$

证 令

$$\varphi(t)=\varphi(-H), t\in[-2H,-H],\quad V_1 = x^{\mathrm T}(t)Px(t),$$

$$V_2 = \frac1H\sum_{i=1}^{l}\alpha_i\int_0^{h_i(t)}\int_{t-s}^{t}x^{\mathrm T}(\tau)A_0^{\mathrm T}A_i^{\mathrm T}A_iA_0x(\tau)\mathrm d\tau\mathrm ds + \frac1H\sum_{i=1}^{l}\sum_{j=1}^{l}\beta_i\int_0^{h_i(t)}\int_{t-s}^{t}x^{\mathrm T}(\tau-h_j(\tau))A_j^{\mathrm T}A_i^{\mathrm T}A_iA_jx(\tau-h_j(\tau))\mathrm d\tau\mathrm ds + \frac1H\sum_{i=1}^{l}\gamma_ik_0^2b_0^{\mathrm T}A_i^{\mathrm T}A_ib_0c_0^{\mathrm T}c_0\int_0^{h_i(t)}\int_{t-s}^{t}x^{\mathrm T}(\tau)x(\tau))\mathrm d\tau\mathrm ds + \frac1H\sum_{i=1}^{l}\sum_{j=1}^{m}\delta_ib_j^{\mathrm T}A_i^{\mathrm T}A_ib_jk_j^2c_j^{\mathrm T}c_j\int_0^{h_i(t)}\int_{t-s}^{t}x^{\mathrm T}(\tau-h_{j+l}(\tau))x(\tau-h_{j+l}(\tau))\mathrm d\tau\mathrm ds,$$

$$V_3 = \sum_{i=1}^{l}\sum_{j=1}^{l}\frac{\beta_i}{1-d_j}\int_{t-h_j(t)}^{t}x^{\mathrm T}(s)A_j^{\mathrm T}A_i^{\mathrm T}A_iA_jx(s)\mathrm ds + \sum_{i=1}^{l}\sum_{j=1}^{m}\frac{\delta_i}{1-d_{j+l}}b_j^{\mathrm T}A_i^{\mathrm T}A_ib_jk_j^2c_j^{\mathrm T}c_j\int_{t-h_{j+l}(t)}^{t}x^{\mathrm T}(s)x(s)\mathrm ds + \sum_{i=1}^{m}\frac{\lambda_i}{1-d_{i+l}}k_i^2b_i^{\mathrm T}b_ic_i^{\mathrm T}c_i\int_{t-h_{j+l}(t)}^{t}x^{\mathrm T}(s)x(s)\mathrm ds.$$

取 Lyapunov 泛函 $V=V_1+V_2+V_3$，则存在常数 $a_5>0, a_6>0$ 使得 $a_5\|\varphi(0)\|^2\leqslant V\leqslant a_6\|\varphi\|^2_{C([-2H,0])}$.

类似于定理 1 的证明易得

$$\left.\frac{\mathrm dV}{\mathrm dt}\right|_{(5)}\leqslant x^{\mathrm T}(t)\Big[A^{\mathrm T}P + PA + \Big(\frac1\varepsilon + \sum_{i=1}^{m}\frac{1}{\lambda_i}\Big)P^2 + \sum_{i=1}^{l}\Big(\frac{1}{\alpha_i}+\frac{l}{\beta_i}+\frac{1}{\gamma_i}+\frac{m}{\delta_i}\Big)\frac{H^2}{1-d_i}P^2\Big]x(t) + x^{\mathrm T}(t)\Big[\sum_{i=1}^{l}\alpha_iA_0^{\mathrm T}A_i^{\mathrm T}A_iA_0 + \sum_{i=1}^{l}\sum_{j=1}^{l}\frac{\beta_i}{1-d_j}A_j^{\mathrm T}A_i^{\mathrm T}A_iA_j\Big]x(t) + \Big(\sum_{i=1}^{m}\frac{\lambda_i}{1-d_{i+l}}k_i^2b_i^{\mathrm T}b_ic_i^{\mathrm T}c_i + \sum_{i=1}^{l}\sum_{j=1}^{m}\frac{\delta_i}{1-d_{j+l}}b_j^{\mathrm T}A_i^{\mathrm T}A_ib_j\,k_j^2c_j^{\mathrm T}c_j\Big)x^{\mathrm T}(t)x(t) + \Big(\sum_{i=1}^{l}\gamma_ik_0^2b_0^{\mathrm T}A_i^{\mathrm T}A_ib_0c_0^{\mathrm T}c_0 + \varepsilon b_0^{\mathrm T}b_0k_0^2c_0^{\mathrm T}c_0\Big)x^{\mathrm T}(t)x(t) =$$

$x^{\mathrm{T}}(t)[A^{\mathrm{T}}P+PA+\bar{A}+\mu I_n+\xi P^2+H^2\eta P^2]x(t)$.

其中

$$\xi=\frac{1}{\epsilon}+\sum_{i=1}^{m}\frac{1}{\lambda_i},$$

$$\eta=\sum_{i=1}^{l}\left(\frac{1}{\alpha_i}+\frac{l}{\beta_i}+\frac{1}{\gamma_i}+\frac{m}{\delta_i}\right)\frac{1}{1-d_i}.$$

当 $H\leqslant\bar{H}$ 时, $\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(5)}\leqslant x^{\mathrm{T}}(t)[A^{\mathrm{T}}P+PA+\bar{A}+\mu I_n+\xi P^2+\bar{H}^2\eta P^2]x(t)$. 由 Schur 补[8]知, $A^{\mathrm{T}}P+PA+\bar{A}+\mu I_n+\xi P^2+\bar{H}^2\eta P^2<0$ 与LMI(6)等价.因此当 $H\leqslant\bar{H}$ 时,若式(6)成立,则存在常数 $\theta>0$ 使 $\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{(5)}\leqslant-\theta\|x(t)\|^2$. 由文[2]知系统(1)绝对稳定. 证毕.

3 仿真算例(Simulation example)

考虑如下时滞系统

$$\begin{cases}\dot{x}(t)=A_0x(t)+A_1x(t-\tau)+b_1f(\sigma_1(t-\tau)),\\ \sigma_1(t)=c_1^{\mathrm{T}}x(t),\ f\in F_{[0,0.5]},\\ x(t)=\varphi(t),\ t\in[-H,0].\end{cases}\tag{7}$$

其中

$$A_0=\begin{bmatrix}-2&0\\0&-1\end{bmatrix},\ A_1=\begin{bmatrix}0&0.5\\0.5&0\end{bmatrix},$$

$$b_1=\begin{bmatrix}0.5\\0.5\end{bmatrix},\ c_1=\begin{bmatrix}1\\-1\end{bmatrix}.$$

对于此系统,由文[6]的判据均不能判定其绝对稳定性,但存在 $\alpha=\beta=1$, 及

$$P=\begin{bmatrix}2.1821&0\\0&1.7813\end{bmatrix}$$

使得LMI(2)成立,因此由定理1知,系统(7)绝对稳定.由定理2及定理3可验证,当 $H\leqslant0.69$ 时系统(7)绝对稳定.

4 结论(Conclusion)

对具有多个状态和控制的时变时滞的Lurie型控制系统,本文基于线性矩阵不等式方法对其稳定性进行分析,得到了系统绝对稳定的几个充分条件,这些条件用线性矩阵不等式表示,具有较低的保守性,较黎卡提方程方法有很大的优越性.

参考文献(References)

[1] Somo Lines A. Stability of Lurie-type functional equation [J]. J. Diff. Eqs., 1977, 26(2): 191-199

[2] Hale J K. Theory of Functional Differential Equation [M]. New York: Springer-Verlag, 1977

[3] Liao Xiaoxin. Absolute Stability of Nonlinear Control Systems [M]. Hong kong: Kluwer Academic Publishers, 1993

[4] Nian Xiaohong. Absolute stability of Lurie control systems with time delay [J]. Journal of Northwest Normal University (Natural Science), 1997, 33(1): 9-14 (in Chinese)

[5] Nian Xiaohong. Delay dependent conditions for absolute stability of Lurie type control systems [J]. Acta Automatica Sinica, 1999, 25(4): 564-566 (in Chinese)

[6] Liu Zurun, Nian Xiaohong. Absolute stability of control systems with time-delay in both state and control input [J]. Journal of Systems Engineering, 2000, 15(1): 103-106 (in Chinese)

[7] Park P G. A delay-dependent stability criterion for systems with uncertain time-invariant delays [J]. IEEE Trans. Automatic Control, 1999, 44(4): 876-877

[8] Boyd S, Ghaoui L E I, Feron E, et al. Linear matrix inequalities in system and control theory [A]. Studies in Applied Mathematics [M]. Philadelphia: SIAM, 1994, 15

本文作者简介

徐炳吉 副教授.博士生.主要研究方向为非线性控制及神经网络等.

廖晓昕 教授.博士生导师.主要研究方向为神经网络及非线性动力系统的稳定性分析与控制等.

刘新芝 教授.博士生导师."长江学者奖励计划"特聘教授.主要研究方向为大型动力系统的稳定性理论,混合动力系统的理论与应用,脉冲系统的定性分析等.