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Robust H-stability of Hopfield neural networks with impulsive effects and design of impulsive controllers

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Abstract: This paper studies the robust H-stability (e.g. in the sense of Hopfield) for Hopfield neural networks (HNN for short) with impulsive effects. By employing the method of Lyapunov functions and Riccati inequality, some sufficient conditions for robust H-stability and robustly asymptotical H-stability are established. On the basis of these results, the author also designs some impulsive controllers to stabilize HNN. Finally, one illustrative example is given.

Key words: Hopfield neural networks; impulse; robust H-stability; Riccati inequality

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脉冲 Hopfield 神经网络的鲁棒 H-稳定性及其脉冲控制器设计

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摘要: 研究了脉冲 Hopfield 神经网络在 Hopfield 意义下的鲁棒稳定性.通过应用 Lyapunov 函数法和 Riccati 不等式方法,得到了脉冲 Hopfield 神经网络鲁棒稳定和鲁棒渐近稳定的充分条件,在此基础上,设计出了易于实施的脉冲控制器来镇定 Hopfield 神经网络.最后,给出了例子.

关键词: Hopfield 神经网络; 脉冲; 鲁棒 H-稳定性; Riccati 不等式

1 Introduction

Hopfield neural networks (HNN)[1,2] has been extensively studied in recent years. There are many stability results of HNN in literatures such as [3 ~ 6]. HNN has also been applied to associative memory, model identification, optimization problems, etc. HNN used today can be classified into two kinds: continuous HNN and discrete HNN. However, sudden and sharp changes occurred instantaneously in this kind of neural network systems can not be well described by using pure continuous HNN or pure discrete HNN models. Therefore, it is important and, in effect, necessary to introduce a new type of neural networks. That is neural networks with impulsive effects, which can be used as an appropriate description of these phenomena of abrupt qualitative dynamical changes of essentially continuous time systems. Literatures [7 ~ 11] have established a series of stability results for impulsive systems. Literatures [12, 13] have discussed the measure-type impulsive neural networks. This paper aims to study the stability of HNN with impulsive effects.

2 Problem formulation

The HNN with impulsive effects considered in this paper can be described:

$$\begin{cases} C_i \dfrac{\mathrm{d}u_i}{\mathrm{d}t} = -\dfrac{u_i}{R_i} + \sum\limits_{j=1}^{n} T_{ij} g_j(u_j) + I_i, \\ \qquad i = 1,2,\cdots,n,\ t_k < t \leqslant t_{k+1}, \\ \Delta u(t_k) = u(t_k^+) - u(t_k) = \phi_k(u(t_k)),\ t = t_k, \end{cases} \tag{1}$$

where the impulsive time instances $\{t_j\}$ satisfy $0 < t_0 < t_1 < \cdots < \infty, \lim\limits_{t\to\infty} t_j = \infty, \phi_k \in C[\mathbb{R}^n, \mathbb{R}^n]$.

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Let E the set of all equilibrium for system (1), e.g.

$$E = \{u : \frac{u_i}{R_i} = \sum_{j=1}^{n} T_{ij}V_j + I_i,\ \phi_k(u) = 0,$$
$$i = 1,2,\cdots,n,\ k = 1,2,\cdots.\}.$$

Denote $\rho(u, E) \overset{\text{def}}{=\!=} \inf_{x \in E} \{ \| u - x \| \}$ as the distance from u to set E.

Definition 2.1[6] System (1) is said to be H-stable (e.g. stable in the sense of Hopfield) if for any $\delta > 0$, and $u_0 \in S_\delta \overset{\text{def}}{=\!=} \{u : \| u \| \leqslant \delta\}$, the solution $u(t, t_0, u_0)$ of (1) satisfies

$$\rho(u(t, t_0, u_0), E) \to 0, \text{ when } t \to \infty.$$

Remark 2.1 In effect, from the Definition 2.1, the H-stability of (1) is the attraction of set E. Hence, the H-stability is different from the L-stability (e.g. stability in the sense of Lyapunov). But it has been applied to many aspects such as associative memory, model identification, optimization problems, etc. So it's significant to discuss the H-stability for HNN or HNN with impulsive effects.

Remark 2.2 If the author considers the relation between H-stability and L-stability for HNN with impulsive effects, and let $x_i = \frac{du_i}{dt}$, $i = 1,2,\cdots,n$, then (1) can be formulated as follows:

$$\begin{cases} \dfrac{dx_i}{dt} = -\dfrac{x_i}{C_iR_i} + \sum\limits_{j=1}^{n} \dfrac{T_{ij}}{C_i} g_j'(u_j)x_j, \\ \qquad i = 1,2,\cdots,n,\ t_k < t \leqslant t_{k+1}, \\ \Delta x(t_k) = x(t_k^+) - x(t_k) = \varphi_k(x(t_k)), \\ \qquad k = 1,2,\cdots,\ x(t_0^+) = x_0. \end{cases} \tag{2}$$

where $\varphi_k \in C[\mathbb{R}^n, \mathbb{R}^n]$, φ_k can be determined by transformation $x_i = \frac{du_i}{dt}$ and functions ϕ_k, $k = 1,2,\cdots$, and $\varphi_k(0) = 0$.

It is not very difficult to see the L-stability of (2) implies the H-stability of (1). Hence, the author just needs to pay the attention to L-stability of (2).

Because the neural cell functions $g_i(i = 1,2,\cdots,n)$ are nonlinear, it is difficult to obtain their formulations. Suppose that the author just knows their boundary.

Let $| g_i(u_i) | \leqslant \eta_i$, η_i be positive constants, $i = 1, 2,\cdots,n$. Considering this case, from [6], the author knows system (1) is dissipative in the following set.

$$\Omega =$$
$$\Big\{u : \sum_{i=1}^{n} \frac{1}{R_i}\Big[|u_i| - \frac{1}{2}\Big(\sum_{j=1}^{n} |T_{ij}| \eta_j R_i + |I_i| R_i\Big)\Big]^2 \leqslant$$
$$\sum_{i=1}^{n} \frac{1}{R_i}\Big[\frac{1}{2}\Big(\sum_{j=1}^{n} |T_{ij}| \eta_j R_i + |I_i| R_i\Big)\Big]^2\Big\}.$$

In the following, it just needs to consider the stability problem of (2) in the region Ω. Denote

$$p_{ij} \overset{\text{def}}{=\!=} \inf_{u_j \in \Omega} \Big\{\frac{T_{ij}}{C_i} g_j'(u_j)\Big\},$$

$$q_{ij} \overset{\text{def}}{=\!=} \sup_{u_j \in \Omega} \Big\{\frac{T_{ij}}{C_i} g_j'(u_j)\Big\},$$

and

$$A \in N[P, Q] =$$
$$\{(a_{ij}) \in \mathbb{R}^{n \times n} : p_{ij} \leqslant a_{ij} \leqslant q_{ij}, i, j = 1,2,\cdots,n\},$$

where $P = (p_{ij})_{n \times n}$, $Q = (q_{ij})_{n \times n} \in \mathbb{R}^{n \times n}$. By Lemma 2.1 in [14], $A = A_0 + E\Sigma F$, where

$$A_0 = \frac{1}{2}(P + Q),$$

$$H = (h_{ij})_{n \times n} = \frac{1}{2}(Q - P) \geqslant 0,$$

$$\Sigma \in \Sigma^* =$$
$$\{\Sigma \in \mathbb{R}^{n^2 \times n^2} : \Sigma = \text{diag}(\varepsilon_{11}, \cdots, \varepsilon_{1n}, \cdots, \varepsilon_{n1}, \cdots, \varepsilon_{nn}),$$
$$| \varepsilon_{ij} | \leqslant 1; i, j = 1,2,\cdots,n\},$$

$$E \cdot E^{\mathrm{T}} = \text{diag}\Big\{\sum_{j=1}^{n} h_{1j}, \sum_{j=1}^{n} h_{2j}, \cdots, \sum_{j=1}^{n} h_{nj}\Big\},$$

$$F^{\mathrm{T}} \cdot F = \text{diag}\Big\{\sum_{j=1}^{n} h_{j1}, \sum_{j=1}^{n} h_{j2}, \cdots, \sum_{j=1}^{n} h_{jn}\Big\}.$$

Hence, the system (2) becomes an uncertain HNN with impulsive effects:

$$\begin{aligned} &\frac{dx}{dt} = (D + A_0 + E\Sigma F)x,\ t_k < t \leqslant t_{k+1}, \\ &\Delta x(t_k) = x(t_k^+) - x(t_k) = \varphi_k(x(t_k)), \\ &k = 1,2,\cdots,\ x(t_0^+) = x_0. \end{aligned} \tag{3}$$

where matrix

$$D = \text{diag}\Big\{-\frac{1}{C_1R_1}, -\frac{1}{C_2R_2}, \cdots, -\frac{1}{C_nR_n}\Big\}.$$

Definition 2.2 The HNN (1) with impulsive effects is called robustly H-stable, robustly asymptotically H-stable if for any $A \in N\ [P, Q]$, the equilibrium $x = 0$ of (3) is L-stable, asymptotically L-stable, respectively.

Definition 2.3 Let $V : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^+$, then V is said to belong to class v_0 if

1) V is continuous in $(t_k, t_{k+1}] \times \mathbb{R}^n$ and for each x

$\in \mathbb{R}^n, k = 1,2,\cdots, \lim_{(t,y)\to(t_k^+,x)} V(t,y) = V(t_k^+,x)$ exists;

2) $V(t,x)$ is locally Lipschitzian in x.

The author denotes by K the class of functions $\phi:\mathbb{R}^+\to\mathbb{R}^+$ which are continuous, strictly increasing and $\phi(0) = 0$, K_0 the class of continuous functions $\Psi:\mathbb{R}^+\to\mathbb{R}^+$ such that $\Psi(s) = 0$ if and only if $s = 0$, and PC the class of functions $\varphi:\mathbb{R}^+\to\mathbb{R}^+$, where φ is continuous everywhere except $t_k(k = 1,2,\cdots)$ at which φ is left continuous and the right limit $\varphi(t_k^+)$ exists.

3 Main results

3.1 Robust H-Stability for HNN with impulsive effects

Lemma 3.1 Let $\Sigma \in \Sigma^*$. Then for any positive constant λ and for any $\xi \in \mathbb{R}^{n^2}$, $\eta \in \mathbb{R}^{n^2}$, the inequality holds:

$$2\xi^{\mathrm{T}}\Sigma\eta \leqslant \lambda^{-1}\xi^{\mathrm{T}}\xi + \lambda\eta^{\mathrm{T}}\eta. \tag{4}$$

Proof Using the Schwartz inequality and $\Sigma\cdot\Sigma^{\mathrm{T}} = \Sigma^{\mathrm{T}}\cdot\Sigma \leqslant I$, The authar can easily get (4).

Theorem 3.1 Assume that there exist constants $\mu > 0$, α and a positive definite matrix X such that

1) there exists a ρ_0 with $0 < \rho_0 < \rho$ such that $x \in S_{\rho_0}$ implies that $x + I_k(x) \in S_\rho$ for all $k \in \mathbb{N}$.

2) the Riccati inequality holds:

$$X(D+A_0)+(D+A_0)^{\mathrm{T}}X+\mu^{-1}XEE^{\mathrm{T}}X+\mu F^{\mathrm{T}}F \leqslant \alpha X, \tag{5}$$

3) $$(x_k + \varphi_k(x_k))^{\mathrm{T}}X(x_k + \varphi_k(x_k)) \leqslant \Psi_k(x_k^{\mathrm{T}}Xx_k), \tag{6}$$

where $\Psi_k \in K_0$, $x_k = x(t_k)$, $k \in \mathbb{N}$.

4) there exist constants $\sigma > 0$, γ_k, $k \in \mathbb{N}$, such that for all $z \in (0,\sigma)$

$$\sup_{z\in(0,\sigma)}\left\{\alpha(t_{k+1} - t_k) + \ln\frac{\Psi_k(z)}{z}\right\} \leqslant -\gamma_k. \tag{7}$$

Then system (3) is robustly stable in the sense of Lyapunov if $\gamma_k \geqslant 0$ for all $k \in \mathbb{N}$, and asymptotically stable if in addition $\sum_{k=1}^{n}\gamma_k = +\infty$.

Proof To prove this theorem, the author only needs to testify that all the conditions of Theorem 2.1 or Theorem 2.2 in [10] are satisfied. Let $V(t,x) = x^{\mathrm{T}}Xx$. Clearly, V belongs to V_0 and

$$\lambda_{\min}(X)\cdot\|x\|^2 \leqslant V \leqslant \lambda_{\max}(X)\cdot\|x\|^2, \tag{8}$$

where $\lambda_{\min}(X)$, $\lambda_{\max}(X)$ is the minimum and maximum eigenvalue of matrix X.

Let $a(s) = \lambda_{\max}(X)\cdot s^2$ and $b(s) = \lambda_{\min}(X)\cdot s^2$, $s \in \mathbb{R}$, then $a, b \in K$, such that

$$b(\|x\|) \leqslant V(t,x) \leqslant a(\|x\|), \tag{9}$$

where $(t,x) \in \mathbb{R}^+\times S_\rho$.

Using Lemma 3.1 and condition 2), for some $\mu > 0$ and any $t \in (t_k, t_{k+1}]$, the author gets

$$\begin{aligned}&D^+V(t,x) = \\ &x^{\mathrm{T}}((A_0 + D)^{\mathrm{T}}X + X(A_0 + D))x + 2x^{\mathrm{T}}XE\Sigma Fx \leqslant \\ &x^{\mathrm{T}}((A_0 + D)^{\mathrm{T}}X + X(A_0 + D) + \\ &\mu^{-1}XEE^{\mathrm{T}}X + \mu F^{\mathrm{T}}F)x \leqslant \\ &\alpha\cdot x^{\mathrm{T}}Xx = \alpha\cdot V(t,x).\end{aligned} \tag{10}$$

Hence, if $\alpha \geqslant 0$, then by Theorem 2.1 in [10] and the condition 3) and 4), the author can conclude that this theorem is true. If $\alpha < 0$, then by Theorem 2.2 in [10] and the condition 3) and 4), then this theorem is true.

In the following, the author considers the HNN with linear impulsive effects. The corresponding systems can be described as:

$$\begin{cases}\dfrac{\mathrm{d}x}{\mathrm{d}t} = (D + A_0 + E\Sigma F)x,\ t_k < t \leqslant t_{k+1}, \\ \Delta x(t_k) = B_k\cdot x(t_k),\ k = 1,2,\cdots,\ x(t_0^+) = x_0.\end{cases} \tag{11}$$

where matrices $B_k \in \mathbb{R}^{n\times n}$, $k \in \mathbb{N}$ all are constant matrices.

In order to obtain the robust stability results for (11), the author first introduces the following lemma.

Lemma 3.2 Let $X \in \mathbb{R}^{n\times n}$ be a positive definite matrix and $Y \in \mathbb{R}^{n\times n}$ a symmetric matrix. Then for any $x \in \mathbb{R}^n$, the inequality holds:

$$x^{\mathrm{T}}Yx \leqslant \lambda_{\max}(X^{-1}Y)\cdot x^{\mathrm{T}}Xx. \tag{12}$$

Proof Since X is a positive definite matrix, there exists a nonsingular matrix C such that $X = C^{\mathrm{T}}C$. Denote $J = \dfrac{x^{\mathrm{T}}Yx}{x^{\mathrm{T}}Xx}$. Making a nonsingular transformation $z = Cx$, then

$$J = \frac{z^{\mathrm{T}}(C^{-1})^{\mathrm{T}}Y(C^{-1})z}{z^{\mathrm{T}}z} \leqslant \lambda_{\max}((C^{-1})^{\mathrm{T}}Y(C^{-1})). \tag{13}$$

Moreover

$$\begin{aligned}&(C^{-1})^{\mathrm{T}}Y(C^{-1}) = C\cdot C^{-1}\cdot(C^{-1})^{\mathrm{T}}YC^{-1} = \\ &C\cdot X^{-1}Y\cdot C^{-1},\end{aligned}$$

hence,

$$\lambda_{\max}(X^{-1}Y) = \lambda_{\max}((C^{-1})^{\mathrm{T}}Y(C^{-1})). \quad (14)$$

By (13) and (14), the author knows (12) is true.

Theorem 3.2 Assume that the assumptions 1), 2) of Theorem 3.1 hold. Suppose further that

5) denote $\beta_k = \lambda_{\max}(X^{-1}(1 + B_k)^{\mathrm{T}}X(I + B_k))$, there exist some γ_k such that

$$\alpha(t_{k+1} - t_k) + \ln\beta_k \leqslant -\gamma_k,\ k \in \mathbb{N}. \quad (15)$$

where I is the identity matrix.

Then system (11) is robustly stable in the sense of Lyapunov if $\gamma_k \geqslant 0$ for all $k \in \mathbb{N}$, and asymptotically stable if in addition $\sum_{k=1}^{n}\gamma_k = +\infty$.

Proof Let $V(t,x) = x^{\mathrm{T}}Xx$. Then, from the condition 2), the (10) still holds. Moreover, by using Lemma 3.2 and 5), the author gets

$$V(t_k^+, x_k^+) = x_k^{\mathrm{T}}(I + B_k)^{\mathrm{T}}X(I + B_k)x_k \leqslant \beta_k \cdot x_x^{\mathrm{T}}Xx_k = \beta_k \cdot V(t_k, x_k). \quad (16)$$

The following proof is similar to that of Theorem 3.1, the author omits it here.

The above results can also be used to HNN with no impulsive effects. the author gives the following corollary, which is not difficult to be proved by using Theorem 3.1 and theorems in [15].

Corollary 3.1 Assume one of the following conditions holds,

1) there exist constants $\mu > 0$ and $\alpha > 0$ and a positive definite matrix X such that the Riccati inequality holds:

$$X(D + A_0) + (D + A_0)^{\mathrm{T}}X + \mu^{-1}XEE^{\mathrm{T}}X + \mu F^{\mathrm{T}}F \leqslant -\alpha X. \quad (17)$$

2) there exist constants $\mu > 0$ and $\varepsilon > 0$ such that there is only a positive definite matrix X satisfying the Riccati equation

$$X(D + A_0) + (D + A_0)^{\mathrm{T}}X + \mu^{-1}XEE^{\mathrm{T}}X + \mu F^{\mathrm{T}}F + \varepsilon I = 0. \quad (18)$$

3) $D + A_0$ is Hurwitz matrix and satisfies

$$\| F(sI - D - A_0)^{-1}E \|_{\infty} < 1. \quad (19)$$

Then the HNN (2) with no impulsive effects is robustly asymptotically L-stable and (1) is robustly asymptotically H-stable.

3.2 Impulsive controller design for HNN

By approaching Theorem 3.2, the author can design the impulsive controller $\Delta x(t_k) = B_k x_k$ for HNN so that the HNN is robustly H-stable and robustly asymptotically H-stable.

Theorem 3.3 Assume that α is the smallest real number satisfying the Riccati inequality

$$(D + A_0)^{\mathrm{T}} + (D + A_0) + EE^{\mathrm{T}} + F^{\mathrm{T}}F \leqslant \alpha I. \quad (20)$$

Then, the impulsive controller $\Delta x(t_k) = B_k x_k$ that ensures the HNN (1) robust H-stability (robustly asymptotical H-stability, in this case: $\sum_{k=1}^{n}\gamma_k = +\infty$) can be designed according to the following laws:

$$\max\{|1 + \lambda_{\min}(B_k)|, |1 + \lambda_{\max}(B_k)|\} \leqslant e^{-\frac{1}{2}(\alpha(t_{k+1}-t_k)+\gamma_k)},\ \gamma_k \geqslant 0,\ k \in \mathbb{N}. \quad (21)$$

Proof In the Theorem 3.2, let $X = I$, then the author can get the results.

In order to design and use conveniently, let

$$B_k = \mathrm{diag}(d_{k1}, d_{k2}, \cdots, d_{kn}),\ k \in \mathbb{N}.$$

That is to say, the author simply sets $\Delta x_i(t_k^+) = d_{ki}x_i(t_k), i = 1,2,\cdots,n$, and can ensure the H-stability for HNN (1). From Theorem 3.3, The author can get the following convenient impulsive controller for HNN (1).

Corollary 3.2 Suppose (20) is satisfied. If

$$|1 + d_{ki}| \leqslant e^{-\frac{1}{2}(\alpha(t_{k+1}-t_k)+\gamma_k)},\quad \gamma_k \geqslant 0,\ i = 1,2,\cdots,n,\ k \in \mathbb{N}, \quad (22)$$

then impulsive controller $\Delta x_i(t_k^+) = d_{ki}x_i(t_k), i = 1, 2,\cdots,n$, can ensure the robust H-stability for HNN (1) and robustly asymptotical H-stability for HNN (1) if in addition $\sum_{k=1}^{n}\gamma_k = +\infty$.

4 Example

Consider a HNN:

$$\begin{cases} C_1\dfrac{\mathrm{d}u_1}{\mathrm{d}t} = -\dfrac{1}{R_1}u_1 + T_{11}g_1(u_1) + T_{12}g_2(u_2) + I_1, \\ C_2\dfrac{\mathrm{d}u_2}{\mathrm{d}t} = -\dfrac{1}{R_2}u_2 + T_{21}g_1(u_1) + T_{22}g_2(u_2) + I_2. \end{cases} \quad (23)$$

where

$$C_i = \frac{1}{2},\ R_i = \frac{1}{3},\ i = 1,2;$$

$$q_{ii} = 3,\ p_{ii} = -1,\ i = 1,2;$$

$$p_{12} = -3.5,\ q_{12} = 3,\ p_{21} = -3,\ q_{21} = 4,$$

then

$$(D + A_0)^{\mathrm{T}} + (D + A_0) + EE^{\mathrm{T}} + F^{\mathrm{T}}F = \begin{pmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{pmatrix}$$

is a positive definite matrix.

Now the author designs the impulsive controller $\Delta x_i(t_k^+) = d_{ki}x_i(t_k), i = 1,2,\cdots,n$, for (23) by using Corollary 3.2. In this case, it is easy to get $\alpha = 1$ and for any $\gamma_k \geqslant 0$, let

$$-1 - \mathrm{e}^{-\frac{1}{2}(t_{k+1}-t_k+\gamma_k)} \leqslant d_{ki} \leqslant -1 + \mathrm{e}^{-\frac{1}{2}(t_{k+1}-t_k+\gamma_k)},$$
$$i = 1,2,\cdots,n, \tag{24}$$

then (24) can guarantee the robust H-stability of (23) with impulsive effects $\Delta x(t_k^+) = B_k x(t_k)$, where

$$B_k = \mathrm{diag}\,(d_{k1}, d_{k2}, \cdots, d_{kn}), k \in \mathbb{N}.$$

If, in addition, $\sum_{k=1}^{n} \gamma_k = +\infty$, then, (24) can also guarantee the robustly asymptotical H-stability of (23).

5 Conclusions

In this paper, the author has formulated and studied the Hopfield neural networks with impulsive effects. By establishing the relation between H-stability and L-stability and employing the method of Lyapunov functions and Riccati inequality, some sufficient conditions for robust stability and robustly asymptotical stability in the sense of Hopfield are established for this kind of HNN with impulsive effects. On the basis of these results, the author has designed some impulsive controllers to stabilize HNN. In addition, the robust stability results for HNN obtained in this paper can be extended to general uncertain impulsive systems. The author will discuss them in future papers.

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