Article ID: 1000 - 8152(2003)02 - 0173 - 07

Robust output feedback tracking for a class of uncertain nonlinear systems

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Abstract: An output feedback tracking strategy for a class of uncertain nonlinear systems in strict feedback form is proposed. The uncertain nonlinearities of the system do not need to obey the linear growth conditions. A high-gain differential observer is used to estimate the derivatives of the tracking error. Instead of these signals themselves, their signs are used in a special variable structure observer. The controller is then designed via backstepping procedure. It can guarantee the boundedness of all signals in the whole closed-loop system. The tracking error can be limited in any desirable range.

Key words: robust control; uncertain nonlinear system; output feedback; variable structure observer; backstepping procedure

CLC number: TP13 **Document code**: A

一类不确定非线性系统的鲁棒输出反馈跟踪

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摘要: 讨论了一类严格反馈型不确定非线性系统的输出反馈跟踪问题.所讨论的不确定性不满足线性增长条件.先设计高增益观察器估计输出偏差的微分,并将由此获得的信号的符号用于变结构观察器,随后用 backstepping 过程设计输出反馈控制器.得到的控制器可以保证闭环系统的所有信号都是有界的,且跟踪误差可以任意地小.

关键词: 鲁棒控制; 不确定非线性; 输出反馈; 变结构观察器; backstepping 过程

1 Introduction

The control problem has attracted much attention. Concerning how to achieve robust stabilization or tracking via feedback control when a nonlinear system embodies uncertainties, such as unmodeled dynamics, unknown parameters or exogenous disturbance. Compared with the rapid progress in state feedback control[1,2], there are very few results of robust output feedback stabilization or tracking for uncertain nonlinear systems described by state space models[3~6]. The aim of this paper is to investigate the problem of robust output feedback tracking for a class of SISO uncertain nonlinear systems in strict feedback form. The uncertain nonlinear functions involved need not obey the linear growth conditions, provided that the nonlinear functions together with a sufficient number of their time derivatives are piecewise bounded by some known functions. The key contribution of the method is to introduce a variable structure observer with special form. What is needed here is the signs of the signals obtained by the high-gain differential observer. Robust output feedback control law is obtained by a backstepping procedure with the use of signals from the variable structure observer. The controller can guarantee boundedness for all signals in the closed-loop system, and make the tracking error to be arbitrarily small by choosing suitable parameters of the variable structure observer from a solvable set of inequalities.

2 Problem statement

Consider the following SISO system

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(x, d_i), \\ \dot{x}_n = u + f_n(x, d_n), \\ y = x_1, \ i = 1,2,\cdots,n-1, \end{cases} \tag{1}$$

Received date: 2001 - 05 - 24; Revised date: 2002 - 12 - 12.
Foundation item: supported by the National Natural Science Foundation of China (69934010).

where $x(t) = [x_1 \ \cdots \ x_n]^T$ is an n-th order state vector, functions $f_i(x, d_i)$'s represent the system nonlinearities with uncertainties, $d_i(t)$'s are uncertainties that include unknown but bounded parameters, unmodeled dynamics and exogenous disturbances. Assuming:

Assumption 1 In system (1), functions $f_i(x, d_i)$, $i = 1,2,\cdots,n$ and their continuous time derivatives from 1st to $(n-i)$ th order satisfy

$$|f_i^{(j)}(x, d_i)| \leqslant h_{ij}(x_1, \cdots, x_{i+j}),$$
$$\forall j = 0,1,\cdots,n-i$$

except at some limited number of discontinuous points. Here $h_{ij}(x_1, \cdots, x_{i+j})$'s are known sufficiently smooth functions.

Under this assumption, system (1) belongs to a class of nonlinear system with strict feedback form[2,7]. The aim of this paper is to design a tracking control via output and some signals constructed by output. First of all, we have the following assumption:

Assumplion 2 The reference trajectory $y_r(t)$ and all its first nth-order time derivatives are bounded, or, there exists a positive constant γ such that

$$|y_r^{(i)}(t)| \leqslant \gamma, \ \forall t \geqslant 0, i = 0,\cdots n.$$

Assumption 3 There exist some known positive constants α_i such that $|x_i(0)| \leqslant \alpha_i$ is valid for the initial values $x_i(0)$ of the system (1).

Remark 1 Assumption 1 is somewhat restrictive. But there still exists a wide class of nonlinear systems in strict feedback form satisfying such an assumption. Some abruptly changed exogenous disturbances can be considered. And in fact, the systems discussed in [8] satisfies this assumption.

Now all the states of system (1) are to be estimated. It is rather difficult to use high-gain observer directly due to the severe nonlinearities in the system. So the variable structure observer of special form as follows is proposed:

$$\begin{cases} \dot{\hat{x}}_i = \hat{x}_{i+1} + g_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_i) + L_i\sigma_i(z_i), \\ \dot{\hat{x}}_n = u + g_n(\hat{x}) + L_n\sigma_n(z_n), \\ i = 1,2,\cdots,n-1. \end{cases} \tag{2}$$

Functions $g_i(\cdot)$ are to be chosen. Variable structure gains L_i are positive constants to be determined, $i = 1, \cdots, n$. Functions $\sigma_i(\cdot)$'s are defined as

$$\sigma_i(x) = \begin{cases} \operatorname{sgn}(x), & |x| > k_i, \\ x/k_i, & |x| \leqslant k_i. \end{cases} \tag{3}$$

Auxiliary signals $z_i(t)$'s are defined in a recursive form as follows:

$$\begin{cases} z_1(t) = x_1(t) - \hat{x}_1(t), \\ z_{i+1}(t) = \dfrac{s}{\mu s + 1}(z_i(t)) + \dfrac{1}{(\mu s + 1)^i}(L_i\sigma_i(z_i(t))), \\ i = 1,\cdots,n-1, \end{cases} \tag{4}$$

where the positive constant μ and the non-negative constant $k_i, i = 1,\cdots,n$ will be determined later. The initial conditions $z_{i+1}(0), i = 1,\cdots,n-1$ are chosen to be zero.

Remark 2 In this paper, a system

$$r(t) = \frac{s^m}{(\mu s + 1)^n} w(t), n \geqslant m$$

means that

$$\begin{cases} \mu\dot{y}_i = -y_i + y_{i+1}, \ i = 1,\cdots,n-m-1, \\ \mu\dot{y}_j = -y_j + \tilde{y}_{j+1}, \\ \mu\dot{\tilde{y}}_j = \tilde{y}_{j+1} - y_j, \ j = n-m,\cdots,n, \\ r(t) = y_1(t), \end{cases} \tag{5}$$

with $\tilde{y}_{n+1}(t) := w(t)$ and initial values $y_i(0) = 0$, $i = 1,\cdots,n$. It is easy to see that the first $(n-m+k)$th time derivatives of the signal $r(t)$ exist if the input $w(t)$ is kth-order differentiable. Here k is any non-negative integer.

Several simple propositions are introduced:

Proposition 1 Let $r(t) = \dfrac{1}{(\mu s + 1)^m}$, where $\mu > 0$ is a constant and m is a non-negative integer; suppose there exists a positive constant M such that the input satisfies $|w(t)| \leqslant M$, then

1) $|r(t)| \leqslant M$;

2) Furthermore, if there exist positive constants p and q satisfying

$$w(t) \geqslant p + q \ (\text{or } w(t) \leqslant -p-q),$$
$$\forall t \in [t_0, t_0 + T], \ \forall t_0 > 0,$$

then for any given positive constant $r < q$, there exists the positive constant ζ which is not related to μ such that

$$r(t) \geqslant p + l \ (\text{or } r(t) \leqslant -p-l),$$
$$\forall t \in [t_0 + \mu\zeta, t_0 + T],$$

where T is a constant larger than $\mu\zeta$.

Proposition 2 Let $r(t) = \dfrac{s}{\mu s + 1} w(t)$, and suppose $w(t)$ is differentiable, then

$$r(t) = \frac{s}{\mu s + 1} w(0) + \frac{1}{\mu s + 1}\dot{w}(t).$$

Proposition 3 For any constant L, let $r(t) = \frac{s^m}{(\mu s+1)^n}L$, $n \geqslant m$ and n, m are non-negative integers, then there exists a positive constant κ which is not related to μ such that $|r(t)| \leqslant \kappa e^{-\frac{t}{2\mu}} \frac{L}{\mu^m}$.

These propositions can be easily proved by linear system theory.

Setting $e = x - \hat{x}$, it can be obtained

$$\begin{cases} \dot{e}_i = e_{i+1} + f_i - g_i - L_i\sigma_i(z_i), \\ \dot{e}_n = f_n - g_n - L_n\sigma_n(z_n), \\ i = 1,\cdots,n-1. \end{cases} \tag{6}$$

After introducing the variable structure observer, the objective of this work can be defined as follows:

For a given positive constant ε and any smoothly bounded reference trajectory $y_r(t)$, an output feedback controller

$$u = u(\hat{x}, y_r, \cdots, y_r^{(n)})$$

with suitable parameters μ and $k_i, L_i (i = 1,\cdots,n)$ is said to be tracking with accuracy ε, if it makes all the signals in the closed loop system bounded and there exists a positive constant T such that

$$|x_1(t) - y_r(t)| < \varepsilon, \ \forall t \geqslant T. \tag{7}$$

3 Controller design

The controller for the observer (2) will be designed in this section firstly. All the signals of the observer can be obtained directly; the functions $g_i(\cdot)$'s and the constants L_i's are to be chosen and $|L_i\sigma_i(z_i)| \leqslant L_i, i = 1,\cdots,n$. Therefore, the control law can be constructed as a special case of a class of strict feedback system with bounded uncertainties, such as investigated by [1] and [2]. So, taking the observer (2) as the controlled plant, the feedback controller can easily be designed by backstepping procedure as follows:

Consider the following coordinate transformation for observer (2)

$$\bar{x}_i = \hat{x}_i - v_i(\hat{x}_1,\cdots,\hat{x}_{i-1}, y_r,\cdots,y_r^{(i-1)}, \varepsilon_i), \tag{8}$$

with sufficiently smooth functions $v_i(\hat{x}_1,\cdots,\hat{x}_{i-1}, y_r, \cdots, y_r^{(i-1)})$ as:

$$\begin{cases} v_1 = y_r(t), \\ v_{i+1} = -\lambda_i\bar{x}_i - \frac{\bar{x}_i}{4\varepsilon_i}\left[\sum_{j=1}^{i-1}\left(\frac{\partial v_i}{\partial \hat{x}_j}L_j\right)^2 + L_i^2\right] - \bar{x}_{i-1} - g_i + \\ \qquad \sum_{j=1}^{i-1}\frac{\partial v_i}{\partial \hat{x}_j}(\hat{x}_{j+1} + g_j) + \sum_{j=1}^{i-1}\frac{\partial v_i}{\partial y_r^{(j-1)}}y_r^{(j)} + y_r^{(i)}, \\ i = 1,\cdots,n-1, \end{cases} \tag{9}$$

where $\lambda_i(\varepsilon_i, i = 1,\cdots,n)$ are arbitrary positive constants, the observer (2) can be rewritten in the new coordinates:

$$\begin{cases} \dot{\bar{x}}_i = \bar{x}_{i+1} - \lambda_i\bar{x}_i - \frac{\bar{x}_i}{4\varepsilon_i}\left[\sum_{j=1}^{i-1}\left(\frac{\partial v_i}{\partial \hat{x}_j}L_j\right)^2 + L_i^2\right] - \\ \qquad \bar{x}_{i-1} + \sum_{j=1}^{i-1}\frac{\partial v_{i1}}{\partial \hat{x}_j}L_j\sigma_j + L_i\sigma_i, \\ \qquad i = 1,2,\cdots,n-1. \\ \dot{\bar{x}}_n = u + g_n - \sum_{j=1}^{n-1}\frac{\partial v_n}{\partial \hat{x}_j}(\hat{x}_{j+1} + g_j + L_j\sigma_j) - \\ \qquad \sum_{j=1}^{n-1}\frac{\partial v_n}{\partial y_r^{(j-1)}}y_r^{(j)} - y_r^{(n)} + L_n\sigma_n. \end{cases} \tag{10}$$

Consequently, by choosing the controller

$$u = -\lambda_n\bar{x}_n - \bar{x}_{n-1} - g_n + \sum_{j=1}^{n-1}\frac{\partial v_n}{\partial \hat{x}_j}(\hat{x}_{j+1} + g_j + L_j\sigma_j) + \sum_{j=1}^{n-1}\frac{\partial v_n}{\partial y_r^{(j-1)}}y_r^{(j)} + y_r^{(n)} - L_n\sigma_n \tag{11}$$

and using the following Lyapunov function $V(\bar{x}) = \frac{1}{2}\sum_{i=1}^{n}\bar{x}_i^2$, it can be concluded that the derivative of $V(\bar{x})$ along the trajectory of the observer (2) satisfies $\dot{V} \leqslant -\lambda V + \lambda\bar{\varepsilon}$, that is

$$V(\bar{x}) \leqslant \bar{\varepsilon} + (V(0) - \bar{\varepsilon})e^{-\lambda t}, \ \forall t \geqslant 0.$$

Here

$$\bar{\varepsilon} = \frac{1}{\lambda}\sum_{i=1}^{n-1}(i\varepsilon_i), \ \lambda = \min_{i=1,\cdots,n} \lambda_i. \tag{12}$$

$V(0)$ is an initial value of $V(\bar{x})$ at $t = 0$. Noting that the initial conditions of the observer can be set arbitrarily, and the initial values at $t = 0$ of the reference trajectory together with all its first n-th order time derivatives can be used directly, the following equation can be satisfied step by step:

$$\hat{x}_i(0) = v_i(\hat{x}_1,\cdots,\hat{x}_{i-1}, y_r,\cdots,y_r^{(i-1)})|_{t=0}, \quad i = 1,\cdots,n. \tag{13}$$

This leads to $V(0) = 0$, thus it can be obtained that $V(\bar{x}) \leqslant \bar{\varepsilon}$, $\forall t \geqslant 0$. Since $\lambda_i(\varepsilon_i, i = 1,\cdots,n)$ can be chosen arbitrarily, so is $\bar{\varepsilon}$.

Based on the above analysis and noting that v_i's are expressed in a special low-triangle form, the following lemma can be directly verified:

Lemma 1 For any positive constant $\bar{\varepsilon}$ in (12), there exist positive $h_i(L_1,\cdots,L_{i-2})$ and $a_i(L_1,\cdots,$

$L_{i-1})(i = 1,2,\cdots,n)$ such that the closed loop system formed by system (1) together with the observer (2) and the controller (11) satisfies:

$$\begin{cases} |\hat{x}_i(0)| \leqslant h_i(L_1,\cdots,L_{i-2}), \\ |\hat{x}_i(t)| \leqslant a_i(L_1,\cdots,L_{i-1}), \forall t \geqslant 0, \\ i = 1,\cdots,n \end{cases} \tag{14}$$

and $|\hat{x}_1(t) - y_r(t)| \leqslant \sqrt{2\bar{\varepsilon}}, \forall t \geqslant 0$.

Remark 3 The parameters $L_1,\cdots,L_n$ are to be chosen. They are contained in the functions $a_i(\cdot)$ and $h_i(\cdot)$ in the controller (11).

With above assertions, the differential observer (4) is analyzed below. Appling Propositions 2, we can obtain from (4) and (6) that

$$z_1 = e_1,$$

$$z_i = \frac{1}{(\mu s + 1)^{i-1}} e_i + \sum_{j=1}^{i-1} \frac{s^{i-1-j}}{(\mu s + 1)^{i-1}}(f_j - g_j) + \sum_{j=1}^{i-1} \frac{s^{i-j}}{(\mu s + 1)^{i-1}} e_j(0),\ i = 1,\cdots,n.$$

Define

$$0 = \bar{t}_0 < \bar{t}_1 < \bar{t}_2 < \cdots < \bar{t}_v < \cdots \tag{15}$$

to be discontinuous points of the functions f_i 's and define $f_j^{(i-1-j)}$'s to be any values at instant $\bar{t}_v$, then applying Proposition 2 repeatedly, we yield

$$\frac{s^{i-1-j}}{(\mu s + 1)^{i-1}} f_j = \frac{1}{(\mu s + 1)^{i-1}} f_j^{(i-1-j)} + \sum_{k=1}^{i-2-j} \frac{s^{i-1-j-k}}{(\mu s + 1)^{i-1}} f_j^{(k)}\Big|_{t=0} + \sum_{k=1}^{i-2-j}\sum_{v=1}^{l} \frac{s^{i-1-j-k}}{(\mu s + 1)^{i-1}}\left(f_j^{(k)}\big|_{t=\bar{t}_v^+} - f_j^{(k)}\big|_{t=\bar{t}_v^-}\right),$$

$$\forall i = 2,\cdots,n,\ j = 1,\cdots,i-1,\ l = \max\{v \mid \bar{t}_v < t\}. \tag{16}$$

By Assumption 1 and applying the property (1) of Proposition 1, from (14) it can be concluded that there exists $\bar{q}_{ij}(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1})$ such that

$$\left|\frac{1}{(\mu s + 1)^{i-1}} f_j^{(i-1-j)}\right| \leqslant \bar{q}_{ij}(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1}),$$
$$\forall\ |e_k| \leqslant b_k,\ k = 1,\cdots,i-1$$

is satisfied for arbitrary positive constants b_i. From the above inequality and (16), it is clear that $g_i(\cdot)$'s can always be selected (in particular choose $g_i = 0$) such that there exist $q_{ij}(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1})$'s satisfying

$$\left|\sum_{j=1}^{i-1} \frac{s^{i-1-j}}{(\mu s + 1)^{i-1}}(f_j - g_j)\right| \leqslant q_i(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1}) + \left|\sum_{j}^{i-1}\sum_{k=1}^{i-2-j} \frac{s^{i-1-j-k}}{(\mu s + 1)^{i-1}}(f_j^{(k)}\big|_{t=0}\right| + \left|\sum_{j=1}^{i-1}\sum_{k=1}^{i-2-j}\sum_{v=1}^{l} \frac{s^{i-1-j-k}}{(\mu s + 1)^{i-1}} f_j^{(k)}\big|_{t=\bar{t}_v^+} - f_j^{(k)}\big|_{t=\bar{t}_v^-})\right|,$$
$$\forall\ |e_k| \leqslant b_k,\ k = 1,\cdots,i-1. \tag{17}$$

From the discussions above, by applying Proposition 3 and Lemma 1 together with Assumption 3, the following lemma can be obtained:

Lemma 2 For some positive $c_i(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1})$ satisfying

$$c_i(L_1,L_{i-2},b_1,\cdots,b_{i-1}) \geqslant |f_{i-1} - g_{i-1}|,$$
$$i = 2,\cdots,n$$

and for any given $\frac{1}{3}\min\{\bar{t}_{v+1} - \bar{t}_v\} > \delta_i > 0$, there exists a positive constant $\bar{M}$ such that

$$\left|z_i - \frac{1}{(\mu s + 1)^{i-1}} e_i\right| \leqslant c_i(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1}),$$
$$\forall t \in [\bar{t}_l + \delta_i, \bar{t}_{l+1}),\ i = 2,\cdots,n$$

for any μ satisfying $\frac{1}{\mu} > \bar{M}$ with $l = 0,1,\cdots$ and $\bar{t}_l$ in Eq.(15).

Combining the above Lemma and Proposition 1, the following lemma can be proved:

Lemma 3 For any given positive constant τ_i, suppose that

$$\begin{cases} b_i \geqslant |e_i(t)|, \\ b_i \geqslant c_i(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1}) + d_i, \end{cases} \tag{18}$$

and

$$e_i(t) \geqslant c_i + d_i \quad \text{or} \quad -e_i(t) \geqslant c_i + d_i,$$
$$\forall t \in [\bar{t}_0, \bar{t}_0 + T_i] \subseteq [\bar{t}_v + \delta_i, \bar{t}_{v+1}),\ T_i > \tau_i$$

for some $\bar{t}_v$ defined in (15), then there exists a positive constant $M(\tau_2,\cdots,\tau_n)$, such that

$$\sigma_i(z_i) = \mathrm{sgn}\,(e_i),\ \forall t \in [\bar{t}_0 + \tau_i, \bar{t}_0 + T_i] \tag{19}$$

is satisfied for the signals $z_i(t)$ in observer (2) under the feedback control (11) and any μ satisfying $\frac{1}{\mu} \geqslant$

M. Here $d_i \geq 2^{i-1}k_i$, k_i is defined in (3), δ_i and $c_i(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1})(i = 2,\cdots,n)$ are obtained in Lemma 1.

Proof Suppose $e_i(t) \geq c_i + d_i$ (Similar proof can be conducted for $-e_i(t) \geq c_i + d_i$), $\forall t \in [\bar{t}_0, \bar{t}_0 + T_i]$, applying the Proposition 1 and using (18), it can be obtained that there exists a positive constant ζ_i not related to μ such that $\frac{1}{(\mu s + 1)^{i-1}}e_i \geq c_i + \frac{d_i}{2^{i-1}} \geq c_i + k_i$, $\forall t \in [\bar{t}_0 + \mu\zeta_i, \bar{t}_0 + T_i]$. By applying Lemma 2, it can be concluded that for given $\bar{\delta}_i = \frac{\tau_i}{2}$ there exists a positive constant $\bar{M}_i$ such that

$$\left| z_i - \frac{1}{(\mu s + 1)^{i-1}}e_i \right| \leq c_i(L_1,\cdots,L_{i-2},b_1,\cdots,b_{i-1}),$$

$\forall t \geq \bar{\delta}_i$ for any $\frac{1}{\mu} > \bar{M}_i$. Note that ζ_i is not related to μ and take $M = \max\limits_{i=2,\cdots,n}\left\{\bar{M}_i, \frac{2\tau_i}{\zeta_i}\right\}$. Thus, it can be obtained from (3) that (19) is valid. Lemma 3 is proved completely.

What remains to be done is to choose the parameters in (2). From the above analysis, a set of inequalities as follows can be constructed:

$$\begin{cases} b_1 \geq \alpha_1, \\ b_{i+1} \geq 2L_{i+1}(\tau_{i+1} + \delta_{i+1}) + \alpha_{i+1}, \\ b_1 \geq k_1, \\ b_{i+1} \geq 2L_{i+1}(\tau_{i+1} + \delta_{i+1}) + \\ \qquad c_{i+1}(L_1,\cdots,L_{i-1},b_1,\cdots,b_i) + d_{i+1} + 1, \\ L_i \geq 2b_{i+1} + 1, \\ (L_i - 2b_{i+1})(\Delta - \delta_i - \tau_i) \geq 2L_i(\delta_i + \tau_i), \\ L_n \geq c_n(L_1,\cdots,L_{n-1},b_1,\cdots,b_n) + 1, \\ \frac{1}{\mu} \geq M(\tau_2,\cdots,\tau_n), \end{cases} \tag{20}$$

with $\Delta := \frac{1}{2}\min\{\bar{t}_{v+1} - \bar{t}_v\}$. Here $i = 1,\cdots,n-1$, δ_i and $c_i(L_1,\cdots,L_{i-1},b_1,\cdots,b_i)$ are obtained from Lemma 2, d_i and $M(\tau_2,\cdots,\tau_n)$ is obtained from Lemma 3, and t_v defined in (15).

From the structure of (20), the following lemma exists:

Lemma 4 There exists a positive constant μ_0 such that the Ineqs. (20) are solvable for arbitrary given positive constants $d_i, i = 1,\cdots,n-1$ and any μ satisfying $0 < \mu \leq \mu_0$.

Now, the following theorem can be established:

Theorem 1 For any given positive constant ε, let $k_1 = \frac{\varepsilon}{2}$ in (3) and $\bar{\varepsilon} = \frac{\varepsilon^2}{8}$ in Lemma 1, then for arbitrary positive constants $k_i, i = 2,\cdots,n$, the controller (11) with any solution of (20) can fulfil the tracking task with accuracy ε.

Proof Obviously, Ineqs. (20) are solvable by Lemma 4. For the controller (11) with any solution of (20), what follows will be the proof by contradiction that $|e_i| \leq b_i, i = 1,\cdots,n$.

If this is not true, it can be defined that

$$\begin{cases} t_i = \inf\{t \,\|\, e_i(t) \,\| > b_i\},\ i = 1,\cdots,n, \\ T := \min\limits_{i=1,\cdots,n} t_i \end{cases} \tag{21}$$

with bounded T. So, it can be set $T \in [\bar{t}_v, \bar{t}_{v+1}]$ for some v with $\bar{t}_v$ defined in (15). From (20), there exist $b_i \geq \alpha_i \geq e_i(0)$, thus it is obvious that

$$|e_i(t)| \leq b_i,\ \forall t \leq T,\ i = 1,\cdots,n \tag{22}$$

is satisfied from (21). If $T = t_1$, since $e_1(t)$ is continuous and $b_1 \geq \alpha_1 \geq e_1(0)$, it can be concluded from (21) that

$$|e_1(T)| = b_1,\ e_1(T)\dot{e}_1(T) > 0. \tag{23}$$

Let Lyapunov function, from (17), (20) and (22), the derivative of V_1 along the trajectory of (6) satisfies

$$\dot{V}_1 \leq |e_1|(b_2 + b_2) - L_1|e_1| \leq -|e_1|,$$

$$\forall |e_1| \geq k_1,\ \forall t \leq T,$$

which contradict (23). If $T = t_l$, $l \neq 1$, then

$$|e_l(0)| \leq \alpha_l \leq b_l - 2L_l(\tau_l + \delta_l)$$

can be obtained by (20). Since $e_l(t)$ is continuous, from (21)

$$T_1 = \max\{t \,\|\, e_l(t) \,\| = b_l - 2L_l(\tau_l + \delta_l),\ t \in [0, T]\}$$

can be taken, and

$$\begin{cases} |e_l(T)| = b_l,\ |e_l(T_1)| = b_l - 2L_l(\tau_l + \delta_l), \\ |e_l(t)| \geq b_l - 2L_l(\tau_l + \delta_l),\ \forall t \in [T_1, T] \end{cases} \tag{24}$$

are satisfied. At the same time, the sign of $e_l(t)$ is invariant for $t \in [T_1, T]$. Therefore, from (20) and (6) it can be obtained

$$|\dot{e}_l(t)| = |e_{l+1}(t) + f_l - g_l + L_l\sigma_l(z_l)| \leq 2L_l. \tag{25}$$

From (22), it can be observed that conditions of Lemma 3 are satisfied when $t \in [T_1, T]$. Now, if $T_1 > \bar{t}_{v-1} + \frac{\Delta}{3}$ with $\Delta = \min\{t_{v+1} - t_v\}$, that is, there is at most one discontinuous point in $[T_1, T]$, then from (20) and Lemma 3, it can be asserted that $\sigma_l(z_l) = \mathrm{sgn}(e_l)$ is not valid at most with measure $\delta_l + \tau_l$ in $[T_1, T]$. Thus, it can be obtained from (24) that

$$| e_l(t) | < | e_l(T_1) | + 2L_l(\delta_l + \tau_l) = b_l$$

is valid for any $t \in [T_1, T]$. This contradicts (24). So $T_1 \leqslant \bar{t}_{v-1} + \frac{\Delta}{3}$, this leads to $[\bar{T}_1, T] \subseteq [T_1, T]$ with $\bar{T}_1 := \bar{t}_v - \frac{\Delta}{2}$. Thus, the conditions of Lemma 3 are satisfied when $t \in [\bar{T}_1, T]$. By using (20) and applying Lemma 3 again, it can be sure that $\sigma_l(z_l) = \mathrm{sgn}(e_l)$ is not valid at most with measure $\delta_l + \tau_l$ in $[\bar{T}_1, T]$. This means that

$$\begin{aligned} &| e_l(\bar{T}_1) | \geqslant \\ &| e_l(T) | - 2L_l(\delta_l + \tau_l) + \\ &(L_l - 2b_{l+1})(T - \bar{T}_1 - \delta_l - \tau_l), \end{aligned}$$

and from (6) and (20), and by (20) it leads to $| e_l(\bar{T}_1) | > b_l$, which contradicts (21).

From the above discussion, it follows that $| e_i | \leqslant b_i$, $i = 1, \cdots, n$. This leads to $\dot{V}_1 \leqslant -| e_1 |, \forall\ | e_1 | \geqslant k_1$, and by applying Lemma 1 we can achieve that there exists a positive constant T_0 such that

$$| y(t) - y_r(t) | < \varepsilon, \forall t \geqslant T_0$$

is satisfied. Theorem 1 is proved completely.

The forgoing discussion shows that Theorem 1 is valid under the influence of discontinuous exogenous disturbance. This is made possible because only the signs of signals z_i's are used. When the exogenous disturbances are abruptly changed, the large errors between e_i and z_i may occur in the high gain observer, but their signs will become equally swift.

4 Simulation example

Consider the following system

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x, d_1), \\ \dot{x}_2 = x_3 + f_2(x, d_2), \\ \dot{x}_3 = u + f_3(x, d_3), \\ y = x_1. \end{cases} \tag{26}$$

The reference trajectory is $y_r(t) = 0.7 + 1.3\sin t$, and the uncertain nonlinear functions are taken as follows

$$\begin{cases} f_1 = 0.5 + 0.5\sin t, \\ f_2 = x_2 + 0.1x_1^2\cos 0.5t + x_1\sin t + \cos 0.5t, \\ f_3 = x_3 + 0.25\mathrm{e}^{x_1} + x_2\cos x_1 + \sin 1.5t. \end{cases}$$

The initial values are $x_1(0) = 0.1, x_2(0) = 0.4, x_3(0) = 0.4$. In the following design, suppose the uncertain nonlinear functions satisfy

$$\begin{cases} | f_1 | \leqslant 1,\ | \dot{f}_1 | \leqslant 1,\ | \ddot{f}_1 | \leqslant 1, \\ | f_2 | \leqslant 0.6x_2^2 + 2x_1^2 + 3.5, \\ | \dot{f}_2 | \leqslant | x_3 | + | x_2 | + | 0.1x_2^2 | + | x_1 | + 1, \\ | f_3 | \leqslant | x_3 | + 0.3\mathrm{e}^{x_1} + | x_2 | + 1, \end{cases}$$

for all t and the initial conditions satisfy

$$| x_1(0) | \leqslant 0.1,\ | x_2(0) | \leqslant 0.5,\ | x_3(0) | \leqslant 0.5.$$

In the simulation,

$$\begin{aligned} &\varepsilon_1 = 0.3,\ \varepsilon_2 = 0.6, \\ &k_1 = 0.02,\ \lambda_i = 1, i = 1,2,3, \\ &L_1 = 2,\ L_2 = 7,\ L_3 = 11, \\ &k_2 = 1,\ k_3 = 2,\ \mu = 100 \end{aligned}$$

can be taken. With these parameters, Fig. 1 shows the tracking error.

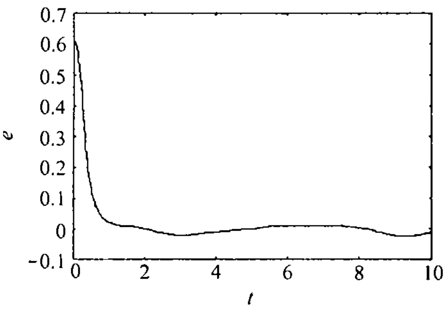

Fig. 1 Tracking error

Remark 4 In the simulation there is no discontinuous point, but the simulation results are almost the same if the $\sin t$ signal in f_1 is replaced by a unit square wave.

5 Conclusions

A robust output feedback-tracking controller for a class of uncertain nonlinear system has been designed. Here, a high-gain differential observer is adopted in a special variable structure observer. This may be considered as one of the key approaches that make the proposed design procedure successful. The other achievements is that only the signs of the signals from the differential observer are used in the variable structure observer. By solving some inequalities, all the parameters in the controller can be obtained. With these parameters, the controller makes all the signals in the closed-loop system bounded, and furthermore, by choosing the suitable solution of the inequalities, any tracking accuracy can be achieved. It is worth mentioning that due to the uniqueness of our variable structure observer and controller, the robustness of the closed system is significantly improved.

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