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Robust stabilization of uncertain Lur'e-Postnikov systems with state delay

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Abstract: Using Lyapunov and a Razumikhin-type method, robust stabilization of uncertain Lur'e-Postnikov systems with delay state is discussed. For uncertain Lur'e-Postnikov system with delay state and with some norm-bounded perturbations, if its coefficient matrices satisfy an algebraic Riccati inequality, then via a linear static and/or dynamic state feedback, quadratic stability for its closed-loop system is guaranteed. Also, using a Razumikhin-type appraoch, a sufficient condition is given for the stabilization of a class of uncertain nonlinear systems with time-varying delay.

Key words: uncertain Lur'e-Postnikov systems; robust stabilization; Riccati equation/inequality; time-delay

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带有时滞状态的不确定 Lur'e-Postnikov 系统的鲁棒镇定

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摘要: 由 Lyapunov 和一种 Razumikhin 型的方法,讨论了带有时滞状态的不确定 Lur'e-Postnikov 系统的鲁棒镇定.证明对带有状态时滞和范数有界扰动的不确定 Lur'e-Postnikov 系统,若其系统矩阵满足某个代数 Riccati 不等式,则可通过某(静态)线性状态反馈或(动态)状态反馈使其闭环系统是二次稳定的.同样,应用一 Razumikhin 型方法,对带有时变时滞的一类不确定非线性系统的能稳定性问题,也给出一个充分条件.

关键词: 不确定 Lur'e-Postnikov 系统; 鲁棒镇定; Riccati 方程或不等式; 时滞

1 Introduction

In recent years, because the uncertainty caused by modelling errors, measurement errors, linearization approximations, may occur in many practical time-delay systems, the problem of robust stability and stabilization for linear time-delay systems with norm-bounded or cone-bounded uncertainties has been of great interest to researchers, see [1 ~ 4]. Moreover, for uncertain nonlinear systems, when an arbitrary nonlinearity Φ, whose components belong to sector $(0.5, \infty)$ is inserted into the plant input, their stability and robust stabilization are interesting topics, see [1,5]. Ohta et al investigated the quadratic stabilization of uncertain Lur'e-Posnikov systems with no time-delay, see [5]; while we extend it to the case of time-delay in the state, see Section 2. Also, the series nonlinearity for control input in [5] is more general than that in [1]. In Section 3, we accept the nonlinear characteristics for control input as that of in [5], and our design of control law bears much difference from that of [1], so our result here could be considered as a kind of generalization of the related one in [1]. All in all, the purpose of this paper is to derive some Riccati equation type conditions for uncertain Lur'e-Postnikov systems with either norm-bounded or cone-bounded uncertainties.

Throughout this paper, the uncertainty in nonlinear characteristics for the control input is described by

$$2u^{\mathrm{T}}R\Phi(u) \geq u^{\mathrm{T}}Ru,\ \forall\, u \in \mathbb{R}^m, \qquad (1)$$

where R is a given $m \times m$ positive-definite matrix, $\mathbb{R}^m$ represents the input space. $\Phi:\mathbb{R}^m \to \mathbb{R}^m$ is any continuous function satisfying (1) and such that $\Phi(0) = 0$, see [5]. The following facts will be essential for the proofs in the Sections 2 and 3, see [2].

Lemma 1.1 Let A, D, E and F be real matrices of

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appropriate dimensions with $F^{\mathrm{T}}F \leqslant I$, here I is the identity matrix. Then we have the following:

1) For any scaler $\varepsilon > 0$, $DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} \leqslant \varepsilon^{-1}DD^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E$.

2) For any matrix $P > 0$ and scaler $\varepsilon > 0$ satisfying $\varepsilon I - EPE^{\mathrm{T}} > 0$,

$$(A + DFE)P(A + DFE)^{\mathrm{T}} \leqslant APA^{\mathrm{T}} + APE^{\mathrm{T}}(\varepsilon I - EPE^{\mathrm{T}})^{-1}EPA^{\mathrm{T}} + \varepsilon DD^{\mathrm{T}}.$$

2 Results for norm-bounded uncertainty

In this section, we consider uncertain time-delay Lur'e-Postnikov systems like the following:

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t - \tau) + [B + \Delta B(t)]\Phi(u), \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $A, A_1 \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $\Phi(\cdot)$ is any function satisfying (1). $\tau > 0$ is a given constant. The time-varying norm-bounded perturbation $[\Delta A(t), \Delta A_1(t), \Delta B(t)]$ is in the following form:

$$[\Delta A(t), \Delta A_1(t), \Delta B(t)] = DF(t)[E_a, E_1, E_b], \tag{3}$$

where D, E_a, E_1, E_b are known matrices of appropriate dimensions. $F(\cdot)$ is any Lebesgue measurable matrix-valued function satisfying $F^{\mathrm{T}}(t)F(t) \leqslant I$. It is assumed that all the states are available for feedback and the pair (A, B) is stabilizable. We have the following result for the quadratic stabilization of system (2). For the definition of quadratic stability/stabilizability, see [4,5].

Theorem 2.1 For system (2) with $\Delta B = 0$, if there exist positive definite matrices P and Q and positive numbers $\varepsilon_1, \varepsilon_2$, such that $\varepsilon_2 I - E_1QE_1^{\mathrm{T}} > 0$ and the following Riccati inequality holds:

$$A^{\mathrm{T}}P + PA + Q - PBR^{-1}B^{\mathrm{T}}P + (\varepsilon_1 + \varepsilon_2)PDD^{\mathrm{T}}P + \varepsilon_1^{-1}E_a^{\mathrm{T}}E_a + P[A_1Q^{-1}A_1^{\mathrm{T}} + A_1Q^{-1}E_1^{\mathrm{T}}(\varepsilon_2 I - E_1QE_1^{\mathrm{T}})^{-1}E_1Q^{-1}A_1^{\mathrm{T}}]P < 0, \tag{4}$$

then this Lur'e-Postnikov system is quadratically stabilizable; furthermore, a suitable stabilizing control law is given by $u(t) = -R^{-1}B^{\mathrm{T}}Px(t)$.

Proof The proof is omitted in order to save the paper length.

Remark 2.1 For system (2), when there is no time-delay, i.e.,

$$A_1 = 0, \Delta A_1 = 0,$$

this uncertain Lur'e-Postnikov system reduces to the system discussed by Ohta et al[5]. Also, the Riccati Ineq.(4) is reduced to

$$A^{\mathrm{T}}P + PA + Q - PBR^{-1}B^{\mathrm{T}}P + \varepsilon_1PDD^{\mathrm{T}}P + \varepsilon_1^{-1}E_a^{\mathrm{T}}E_a < 0,$$

which is similar to the related result in [1] that gave no consideration to the time-delay in the state.

Now, we turn to the case when both $\Delta A(t)$ and $\Delta B(t)$ exist. As in [5], we study quadratic stabilizability via dynamic state feedback. Consider the following system:

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t - \tau) + [B + \Delta B(t)]z, \tag{5a}$$

$$\dot{z} = \Phi(u), \tag{5b}$$

or

$$\dot{x}_e(t) = [A_e + \Delta A_e(t)]x_e(t) + [A_{1e} + \Delta A_{1e}(t)]x_e(t - \tau) + B_e\Phi[u(t)], \tag{6}$$

where

$$x_e = \begin{bmatrix} x \\ z \end{bmatrix},\ A_e = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix},$$

$$\Delta A_e = \begin{bmatrix} \Delta A & \Delta B \\ 0 & 0 \end{bmatrix} = D_eF(t)E_e,$$

$$A_{1e} = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Delta A_{1e}(t) = \begin{bmatrix} \Delta A_1(t) & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} DF(t)E_1 & 0 \\ 0 & 0 \end{bmatrix} = D_eF(t)E_{1e};$$

$$D_e = \begin{bmatrix} D \\ 0 \end{bmatrix},\ E_e = [E_a \quad E_b],$$

$$E_{1e} = [E_1 \quad 0],\ B_e = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

From Theorem 2.1, we have the following result:

Theorem 2.2 For system (6), if there exist positive definite $P_e, Q_e \in \mathbb{R}^{(n+m)\times(n+m)}$, and $\varepsilon_1, \varepsilon_2 > 0$, such that $\varepsilon_2 I - E_{1e}Q_eE_{1e}^{\mathrm{T}} > 0$ and the following Riccati inequality holds:

$$A_e^{\mathrm{T}}P_e + P_eA_e + Q_e - P_eB_eR^{-1}B_e^{\mathrm{T}}P_e + (\varepsilon_1 + \varepsilon_2)P_eD_eD_e^{\mathrm{T}}P_e + \varepsilon_1^{-1}E_e^{\mathrm{T}}E_e +$$

$$P_e[A_{1e}Q_e^{-1}A_{1e}^{\mathrm{T}} + A_{1e}Q_e^{-1}E_{1e}^{\mathrm{T}}(\epsilon_2 I - E_{1e}Q_eE_{1e}^{\mathrm{T}})^{-1}E_{1e}Q_e^{-1}A_{1e}^{\mathrm{T}}]P_e < 0,$$

then this Lur'e-Postnikov system is quadratically stabilizable via linear dynamic state feedback. Furthermore, a suitable stabilizing control law is given by

$$u(t) = -R^{-1}B_e^{\mathrm{T}}P_e x_e(t).$$

3 Results for bounded nonlinear uncertainty

In this section, we consider the following uncertain nonlinear time-delay system:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1x(t-h(t)) + B\Phi(u) + \\ \qquad Ba(x(t),u,x(t-h(t)),t), \\ x(t) = \theta(t), t \in [-H,0], \end{cases} \tag{7}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $B \in \mathbb{R}^{n\times m}$, A and A_1 are $n \times n$ matrices. (A, B) is a completely controllable pair, $h(t)$ is the time-varying delay such that $0 \leqslant h(t) \leqslant H$, for some positive constant H. $\theta \in C[-H,0;\mathbb{R}^n]$ is a given initial function. $\Phi(\cdot)$ is any function satisfying (1). The function $a(x(t),u,x(t-h(t)),t)$ is not completely known, but satisfies the following cone-bounded assumption.

Assumption 3.1 There exist known nonnegative constants k_1, k_2, k_3 such that $\forall (p,q,r,s) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n \times \mathbb{R}$,

$$\| a(p,q,r,s) \| \leqslant k_1 \| p \| + k_2 \| q \| + k_3 \| r \|,$$
$$k_2 < 1/(2\kappa(R)), \tag{8}$$

here and throughout this section $\|\cdot\|$ represents the Euclidean norm for vectors and the induced norm for matrices. $\kappa(R)$ is the condition number of R.

Since (A, B) is a completely controllable, for any α, $\rho > 0$, and any positive definite $Q \in \mathbb{R}^{n\times n}$, the following algebraic Riccati equation has an unique positive definite solution $P \in \mathbb{R}^{n\times n}$:

$$(A + \alpha I)^{\mathrm{T}} + P(A + \alpha I) - \rho PBR^{-1}B^{\mathrm{T}}P + Q = 0. \tag{9}$$

Now, we consider the asymptotic stabilization of system (7) by the control law:

$$u(t) = Kx(t),\ K = -\beta\rho R^{-1}B^{\mathrm{T}}P, \tag{10}$$

where the constant $\beta > 1$.

Theorem 3.1 Consider a system (7) satisfying the Assumption 3.1, and subject to controller (10), with (9) and

$$\rho = \frac{2}{r},\ Q = 2(Q_1 + rk_1^2 I), \tag{11}$$

$$\beta = \frac{\| R \|}{[1 - 2k_2\kappa(R)]} + \frac{1}{[1 - 2k_2\kappa(R)} > 1, \tag{12}$$

where Q_1 is a positive-definite matrix, r is any positive number, and $\kappa(R) = \| R \| \cdot \| R^{-1} \|$. The closed-loop system is globally asymptotically stable, provided that

$$\alpha\lambda_{\min}(P) + \lambda_{\min}(Q_1) - \kappa^{\frac{1}{2}}(P) \| PA_1 \| - rk_3^2\kappa(p) > 0, \tag{13}$$

where $\kappa(P) = \| P \| \cdot \| P^{-1} \|$.

Proof The proof is omitted for the limitation of the length.

Corollary 3.1 Consider the situation as in Theorem 3.1, with the exception that $k_3 = 0$ in Assumption 3.1 and $A_1 = 0$ in (7). Then, the origin of the closed-loop system is globally asymptotically stable.

References:

[1] SUN Y J, HSIEH J G. Robust stabilization for a class of uncertain nonlinear systems with time-varying delay: Razumikhin-type approach [J]. *J of Optimal Theory Application*, 1998, 98(1): 161 – 173.

[2] CAO Y Y, SUN Y X, CHENG C W. Delay-dependent robust stabilization of uncertain systems with multiple state delays [J]. *IEEE Trans on Automatic Control*, 1998, 43(11): 1608 – 1611

[3] SU J H. Further results on the robust stability of linear systems with a single time delay [J]. *Systems & Control Letters*, 1994, 23(5): 375 – 379.

[4] LUO J S, VAN DEN B P P, WEILAND S, et al. Design of performance robustness for uncertain linear systems with state and control delays [J]. *IEEE Trans on Automatic Control*, 1998, 43(11): 1593 – 1596.

[5] OHTA Y, GUROV I, HANEDA H. Quadratic stabilization of uncertain Lur'e-Postnikov systems via linear state feedback [J]. *Control Theory and Advanced Technology*, 1992, 8(2): 353 – 360.

[6] HALE J, VERDUYN S M L. *Introduction to Functional Differential Equations* [M]. New York: Springer-Verlag, 1994.

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