

Design of delay dependent robust controller for uncertain systems with time varying delay

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Abstract: The controller synthesis problem of uncertain systems with time varying delay is investigated. A robust controller with delay compensation is proposed based on Lyapunov function method. The stability criterion of the closed-loop system, which is dependent on the size of the time delay and the size of its derivative, is derived in the form of linear matrix inequalities (LMI). Example shows that the results of using the method in this paper are less conservative than the existing ones.

Key words: uncertain systems; delay; stability; Lyapunov function

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含时变滞后的不确定系统的时滞相关型鲁棒控制设计

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摘要: 研究含时变滞后的不确定系统的控制综合问题. 基于 Lyapunov 方法提出了一种含滞后补偿的鲁棒控制设计方法. 闭环稳定性条件由一组线性矩阵不等式表示. 在这些条件中给出了稳定性和滞后以及其导数之间的关系. 实例显示, 利用提供的方法所给出的结果比以往文献给出的结果保守性小.

关键词: 不确定系统; 滞后; 稳定性; Lyapunov 函数

1 Introduction

The study of stability and stabilization of time delay systems has attracted considerable attention over the past several decades because of their practical applications^[1~16]. In these works, the derived results can be classified into two categories, delay independent results in [1, 5, 6, 11, 13] and delay dependent results in [2~4, 6, 8]. Generally speaking, the delay dependent results are less conservative than the delay independent ones when the time delay is small.

Recently, a number of research works were focused on the study of delay dependent method via memoryless controller for uncertain systems with time delay. When time delay is time varying or constant, some memoryless controller design methods^[2~4, 6, 8] were proposed based on Lyapunov function method and first order transformation^[10]. To reduce the conservatism of the existing re-

sults, Gu^[9] used the discretized Lyapunov functional approach to propose a new design method of robust controller. The given controller can stabilize the original system with larger maximum allowed value of time delay than the existing ones by other methods. However, only systems with polytopic uncertainty and constant delay were addressed in [9]. It is not very easy to extend the method in [9] to the systems with norm-bounded uncertainty and time varying delay.

For the study of stabilization of time delay systems, memoryless controller and memory controller were proposed, see [2~6, 8~13] for the memoryless case and [15, 16] for the memory case. Although the proposed memoryless controllers are easy to implement, they often tend to be more conservative especially when the past information on the system can be employed. By using the past state or past input information, delay dependent co-

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ntrollers were designed in [15,16] and were shown by examples to be less conservative than the memoryless controllers. The shortcoming of the methods [15,16] is that the time delay must be assumed to be known and constant.

In this paper, we investigate the problem of delay dependent robust controller design for the systems with norm-bounded uncertainty and time varying delay. To get the transformed system, a neutral model transformation and first order transformation^[6] are employed simultaneously. Different from the memoryless controllers^[2-4,6,8,9], the given controller has feedback of the current state and past state information. There are two advantages of our method: firstly more information on the state is used to implement the controller, and secondly the time delay can be time varying and the exact value of the time delay is not required to be known. The derived stability criteria are expressed in terms of LMI, which can be effectively solved by using various optimization algorithms^[1].

2 System description and main result

Consider the following uncertain system with time varying delay

$$\begin{aligned} \dot{x}(t) = & [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t - \tau(t)) + \\ & [B + \Delta B(t)]u(t), \quad (1) \\ x(s) = & \phi(s), \quad s \in [-\bar{\tau}, 0], \quad (2) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the system state and the control input, respectively. $\tau(t)$ is the time delay which is continuously differentiable and satisfies $0 \leq \tau(t) \leq \bar{\tau}$ and $\dot{\tau}(t) \leq d < 1$. $\phi(t) \in C_0$ is the initial function. C_0 denotes the set of all continuous functions from $[-\bar{\tau}, 0]$ to \mathbb{R}^n . A , A_1 and B are constant matrices of appropriate dimensions. $\Delta A(t)$, $\Delta A_1(t)$ and $\Delta B(t)$ denote the parameter uncertainties which satisfy

$$\begin{bmatrix} \Delta A(t) & \Delta A_1(t) & \Delta B(t) \end{bmatrix} = DF(t) \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix},$$

where D , E_1 , E_2 and E_3 are known matrices and $F(t)$ is unknown time varying matrix which satisfies

$$\|F(t)\| \leq 1.$$

Next, we give our main result.

Theorem 1 Suppose that scalars $\bar{\tau} > 0$ and $d < 1$ are given. Then the system (1) with the control

$$u(t) = YX^{-1} \left[x(t) + \int_{t-\bar{\tau}/2}^t A_1 x(s) ds \right] \quad (3)$$

is asymptotically stable for any $\tau(t)$ satisfying $0 \leq \tau(t) \leq \bar{\tau}$ and $\dot{\tau}(t) \leq d < 1$, if there exist positive definite matrices X , X_k ($k = 1, 2, 3$), γ_i ($i = 1, 2, 3, 4$) and matrix $Y \in \mathbb{R}^{m \times n}$ and positive scalars ϵ_j ($j = 1, 2, \dots, 7$) such that

$$\begin{bmatrix} \Sigma & -\frac{\bar{\tau}}{2}(A+A_1)A_1Y_1 & \Omega_1^T \\ -\frac{\bar{\tau}}{2}Y_1A_1^T(A+A_1)^T & -\frac{\bar{\tau}}{2}Y_1 & \Omega_2^T \\ \Omega_1 & \Omega_2 & \Xi \end{bmatrix} < 0, \quad (4)$$

$$\begin{bmatrix} -Y_2 & Y_2A^T & Y_2E_1^T \\ AY_2 & -X_1 + \epsilon_5 DD^T & 0 \\ E_1Y_2 & 0 & -\epsilon_5 I \end{bmatrix} < 0, \quad (5)$$

$$\begin{bmatrix} -Y_3 & Y_3A_1^T & Y_3E_2^T \\ A_1Y_3 & -X_2 + \epsilon_6 DD^T & 0 \\ E_2Y_3 & 0 & -\epsilon_6 I \end{bmatrix} < 0, \quad (6)$$

$$\begin{bmatrix} -Y_4 & YB^T & YE_3^T \\ BY & -X_3 + \epsilon_7 DD^T & 0 \\ E_3Y & 0 & -\epsilon_7 I \end{bmatrix} < 0, \quad (7)$$

where

$$\begin{aligned} \Sigma = & X(A+A_1)^T + (A+A_1)X + BY + \\ & Y^TB^T + (\epsilon_1 + \frac{\bar{\tau}}{2}\epsilon_2 + \epsilon_3 + \epsilon_4)DD^T + \\ & \frac{\bar{\tau}}{2}A_1(X_1 + X_2 + X_3)A_1^T + \frac{\bar{\tau}}{2}Y_4, \\ \Omega_1^T = & \begin{bmatrix} XE_2^T & \frac{\bar{\tau}}{2}X & \frac{\bar{\tau}}{2}X & \frac{\bar{\tau}}{2}X & 0 & XE_1^T & Y^TE_3^T \end{bmatrix}, \\ \Omega_2^T = & \begin{bmatrix} -\frac{\bar{\tau}}{2}Y_1A_1^TE_2^T & -\frac{\bar{\tau}^2}{4}Y_1A_1^T & -\frac{\bar{\tau}^2}{4}Y_1A_1^T \\ -\frac{\bar{\tau}^2}{4}Y_1A_1^T & -\frac{\bar{\tau}}{2}Y_1A_1^TE_1^T & 0 & 0 \end{bmatrix}, \\ \Xi = & \text{diag} \left((1-d)\epsilon_3 I \quad \frac{\bar{\tau}}{2}Y_1 \quad \frac{\bar{\tau}}{2}Y_2 \quad \frac{\bar{\tau}}{2}(1-d)Y_3 \right. \\ & \left. \frac{\bar{\tau}}{2}\epsilon_2 I \quad \epsilon_1 I \quad \epsilon_4 I \right). \end{aligned}$$

Proof Define a neutral transformation as

$$z(t) = x(t) + \int_{t-\bar{\tau}/2}^t A_1 x(s) ds. \quad (8)$$

Using (8), we design a controller as

$$u(t) = Kz(t), \quad (9)$$

where K is a constant matrix that will be designed later.

Taking the time derivative of $z(t)$ and combining (1)

and (9), we obtain

$$\begin{aligned} z(t) &= (A + A_1)x(t) + \Delta A(t)x(t) + \\ &A_1x(t - \tau(t)) - A_1x(t - \frac{\bar{\tau}}{2}) + \\ &\Delta A_1(x)x(t - \tau(t)) + (B + \Delta B(t))Kz(t), \end{aligned} \quad (10)$$

$$x(t) = \varphi(t), \quad t \in [-\bar{\tau}, 0].$$

Since the once derivative of $x(t)$ exists for $t \geq 0$, using the Leibniz-Newton formula, we obtain

$$\begin{aligned} x(t - \tau(t)) &= x(t - \frac{\bar{\tau}}{2}) - \int_{t-\tau(t)}^{t-\bar{\tau}/2} \dot{x}(s)ds = \\ x(t - \frac{\bar{\tau}}{2}) &- \int_{t-\tau(t)}^{t-\bar{\tau}/2} \{(A + \Delta A(s))x(s) + (A_1 + \\ &\Delta A_1(s))x(s - \tau(s)) + (B + \Delta B(s))Kz(s)\}ds, \end{aligned} \quad (11)$$

for $t \geq -2\bar{\tau}$. Substituting (11) into (10), we get the following transformed system

$$\begin{aligned} z(t) &= (A + A_1)x(t) + \Delta A(t)x(t) + \\ &\Delta A_1(t)x(t - \tau(t)) - \\ &A_1 \int_{t-\tau(t)}^{t-\bar{\tau}/2} \{(A + \Delta A(s))x(s) + \\ &(A_1 + \Delta A_1(s))x(s - \tau(s)) + (B + \\ &\Delta B(s))Kz(s)\}ds + (B + \Delta B(t))Kz(t), \end{aligned} \quad (12)$$

Construct a Lyapunov functional as

$$V(x_t) = V_1(x_t) + V_2(x_t), \quad (13)$$

where $x_t(s) = x(t + s)$, $s \in [-\tau, 0]$,

$$V_1(x_t) = z^T(t)Pz(t),$$

$$V_2(x_t) =$$

$$\begin{aligned} &\int_{t-\tau(t)}^t x^T(s)Tx(s)ds + \int_{t-\bar{\tau}/2}^t \int_s^t x^T(v)Qx(v)dvds \oplus \\ &\int_{t-\bar{\tau}/2}^{t-\tau(t)} \int_s^{t-\tau(t)} x^T(v)Gx(v)dvds + \\ &\frac{\bar{\tau}}{2} \int_{t-\tau(t)}^t x^T(s)Gx(s)ds \oplus \\ &\int_{t-\bar{\tau}/2}^{t-\tau(t)} \int_s^{t-\tau(t)} x^T(v - \tau(v))Wx(v - \tau(v))dvds + \\ &\frac{\bar{\tau}}{2} \int_{t-\tau(t)}^t x^T(s - \tau(s))Wx(s - \tau(s))ds + \\ &\frac{\bar{\tau}}{2(1-d)} \int_{t-\tau(t)}^t x^T(s)Wx(s)ds \oplus \\ &\int_{t-\bar{\tau}/2}^{t-\tau(t)} \int_s^{t-\tau(t)} z^T(v)Rz(v)dvds + \\ &\frac{\bar{\tau}}{2} \int_{t-\tau(t)}^t z^T(s)Rz(s)ds, \end{aligned} \quad (14)$$

where \oplus is '+' as $\tau(t) < \frac{\bar{\tau}}{2}$ and '-' as $\tau(t) \geq \frac{\bar{\tau}}{2}$.

P, T, Q, G, W and R are symmetric positive definite matrices.

It is easy to show that as $\tau(t) \geq \frac{\bar{\tau}}{2}$

$$\begin{aligned} &\int_{t-\bar{\tau}/2}^{t-\tau(t)} \int_s^{t-\tau(t)} x^T(v)Gx(v)dvds \leq \\ &\frac{\bar{\tau}}{2} \int_{t-\tau(t)}^t x^T(s)Gx(s)ds, \\ &\int_{t-\bar{\tau}/2}^{t-\tau(t)} \int_s^{t-\tau(t)} z^T(v)Rz(v)dvds \leq \\ &\frac{\bar{\tau}}{2} \int_{t-\tau(t)}^t z^T(s)Rz(s)ds, \end{aligned}$$

and

$$\begin{aligned} &\int_{t-\bar{\tau}/2}^{t-\tau(t)} \int_s^{t-\tau(t)} x^T(v - \tau(v))Wx(v - \tau(v))dvds \leq \\ &\frac{\bar{\tau}}{2} \int_{t-\bar{\tau}/2}^t x^T(s - \tau(s))Wx(s - \tau(s))ds, \end{aligned}$$

thus, $V_2(x_t)$ is positive definite. Moreover, $V_1(x_t)$ and $V_2(x_t)$ are continuously twice differentiable in x and once in t .

Next, to prove the negative of time derivative of $V(x_t)$, we will consider two cases. Whenever the time delay satisfies the case $\tau(t) < \frac{\bar{\tau}}{2}$ or $\tau(t) \geq \frac{\bar{\tau}}{2}$, we can prove that under the conditions (5), (6) and (7)

$$\dot{V}(x_t) \leq \frac{2}{\bar{\tau}} \int_{t-\bar{\tau}/2}^t [z^T(t) \quad x^T(s)] H \begin{bmatrix} z(t) \\ x(s) \end{bmatrix} ds, \quad (15)$$

where

$$\begin{aligned} H &= \begin{bmatrix} \Sigma_1 + T_1 + \frac{\bar{\tau}}{2}R & -\frac{\bar{\tau}}{2}P(A + A_1)A_1 - \frac{\bar{\tau}}{2}T_1A_1 \\ -\frac{\bar{\tau}}{2}A_1^T(A + A_1)^TP - \frac{\bar{\tau}}{2}A_1^TT_1 & -\frac{\bar{\tau}}{2}Q + \frac{\bar{\tau}}{2}\epsilon_2^{-1}A_1^TE_1^TE_1A_1 + \frac{\bar{\tau}^2}{4}A_1^TT_1A_1 \end{bmatrix}, \\ T_1 &= T + \frac{\bar{\tau}}{2}Q + \frac{\bar{\tau}}{2}G + \frac{\bar{\tau}}{2(1-d)}W. \end{aligned}$$

Under the condition (4), $H < 0$ holds. Hence, there exists a constant $\lambda > 0$ such that

$$\dot{V}(t, x_t) < -\lambda z^T(t)z(t). \quad (16)$$

On the other hand, by Schur complements, $H < 0$ also means

$$-Q + \frac{\bar{\tau}^2}{4}A_1^TQA_1 < 0. \quad (17)$$

By Schur complements and matrix theory, we can further prove that a positive scalar $0 < \alpha < 1$ exists such

that

$$\begin{bmatrix} -\alpha Q & \frac{\bar{\tau}}{2} A_1^T Q \\ \frac{\bar{\tau}}{2} Q A_1 & -Q \end{bmatrix} \leq 0. \quad (18)$$

Using lemma in [14], we know that under the condition (17), $z(t)$ is a stable operator. Then, combining (16) and using Theorem 9.8.1 of [7], we can complete our proof. Q.E.D.

Remark It can be found from (3) that only upper bound of the time delay is needed to implement the controller (3) although the controller has feedback of the current state and the past state information.

3 Example

Example Consider the following system^[4]

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [A_1 + \Delta A_1(t)]x(t - \tau(t)) + Bu(t), \quad (19)$$

where

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\|\Delta A(t)\| \leq 0.2, \|\Delta A_1(t)\| \leq 0.2.$$

$$\tau(t) \text{ satisfies } 0 \leq \tau(t) \leq \bar{\tau} \text{ and } \dot{\tau}(t) \leq d < 1.$$

Choose

$$D_1 = D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$E_1 = E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

In [4], it was shown that the maximum allowable value of τ that guarantees the system (19) with $d = 0$ is stable via a memoryless controller is 0.3346. However, by applying Theorem 1, we found that the upper bound of τ which guarantees (19) with $d = 0$ is stable via the controller (3) is 0.5492. In other words, we can design a controller as

$$u(t) = -[0.2132 \quad 311.0461] \cdot$$

$$\left[x(t) + \int_{t-0.2746}^t \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix} x(s) ds \right],$$

which guarantees the closed-loop system is asymptotically stable for any $\tau \in [0 \quad 0.5492]$. If $d = 0.5$, the upper bound of τ which guarantees (19) is stable via the controller (3) is 0.3927. If $d = 0.9$, the upper bound of τ is 0.1298.

4 Conclusions

In this paper, a robust controller with delay compen-

sation was proposed for uncertain systems with time varying delay based on Lyapunov function method. Like the memoryless controller case, it is not required to know the exact value of the time delay although the designed controller in this paper depends on the current state and past state information. Since more information on the state is used, the given controller can provide better performance than the memoryless controllers in the existing references.

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(下转第 268 页)

of the systems can be simplified.

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(上接第 264 页)

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