第 20 卷第 2 期
2003 年 4 月

控制理论与应用
Control Theory & Applications

Vol.20 No.2
Apr. 2003

Article ID: 1000 - 8152(2003)02 - 0265 - 04

Robust control for generalized interconnected systems with similar structures

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Abstract: Two kinds of similar structures for a class of generalized interconnected systems with nonlinear interconnections are defined. There are uncertainties that do not satisfy the matching condition in the interconnections. Sate feedback robust controllers are designed so that the interconnected system is asymptotically stable. Since the controllers also possess similar structures, they are easy to perform.

Key words: similar structure; R-controllable; impulse controllable; normalizable

CLC number: TP13 **Document code**: A

具有相似结构的广义互联系统的鲁棒控制

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摘要: 对于一类具有非线性互联项的广义互联系统,定义了这类系统的两种相似结构.互联项包含不满足匹配条件的不确定性.设计了状态反馈鲁棒控制器使系统渐近稳定.由于控制器也具有相似结构,因此易于实现.

关键词: 相似结构; R-能控; 脉冲能控; 可正常化

1 Introduction

Generalized interconnected systems consist of many subsystems with interconnections. This is a special case of large-scale systems. They are important in large-scale systems and exist in the fields of aerospace, chemical industry, administration, etc.

This paper based on the studies for generalized system[1~4] and similarity[4~6]; and similar structure for a class of generalized interconnected systems with nonlinear interconnections is defined based on sate feedback and restricted system equivalence. The interconnections of the systems in this paper are nonlinear, do not satisfy matching condition and contain uncertainties.

2 Description of the systems

Consider the following generalized interconnected systems

$$E_i \dot{x}_i = A_i x_i + B_i[u_i + f_i(\bar{x}_i, t) + g_i(\bar{x}_i, t)] + h_i(\bar{x}_i, t),\ i = 1,2,\cdots,N, \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ are state, control input of i-th subsystem, respectively; $E_i, A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are constant matrices, rank $E_i < n$;

$$\bar{x}_i = \mathrm{col}(x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_N),$$

$f_i(\bar{x}_i, t)$ are known matching interconnections, $g_i(\bar{x}_i, t)$ are uncertain matching interconnections; $h_i(\bar{x}_i, t)$ are uncertain unmatched interconnections. We assume that

$$f_i(0,t) = 0, g_i(0,t) = 0,$$
$$h_i(0,t) = 0, i = 1,2,\cdots,N.$$

We have the following fundamental assumptions for systems (1) according to [2] and [3]:

Assumption 1[2] For arbitrary initial conditions $E_i x_i^0$ and arbitrary control inputs u_i, systems (1) are regular.

Assumption 2[3] The set of compatible control inputs of systems (1) is nonblank.

Definition 2.1 System $E_i \dot{x}_i = A_i x_i + B_i u_i((E_i,$

Received date: 2000 - 10 - 24; Revised date: 2002 - 04 - 01.
Foundation item: supported by the National Natural Science Foundation of China (79970114, 69974007, 69934010).

A_i, B_i) for simplicity) is called the i-th nominal subsystem of systems (1), $i = 1,2,\cdots,N$.

3 Similar structure I & controllers

Definition 3.1 We call systems (1) possess similar structure I if there exist matrices E, A, B, K_i and nonsingular matrices T_i, S_i so that

$$\begin{cases} T_iE_iS_i = E, \\ T_i(A_i + B_iK_i)S_i = A, \\ T_iB_i = B, \\ i = 1,2,\cdots,N. \end{cases} \tag{2}$$

Meanwhile, (T_i, S_i, K_i) is called the similar perimeter of i-th subsystem.

Remark 3.1 If condition (2) holds, then there exists proportional state feedback

$$u_i = K_ix_i + v_i,\ i = 1,2,\cdots,N, \tag{3}$$

so that the closed-loop system of (3) and (2) is

$$E_i\dot{x}_i = (A_i + B_iK_i)x_i + B_iv_i. \tag{4}$$

Making nonsingular transformations

$$x_i = S_i\xi_i$$

and premultiplying T_i on both sides of (4), we have

$$E\dot{\xi}_i = A\xi_i + Bv_i, \tag{5}$$

which is denoted as (E, A, B).

In the discussion above, Definition 2.1 means that the nominal subsystems (E_i, A_i, B_i) are restricted system equivalent after proportional state feedback. It is well known that proportional state feedback cannot change stabilizability and impulse controllability of a linear generalized system, so we have the following:

Proposition 3.1 If systems (1) possess similar structure I and a nominal subsystem (E_i, A_i, B_i) is stabilizable and impulse controllable, then system (E, A, B) is stabilizable and impulse controllable.

We assume system (5) is stabilizable and impulse controllable, then there exist matrix K and nonsingular matrices T, S so that

$$TES = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix},$$

$$T(A + BK)S = \begin{pmatrix} A_{(1)} & 0 \\ 0 & I_{n-r} \end{pmatrix},$$

where I_r represents identical matrix with order $r, r = \operatorname{rank} E, A_{(1)}$ is a stable matrix. Hence, for any positive definite matrix Q with order r, the following Lyapunov equation

$$A_{(1)}^{\mathrm{T}}P + PA_{(1)} = -Q$$

has unique positive definite solution P. Let

$$\hat{T}_i = TT_i = \begin{pmatrix} T_{i(1)} \\ T_{i(2)} \end{pmatrix},$$

$$\hat{S}_i^{-1} = S^{-1}S_i^{-1} = \begin{pmatrix} S_{i(1)} \\ S_{i(2)} \end{pmatrix},$$

where $T_{i(1)}, S_{i(1)} \in \mathbb{R}^{r\times n}$.

$\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalue of positive definite matrix, respectively, $\|\cdot\|$ denotes the spectral norm of matrix, α_i denotes the arbitrary scale that satisfies

$$0 < \alpha_i < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)\,\|\hat{T}_i\|\cdot\|\hat{T}_i^{-1}\|}.$$

We design controllers for systems (1) as follows:

$$u_i = u_i^1 + u_i^2 + u_i^3,\ i = 1,2,\cdots,N, \tag{6}$$

where

$$u_i^1 = (K_i + K\hat{S}_i^{-1})x_i,$$

$$u_i^2 = -f_i(\bar{x}_i, t),$$

$$u_i^3 =$$

$$\begin{cases} -\dfrac{B_i^{\mathrm{T}}T_{i(1)}^{\mathrm{T}}PS_{i(1)}x_i}{\|B_i^{\mathrm{T}}T_{i(1)}^{\mathrm{T}}PS_{i(1)}x_i\|}\rho_i(E_ix_i), & B_i^{\mathrm{T}}T_{i(1)}^{\mathrm{T}}PS_{i(1)}x_i \neq 0, \\ 0, & B_i^{\mathrm{T}}T_{i(1)}^{\mathrm{T}}PS_{i(1)}x_i = 0. \end{cases}$$

Theorem 3.1 The closed-loop systems of t robust controllers (14) and systems (1) are asymptotically stable if

1) Systems (1) possess similar structure I;

2) A nominal subsystem (E_i, A_i, B_i) is stabilizable and impulse controllable;

3) There exist continuous functions $\rho_i(\cdot)$ that satisfy $\rho_i(0) = 0$, $\|\Delta g_i(\bar{x}_i, t)\| \leqslant \rho_i(E_ix_i)$;

4) $\|\Delta h_i(\bar{x}_i, t)\| \leqslant \alpha_i\|E_ix_i\|$.

Proof Omitted.

4 Similar structure II & controllers

Definition 4.1 We call systems (1) possess similar structure II if there exist matrices E, A, B, L_i, K_i and nonsingular matrices T_i, S_i that satisfy

$$\begin{cases} T_i(E_i + B_iL_i)S_i = E, \\ T_i(A_i + B_iK_i)S_i = A, \\ T_iB_i = B,\ i = 1,2,\cdots,N. \end{cases} \tag{7}$$

Meanwhile, (T_i, S_i, L_i, K_i) is called the similar perimeter of i-th subsystem.

Remark 4.1 By Definition 4.1, if condition (7) holds, then there exists derivative-proportional state feedback

$$u_i = -L_i\dot{x}_i + K_ix_i + v_i,\ i = 1,2,\cdots,N, \quad (8)$$

so that the closed-loop system of (8) and (1) is

$$(E_i + B_iL_i)\dot{x}_i = (A_i + B_iK_i)x_i + B_iv_i. \quad (9)$$

Making nonsingular transformation

$$x_i = S_iz_i$$

and premultiplying T_i on both sides of (9), we have

$$E\dot{z}_i = Az_i + Bv_i, \quad (10)$$

which is denoted as (E, A, B).

By the discussion above, Definition 4.1 means that the nominal subsystems (E_i, A_i, B_i) are restricted system equivalent after derivative-proportional state feedback. Meanwhile, if (7) holds, then

$$\text{rank }(E,B) = \text{rank }[T_i(E_i + B_iL_i)S_i, T_iB_i] =$$

$$\text{rank } T_i[(E_i + B_iL_i), B_i]\begin{pmatrix} S_i & 0 \\ 0 & I_n \end{pmatrix} =$$

$$\text{rank } T_i(E_i, B_i)\begin{pmatrix} I_n & 0 \\ L_i & I_n \end{pmatrix}\begin{pmatrix} S_i & 0 \\ 0 & I_n \end{pmatrix} =$$

$$\text{rank }(E_i, B_i),$$

so we obtains the following:

Proposition 4.1 If systems (1) possess similar structure II and a nominal subsystem (E_i, A_i, B_i) is normalizable, then system (E, A, B) is normalizable.

Furthermore, we know from [1] that derivative-proportional state feedback cannot change stabilizability of a linear generalized system, so we have the following:

Proposition 4.2 If systems (1) possess similar structure II and a nominal subsystem (E_i, A_i, B_i) is stabilizable, then system (E, A, B) is stabilizable.

We assume system (10) is normalizable and stabilizable, then there exists derivative-proportional state feedback

$$v_i = -L\dot{z}_i + Kz_i + w_i,$$

so that

$$\text{rank }(E + BL) = n$$

and the closed-loop system

$$(E + BL)\dot{z}_i = (A + BK)z_i + Bw_i \quad (11)$$

are stable. Here w_i are new control inputs. Premultiply

$$T = (E + BL)^{-1}$$

on both sides of (11) and denote

$$\tilde{A} = (E + BL)^{-1}(A + BK),$$

we obtain

$$\dot{z}_i = \tilde{A}z_i + TBw_i, \quad (12)$$

where $\tilde{A}$ is a stable matrix by the stability of system (11), hence, for any positive definite matrix Q with order n, the following Lyapunov equation

$$\tilde{A}^{\mathrm{T}}P + P\tilde{A} = -Q$$

has unique positive definite solution P. Let

$$\hat{T}_i = TT_i,$$

β_i denote the arbitrary scales that satisfy

$$0 < \beta_i < \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)\ \|\hat{T}_i\| \cdot \|S_i\|}.$$

We design controllers for systems (1) as follows:

$$u_i = u_i^1 + u_i^2 + u_i^3,\ i = 1,2,\cdots,N, \quad (13)$$

where

$$u_i^1 = -(L_i + LS_i^{-1})\dot{x}_i + (K_i + KS_i^{-1})x_i,$$

$$u_i^2 = -f_i(\bar{x}_i, t),$$

$$u_i^3 = \begin{cases} -\dfrac{B_i^{\mathrm{T}}\hat{T}_i^{\mathrm{T}}PS_i^{-1}x_i}{\|B_i^{\mathrm{T}}\hat{T}_i^{\mathrm{T}}PS_i^{-1}x_i\|}\rho_i(x_i), & B_i^{\mathrm{T}}\hat{T}_i^{\mathrm{T}}PS_i^{-1}x_i \neq 0, \\ 0, & B_i^{\mathrm{T}}\hat{T}_i^{\mathrm{T}}PS_i^{-1}x_i = 0. \end{cases}$$

Theorem 4.1 The closed-loop systems of t robust controllers (13) and systems (1) are asymptotically stable if

1) Systems (1) possesses similar structure II;

2) A nominal subsystem (E_i, A_i, B_i) is normalizable and stabilizable;

3) There exist functions $\rho_i(\cdot)$ that satisfy

$$\|\Delta g_i(\bar{x}_i, t)\| \leqslant \rho_i(x_i);$$

4) $\|\Delta h_i(\bar{x}_i, t)\| \leqslant \beta_i \|x_i\|$, where β_i is determined by (15).

Proof The derivative of Lyapunov function

$$V = \sum_{i=1}^{N} x_i^{\mathrm{T}}Px_i$$

for the closed-loop systems is negative definite if 1) ~ 4) hold.

Remark 4.2 Consider controllers (6) and (13), u_i^1 and u_i^3 possess similar structures for the existence of similar parameter (T_i, S_i, K_i) or (T_i, S_i, L_i, K_i)

5 Conclusion

From the research that the similar structures of systems led to the similar structures of the controllers, the design of the systems can be simplified.

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