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Observer-based robust fuzzy control of nonlinear systems with parametric uncertainties

TONG Shao-cheng, ZHANG Xin-zheng

(1. Department of Basic Mathematics, Liaoning Institute of Technology, Liaoning Jinzhou 121001, China;
2. Automation Research Institute, Guangdong Polytechnic University, Guangdong Guangzhou 510090, China)

Abstract: This paper addresses the robust fuzzy control problem for nonlinear systems in the presence of parametric uncertainties and the state variables unavailable for measurement. Sufficient conditions are derived for robust stabilization in the sense of Lyapunov asymptotic stability and are formulated in the format of linear matrix inequalities (LMIs). The effectiveness of the proposed fuzzy controller and fuzzy observer design methodology is finally demonstrated through numerical simulations on an inverted pendulum system.

Key words: fuzzy control; robust control; fuzzy observer; parametric uncertainty; robust stability; linear matrix inequality

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基于观测器的参数不确定非线性系统的模糊鲁棒控制

佟绍成[1], 张新政[2]

(1. 辽宁工学院 基础部,辽宁 锦州 121001; 2. 广东工业大学 自动化研究中心,广东 广州 510090)

摘要: 对一类非线性系统进行模糊建模及其模糊观测器设计,研究了在系统的状态不可测且存在参数不确定的模糊鲁棒控制问题,以线性矩阵不等式的形式给出了模糊控制系统具有李雅普诺夫意义下稳定的充分条件,最后把所提出的方法应用到倒立摆系统进行仿真,仿真结果验证了该控制方法的有效性.

关键词: 模糊控制; 鲁棒控制; 模糊观测器; 参数不确定; 鲁棒镇定; 线性矩阵不等式

1 Introduction

Recently, stability analysis and systematic design are among the most issues for fuzzy control systems. There have been significant research efforts on these issues. With the development of fuzzy systems, it is known that qualitative knowledge of a system can also be represented in the nonlinear functional form. On the basis of this idea, fuzzy model based control design methods have been proposed in fuzzy control field. Usually, the nonlinear system is represented by a Takagi-Sugeno (T-S) fuzzy model; then, the control design is carried out on the basis of the fuzzy model via the so called parallel distributed compensation scheme, and the stability is derived by the Lyapunov direct method. Since uncertainty is usually a source of instability, Tanaka and his colleagues presented the stability analysis for a class of uncertain nonlinear systems[1]. Afterwards, there appear many publications to deal with the same problem[2]. In practical situation, however, not all states are available, and the output feedback control design becomes necessary. Recently, some researchers[3,4] studied fuzzy observer design for T-S fuzzy-model-based control systems, and proved that a state feedback controller with the observer always yields a stabilizing output feedback controller provided that the stabilizing property of the control and asymptotic convergence of the observer are guaranteed by the Lyapunov method. However, in the existing fuzzy observer design, the parametric uncertainties for T-S fuzzy control system are not considered. So the robustness of the whole control system cannot be guaranteed.

This paper proposes some new solutions to the robust stabilization problem for a class of nonlinear system with time-varying but norm-bounded parametric uncertainties

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in spite that their state variables are not available for measurement. The stability conditions are developed, subject to parametric uncertainties. The overall proposed design methodology presents a systematic and effective framework for an inverted pendulum system.

2 Preliminaries

In order to consider parametric uncertainties in T-S fuzzy systems, we proposed the continuous-time T-S fuzzy system in which the ith rule is formulated in the following form:

T-S fuzzy model:

Plant rule i:

IF $z_1(t)$ is M_1^i and $z_2(t)$ is $M_2^i, \cdots$, and $z_n(t)$ is M_n^i, THEN

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t), \\ y(t) = C_i x(t), \\ i = 1,2,\cdots,q, \end{cases} \tag{1}$$

where M_j^i is a fuzzy set ($j = 1,2,\cdots,n$), $z(t) = [z_1(t),\cdots,z_n(t)]^{\mathrm{T}}$ is some measurable system variables, i.e., the premise variables. $x(t) \in \mathbb{R}^s$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, and $y(t) \in \mathbb{R}^l$ is the output vector, and $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$ and $C_i \in \mathbb{R}^{l\times n}$ are system matrix, input matrix and output matrix, respectively, ΔA_i and ΔB_i are uncertain time-varying matrices with appropriate dimensions, which represent parametric uncertainties in the plant model, and q is the number of rules of this T-S fuzzy model.

The defuzzified output of this T-S fuzzy system (1) is represented as follows[2]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{q}\mu_i(z(t))[A_i x(t) + B_i u(t)] + \\ \qquad \sum_{i=1}^{q}\mu_i(z(t))[\Delta A_i(t) + \Delta B_i u(t)], \\ y(t) = \sum_{i=1}^{q}\mu_i(z(t))C_i x(t). \end{cases} \tag{2}$$

Next, fuzzy state observer for T-S fuzzy model with parametric uncertainties (1) is formulated as follows:

Observer rule i:

IF $z_1(t)$ is M_1^i and $z_2(t)$ is $M_2^i, \cdots$, and $z_n(t)$ is M_n^i, THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_i u(t) + G_i[y(t) - \hat{y}(t)], \\ \hat{y}(t) = C_i\hat{x}(t), \\ i = 1,2,\cdots,q, \end{cases} \tag{3}$$

where $G_i \in \mathbb{R}^{n\times l}$ is constant observer gain to be determined.

The defuzzified output of (3) is represented as follows

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{q}\mu_i(z(t))A_i\hat{x}(t) + \sum_{i=1}^{q}\mu_i(z(t))B_i u(t) + \\ \qquad \sum_{i=1}^{q}\mu_i(z(t))G_i[y(t) - \hat{y}(t)], \\ \hat{y}(t) = \sum_{i=1}^{q}\mu_i(z(t))C_i\hat{x}(t). \end{cases} \tag{4}$$

Assumption 1 The parameter uncertainties considered here are norm-bounded, in the form:

$$\begin{cases} [\Delta A_i, \Delta B_i] = D_i F_i(t)[E_{1i}, E_{2i}], \\ F_i^{\mathrm{T}}(t)F_i(t) \leqslant I, \end{cases}$$

where D_i, E_{1i}, and E_{2i} are known real constant matrices of appropriate dimension, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements, I is the identity matrix of appropriate dimension.

Lemma 1 Given constant matrices X and Y of appropriate dimensions, for some $\varepsilon > 0$, the following inequality holds:

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leqslant \varepsilon X^{\mathrm{T}}X + \frac{1}{\varepsilon}Y^{\mathrm{T}}Y.$$

Lemma 2 Given constant matrices D, E, and a symmetric constant matrix S of appropriate dimensions, the following inequality holds:

$$S + DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} < 0,$$

if and only if for some $\varepsilon > 0$,

$$S + [\varepsilon^{-1}E^{\mathrm{T}} \quad \varepsilon D]\begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \varepsilon^{-1}E \\ \varepsilon D^{\mathrm{T}} \end{bmatrix} < 0,$$

where $F^{\mathrm{T}}F \leqslant R$.

3 Output feedback robust stabilization of the T-S fuzzy model

Consider a T-S fuzzy model with parametric uncertainties and observer fuzzy model described by (3) and (4). Define observation error as

$$e(t) = x(t) - \hat{x}(t). \tag{5}$$

The objective is to design a T-S fuzzy-model-based output-feedback controller for robust stabilization of system (5) in the form

$$u(t) = -\sum_{i=1}^{q}\mu_i(z(t))K_i\hat{x}(t). \tag{6}$$

From systems (3),(4),(5) and (6), we have

$$\dot{x}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))(A_i + \Delta A_i - (B_i + \Delta B_i)K_j x(t) + \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))(B_i + \Delta B_i)K_j)e(t), \tag{7}$$

$$\dot{e}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))(A_i - G_iC_j + \Delta B_iK_j)e(t) + \sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(z(t))\mu_j(z(t))(\Delta A_i - \Delta B_iK_j)x(t). \tag{8}$$

The main result on the global asymptotic stability of T-S fuzzy model, with parametric uncertainties and the unavailable state variables summarized in the following theorem:

Theorem 1 If there exist symmetric and positive definite matrices P_1 and P_2, some matrices K_i and G_i, and some scalars ε_{ij}, $(i,j = 1,\cdots,q)$, such that the following LMIs are satisfied, then the T-S fuzzy system (3) is asymptotically stabilizable via the T-S fuzzy-model-based output-feedback controller (6):

$$\begin{bmatrix} \Phi_{ii} & * & * \\ E_{1i}Q - E_{2i}M_i & -(\varepsilon_{ii}^{-1}+1)^{-1}I & * \\ D_i^{\mathrm{T}} & 0 & -(\varepsilon_{ii}+1)^{-1}I \end{bmatrix} < 0 \quad (1 \leqslant i \leqslant q), \tag{9}$$

$$\begin{bmatrix} \Psi_{ij} & * & * & * & * \\ E_{1i}Q - E_{2i}M_i & -\left[\left(\frac{\varepsilon_{ij}}{2}\right)^{-1} + \frac{1}{2}\right]^{-1}I & * & * & * \\ E_{1i}Q - E_{2j}M_i & 0 & -\left[\left(\frac{\varepsilon_{ij}}{2}\right)^{-1} + \frac{1}{2}\right]^{-1}I & * & * \\ D_i^{\mathrm{T}} & 0 & 0 & -\varepsilon_{ij}^{-1}I & * \\ D_j^{\mathrm{T}} & 0 & 0 & 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix} < 0 \ (1 \leqslant i < j \leqslant q), \tag{10}$$

$$\begin{bmatrix} T_{ii} & * & * \\ E_{1i} - E_{2i}K_i & -(\varepsilon_{ii}^{-1} + 1)^{-1}I & * \\ D_i^{\mathrm{T}}P_2 & 0 & -(\varepsilon_{ii} + 1)^{-1}I \end{bmatrix} < 0 \ (1 \leqslant i \leqslant q), \tag{11}$$

$$\begin{bmatrix} \Xi_{ij} & * & * & * & * \\ -E_{2i}K_j & -\left[\left(\frac{\varepsilon_{ij}}{2}\right)^{-1} + \frac{1}{2}\right]^{-1}I & * & * & * \\ -E_{2j}K_i & 0 & -\left[\frac{\varepsilon_{ij}^{-1}}{2} + \frac{1}{2}\right]^{-1} & * & * \\ D_i^{\mathrm{T}} & 0 & 0 & -\varepsilon_{ij}^{-1}I & * \\ D_j^{\mathrm{T}} & 0 & 0 & 0 & -\varepsilon_{ij}^{-1}I \end{bmatrix} < 0 \ (1 \leqslant i < j \leqslant q), \tag{12}$$

where

$$\Phi_{ii} = QA_i^{\mathrm{T}} + A_iQ - M_i^{\mathrm{T}}B_i^{\mathrm{T}} - B_iM_i + I,$$

$$\Psi_{ij} = QA_i^{\mathrm{T}} + A_iQ + QA_j^{\mathrm{T}} + A_jQ - M_j^{\mathrm{T}}B_i^{\mathrm{T}} - B_iM_j - M_i^{\mathrm{T}}B_j^{\mathrm{T}} - B_jM_i,$$

$$T_{ii} = A_i^{\mathrm{T}}P_2 + A_iP_2 - C_i^{\mathrm{T}}N_i^{\mathrm{T}} - N_iC_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}B_iK_i,$$

$$\Xi_{ij} = A_i^{\mathrm{T}}P_2 + P_2A_i + A_j^{\mathrm{T}}P_2 + P_2A_j - N_j^{\mathrm{T}}B_i^{\mathrm{T}} - B_iN_j - N_i^{\mathrm{T}}B_j^{\mathrm{T}} - B_jN_i + \frac{K_i^{\mathrm{T}}B_j^{\mathrm{T}}B_jK_i + K_j^{\mathrm{T}}B_i^{\mathrm{T}}B_iK_j}{4},$$

and $Q = P^{-1}$, $M_i = K_iP^{-1}$, and $N_i = P_2G_i$, where $*$ denotes the transposed elements in the symmetric positions.

5 Computer simulation

To show the effectiveness of the proposed controller design techniques, we simulate the control of the inverted pendulum with parametric uncertainties and while the states are unmeasurable.

The equation of motion for the inverted pendulum device is[4]

$$
\begin{cases}
\dot{x}_1 = x_2, \\
\dot{x}_2 = \dfrac{1}{[(M+m)(J+ml^2) - m^2 l^2 \cos^2 x_1]} \cdot \\
\qquad [-f_1(M+m)x_2 - m^2 l^2 x_2^2 \sin x_1 \cos x_1 + \\
\qquad f_0 m l x_4 \cos x_1 + (M+m) m g \sin x_1 - m l \cos x_1 u], \\
\dot{x}_3 = x_4, \\
\dot{x}_4 = \dfrac{1}{[(M+m)(J+ml^2) - m^2 l^2 \cos^2 x_1]} \cdot \\
\qquad [-f_1 m l x_2 \cos x_1 + (J+ml^2) m l x_2^2 \sin x_1 - \\
\qquad f_0 (J+ml^2) x_4 - m^2 g l^2 \sin x_1 \cos x_1 + \\
\qquad (J+ml^2) u],
\end{cases}
\tag{13}
$$

where $g = 9.8\ \mathrm{m/s^2}$, $M = 1.3282\ \mathrm{kg}$, $m = 0.22\ \mathrm{kg}$, $f_0 = 22.915\ \mathrm{N/m \cdot s^{-1}}$, $f_1 = 0.007056\ \mathrm{N/rad \cdot s^{-1}}$, $l = 0.304\ \mathrm{m}$, $J = 0.004963\ \mathrm{kgm^2}$ in the numerical simulation.

We approximate the system by the following two-rule fuzzy models:

Plant rule 1:

IF x_1 is about 0, THEN

$$\dot{x} = (A_1 + \Delta A_1)x(t) + B_1 u(t),$$
$$y_1(t) = C_1 x(t).$$

Plant rule 2:

IF x_1 is about $\pm\pi/3$, THEN

$$\dot{x} = (A_2 + \Delta A_2)x(t) + B_2 u(t),$$
$$y_2(t) = C_2 x(t).$$

Where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & 0 & a_{43} \end{bmatrix},\quad B_1 = \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},\quad C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_{21}' & a_{22}' & 0 & 0 \\ 0 & 0 & 0 & 1 \\ a_{41}' & a_{42}' & 0 & a_{43}' \end{bmatrix},\quad B_2 = \begin{bmatrix} 0 \\ b_2' \\ 0 \\ b_4' \end{bmatrix}.$$

The parameters of A_1, A_2, B_1 and B_2 are chosen in the same as [2], ΔA_1 and ΔA_2 represent the system parameters uncertainties but bounded, the elements of ΔA_1 and ΔA_2 randomly achieve the values within 30% of their nominal values corresponding to A_1 and A_2, $\Delta B_1 = \Delta B_2 = 0$. Based on Assumption 1, we define

$$D_1 = D_2 = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix},$$

$$E_{11} = E_{12} = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix},$$

$$E_{21} = E_{22} = 0.$$

Membership functions for plant rules are chosen as [4]. By LMI optimization algorithm to solve LMIs (9) ~ (12), we can obtain feedback gain and observer gain matrices which ensure the stability of whole fuzzy system. The initial values of states are chosen $x(0) = [-60°, 0, 0, 0]$, $\hat{x}(0) = [0.9, 0.8, 0.2, 0.2]$. Fig. 1 illustrates the closed-loop system behaviors.

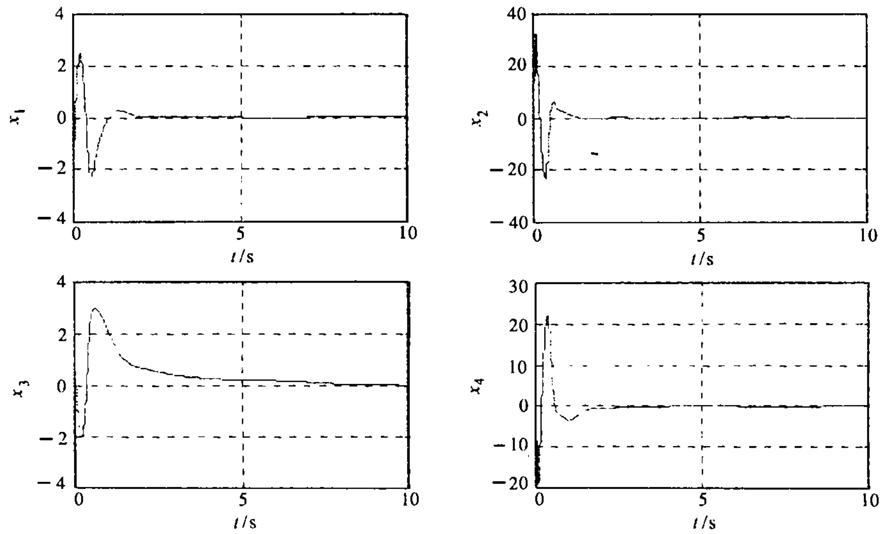

Fig. 1 Closed-loop response for initial conditions

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stabiliser for any relative degree one or two, minimum phase systems of dimension n or less [J]. *Automatica*, 1987,23(1):123 – 125.

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作者简介:

曲怀敬 (1696 —),男,1986年毕业于山东工业大学,山东建筑工程学院信息与电子工程系教师,主要研究方向:模式识别与信息处理自动化;

张 颖 (1967 —),男,1995年在东南大学获博士学位,新加坡制造技术研究院研究员,主要研究方向为:图像信息处理.

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作者简介:

佟绍成 (1960 —),男,辽宁工学院教授,博士,主要研究方向为非线性自适应控制,模糊控制;

张新正 (1963 —),女,广东工业大学教授,博士,主要研究方向为非线性大系统理论.