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Adaptive output tracking control for nonlinear systems with unknown high frequency gain

QU Huai-jing[1], ZHANG Ying[2]

(1. Department of Information and Electrical Engineering, Shandong Institute of Architecture and Engineering, Shandong Jinan 250014, China;
2. Division of Automation Technology, Singapore Institute of Manufacturing Technology, Nanyang Drive 71, 638075, Singapore)

Abstract: An adaptive controller is proposed for output tracking of nonlinear systems with unknown high frequence gain. The control design is achieved by introducing the Nassbaum gain in the backstepping design scheme. It is shown that the proposed controller can achieve asymptotic tracking without any growth conditions and any a priori knowledge on the sign of the high frequency gain.

Key words: adaptive control; high frequency gain; Nassbaum gain; backstepping method of design

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具有未知输入增益的非线性系统的跟踪控制

曲怀敬[1], 张 颖[2]

(1.山东建筑工程学院 信息与电子工程,山东 济南 250014;
2.新加坡制造技术研究院 自动化技术部,新加坡)

摘要: 针对具有未知输入增益的非线性系统,提出了一种可实现系统输出跟踪控制的自适应控制方法.通过在 backstepping 设计中引入一种新的 Nausbaum 增益,按该方法设计的控制器可以在系统输入增益未知的情况下实现系统输出的渐近跟踪.

关键词: 自适应控制; 高频增益; Nassbaum 增益; backstepping 设计方法

1 Introduction

In the adaptive control community, one common concerned is the problem of designing an adaptive controller without the knowledge of the sign of the system high frequency gain. For linear systems, this problem has been well treated by various approaches in [1 ~ 4]. For nonlinear systems, however, little has been done on this problem. Recently in [5], Nassbaum gain incorporating with the backstepping technique was used to design adaptive output stabilizer for high order nonlinear systems with arbitrary relative degree. However, certain sector conditions on system nonlinearities are still needed in [5] to achieve global stability.

In this paper, an adaptive control design scheme using the backstepping technique with Nassbaum gain is proposed to achieve output tracking of nonlinear systems. It is shown that the controllers designed by the proposed scheme can achieve asymptotic tracking for minimum phase nonlinear systems with unknown high frequency gain and arbitrary relative degree. In comparison with the results in [5] the proposed adaptive control design scheme is much simpler and effective both in controller structure and stability analysis. And the sector conditions in [5] are removed.

2 Problem formulation

Consider a class of nonlinear systems that can be transformed into [6]

$$\begin{cases} \dot{x} = A_c x + b(\theta)\sigma(y)u + \sum_{i=1}^{p}\psi_i(y)a_i + \psi_0(y), \\ y(t) = C_c x, \end{cases} \tag{1}$$

where $\sigma(\cdot) \in C(\mathbb{R},\mathbb{R})$, $\psi_i(\cdot) \in C(\mathbb{R},\mathbb{R}^n)$ and

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n},$$

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$$b(\theta)=\begin{bmatrix}0\\ \vdots\\ 0\\ b_m\\ \vdots\\ b_0\end{bmatrix}\in\mathbb{R}^n,\ C_c^{\mathrm{T}}=\begin{bmatrix}1\\ 0\\ \vdots\\ 0\end{bmatrix}\in\mathbb{R}^n,$$

with system relative degree $\rho = n - m$.

The control objective is to force the system output asymptotically track a given reference signal $y_r(t)$ in the presence of unknown a_i and $b_j(i = 1,\cdots,p, j = 0,\cdots, m)$.

Remark 2.1 Unlike previous results in [6] and [7], the parameter b_m, which is referred to as the high frequence gain, is not required with a known sign.

Remark 2.2 As conventionally required in the output tracking problem, the reference signal $y_r(t)$ and its first ρ-th derivatives are assumed to be known and bounded. In addition, $y_r^{(\rho)}(t)$ is piecewise continuous.

3 Backstepping design with a Nassbaum gain and auxiliary signal

In order to obtain the desired adaptive control law by carrying out the backstepping procedures, we defines, as follows, an input-output filter similar to those in [7].

$$\begin{cases}\dot{\xi}_0 = A_0\xi_0 + ky + \psi_0(y),\\ \dot{\xi}_i = A_0\xi_i + \psi_i(y),\ 1\leqslant i\leqslant p,\\ \dot{v}_j = A_0v_j + e_{n-j}\sigma(y)u,\ 0\leqslant j\leqslant m,\end{cases}\tag{2}$$

where

$$k \triangleq [k_1,k_2,\cdots,k_n]^{\mathrm{T}},\tag{3}$$

$$e_{n-j} = [0,\cdots,0,1,0,\cdots,0]^{\mathrm{T}}\in\mathbb{R}^{n\times 1},\tag{4}$$

$$A_0=\begin{bmatrix}-k_1 & 1 & 0 & \cdots & 0\\ -k_2 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -k_{n-1} & 0 & 0 & \cdots & 1\\ -k_n & 0 & 0 & \cdots & 0\end{bmatrix}.\tag{5}$$

The vector k in (3) is chosen such that the matrix A_0 is strictly stable, i.e., all eigenvalues of A_0 have negative real parts.

Thus, we obtain the following expression

$$\dot{y} = \omega^{\mathrm{T}}\theta + \beta + \varepsilon_2 = b_mv_{m,2} + \beta + \bar{\omega}^{\mathrm{T}}\theta + \varepsilon_2,\tag{6}$$

$$\dot{v}_{m,i} = v_{m,i+1} - k_iv_{m,1},\ i = 2,3,\cdots,\rho-1,\tag{7}$$

$$\dot{v}_{m,\rho} = v_{m,\rho+1} - k_\rho v_{m,1} + \sigma(y)u,\tag{8}$$

where

$$\beta \triangleq \xi_{0,2} + \psi_{0,1}\tag{9}$$

$$\theta = [b_m,\cdots,b_0,a_p,\cdots,a_1]^{\mathrm{T}},\tag{10}$$

$$\omega = [v_{m,2},v_{m-1,2},\cdots,v_{0,2},\xi_{p,2}+\psi_{p,1},\cdots,\xi_{1,2}+\psi_{1,1}]^{\mathrm{T}},\tag{11}$$

$$\bar{\omega} = [0,v_{m-1,2},\cdots,v_{0,2},\xi_{p,2}+\psi_{p,1},\cdots,\xi_{1,2}+\psi_{1,1}]^{\mathrm{T}}.\tag{12}$$

In above equations, $v_{i,2}$ denotes the second entry of v_i, $\xi_{i,2}$ the second entry of ξ_i, $\psi_{i,1}(0\leqslant i\leqslant n)$ the first entry of ψ_i, and ε_2 denotes the second entry of ε where ε denotes the state estimation error defined in the same way as in [7].

In order to obtain the desired controller, we take the change of coordinates

$$z_1 = y - y_r,\tag{13}$$

$$z_i = v_{m,i} - y_r^{(i-1)} - \alpha_{i-1},\ i = 2,3,\cdots,\rho,\tag{14}$$

where α_{i-1} is the virtual control at each step and will be determined in later discussions. To illustrate the backstepping procedures using Nassbaum gain, the first two steps of the design are given in details as follows.

Step 1 Define

$$\omega_1 = [\bar{\omega}^{\mathrm{T}}, c_1z_1 + \chi^2z_1 - \dot{y}_r + \beta]^{\mathrm{T}},\tag{15}$$

$$\theta_1 = \left[\frac{\theta^{\mathrm{T}}}{b_m},\frac{1}{b_m}\right]^{\mathrm{T}},\tag{16}$$

where χ is a new variable defined by

$$\chi = \frac{1}{2}z_1^2 + \gamma,\tag{17}$$

$$\dot{\gamma} = (c_1 + \chi^2)z_1^2.\tag{18}$$

It follows from (6),(13) and (14) for $i = 2$ that

$$\dot{z}_1 = -(c_1+\chi^2)z_1 + b_m(z_2 + \alpha_1 + \omega_1^{\mathrm{T}}\theta_1 + \dot{y}_r) + \varepsilon_2.\tag{19}$$

Take the following virtual control law α_1 and adaptive law $\hat{\theta}_1$ for estimating θ_1 in this step without using the sign of b_m,

$$\alpha_1 = -\omega_1^{\mathrm{T}}\hat{\theta}_1 - \dot{y}_r,\tag{20}$$

$$\dot{\hat{\theta}}_1 = N(\chi)\Gamma_1\omega_1z_1,\tag{21}$$

where Γ_1 is a positive matrix of $\mathbb{R}^{(m+p+2)\times(m+p+2)}$ and the Nassbaum gain $N(\chi)$ is chosen as

$$N(\chi) = \chi\cos(\chi).$$

To proceed, we define the Lyapunov function

$$V_1 = \tilde{\theta}_1^{\mathrm{T}}\Gamma_1^{-1}\tilde{\theta}_1 + \frac{1}{4d_1}\varepsilon^{\mathrm{T}}P\varepsilon,\tag{22}$$

where $\tilde{\theta}_1 \triangleq -\hat{\theta}_1 + \theta_1$, P is a positive matrix such that $PA_0^{\mathrm{T}} + A_0^{\mathrm{T}}P = -I$, and d_1 is a real positive number to be determined later.

Using (17), we have

$$| N(\chi)z_1 |^2 \leqslant | \chi^2 z_1^2 | \leqslant \dot{\gamma} = c_1 z_1^2 + \chi^2 z_1^2.$$

It thus readily follows that

$$\dot{V}_1 \leqslant \frac{2N(\chi)}{b_m}\dot{\chi} + \frac{\dot{\gamma}}{| b_m |} - 2N(\chi)z_1 z_2, \quad (23)$$

if d_1 is chosen such that $d_1 \leqslant | b_m | /4$.

Step 2 Now, evaluate the dynamic of the second state z_2. Differentiating (14) for $i = 2$ and using (7), there is

$$\dot{z}_2 = v_{m,3} - k_2 v_{m,1} - \dot{y}_r - \dot{\alpha}_1. \quad (24)$$

Define the Lyapunov function for this step as

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{4d_2}\varepsilon^{\mathrm{T}}P\varepsilon + \frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta}. \quad (25)$$

Using (23), it follows that

$$\dot{V}_2 \leqslant -c_2 z_2^2 + \frac{2N(\chi)}{b_m}\dot{\chi} + \frac{\dot{\gamma}}{| b_m |} + \tilde{\theta}^{\mathrm{T}}\Gamma^{-1}(\tau_2 - \dot{\hat{\theta}}), \quad (26)$$

when the virtual control for this step takes a value of

$$\alpha_2 = -c_2 z_2 + \beta_2 + 2N(\chi)z_1 + d_2\left(\frac{\partial \alpha_1}{\partial y}\right)^2 z_2 + \frac{\partial \alpha_1}{\partial y}\omega^{\mathrm{T}}\hat{\theta}, \quad (27)$$

where

$$\tau_2 = -\Gamma\frac{\partial \alpha_1}{\partial y}\omega z_2, \quad (28)$$

$$\beta_2 \triangleq k_2 v_{m,1} + \frac{\partial \alpha_1}{\partial y}\beta + \sum_{j=1}^{n}\frac{\partial \alpha_1}{\partial \xi_{0,j}}(-k_j\xi_{0,1} + \xi_{0,j+1} + k_j y + \psi_{0,j}(y)) + \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r + \sum_{i=1}^{\rho}\sum_{j=1}^{n}\frac{\partial \alpha_1}{\partial \xi_{i,j}}(-k_j\xi_{i,1} + \xi_{i,j+1} + \psi_{i,j}(y)) + \frac{\partial \alpha_1}{\partial \hat{\theta}_1}\dot{\hat{\theta}}_1. \quad (29)$$

Step $i(i = 3,\cdots,\rho)$ These steps are similar to those in [7], which include defining $V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{4d_i}\varepsilon^{\mathrm{T}}P\varepsilon$, taking

$$\alpha_i = -c_i z_i - d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i - z_{i-1} + \beta_i + \frac{\partial \alpha_{i-1}}{\partial y}\omega^{\mathrm{T}}\hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\Gamma\tau_i - \left(\sum_{k=3}^{i-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}}\right)\Gamma\frac{\partial \alpha_{i-1}}{\partial y}\omega,$$

and choosing $\tau_i = \tau_{i-1} - \Gamma\frac{\partial \alpha_{i-1}}{\partial y}\omega z_i$.

Finally, the actual adaptive controller is obtained by

$$u(t) = \alpha_\rho/\sigma(y), \quad (30)$$

$$\dot{\hat{\theta}} = \tau_\rho. \quad (31)$$

The final Lyapunov function V_ρ satisfies

$$\dot{V}_\rho \leqslant -\sum_{k=2}^{\rho} c_k z_k^2 + \frac{2N(\chi)}{b_m}\dot{\chi} + \frac{\dot{\gamma}}{| b_m |}. \quad (32)$$

This inequality is similar to the one in [4] where a linear system is considered. Following the same procedures as in [4], the stability of the closed-loop system can be concluded as stated in the following theorem.

Theorem 1 For a minimum phase nonlinear system with known relative degree ρ, the adaptive controller given by (30) and (31) can make the output of the system asymptotically track an arbitrary signal with bounded derivatives of up to order ρ while all the signals in the closed-loop system are bounded.

4 Conclusion

This paper studies the problem of designing an adaptive output-feedback controller for output tracking of nonlinear systems with unknown high frequency gain. To solve the problem, a scheme using Nassbaum gain in the backstepping design is proposed. It is shown that the controller obtained by the proposed design scheme can make the output of the system asymptotically track a given signal while all the signals in the whole adaptive control system remain stable. The stability analysis here is obtained without imposing any growth conditions on system nonlinearities. Another feature of the proposed scheme is the way of constructing Nassbaum gain. Unlike the conventional approaches as in all previous results using Nassbaum gains, an augmented tracking error is no longer used here and the dynamics about the Nassbaum gain variable is determined only by a first-order system. This ensures that the order of auxilliary system which is used to construct the Nassbaum gain is reduced to minimum.

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作者简介:

曲怀敬 (1696 —),男,1986年毕业于山东工业大学,山东建筑工程学院信息与电子工程系教师,主要研究方向:模式识别与信息处理自动化;

张 颖 (1967 —),男,1995年在东南大学获博士学位,新加坡制造技术研究院研究员,主要研究方向为:图像信息处理.

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作者简介:

佟绍成 (1960 —),男,辽宁工学院教授,博士,主要研究方向为非线性自适应控制,模糊控制;

张新正 (1963 —),女,广东工业大学教授,博士,主要研究方向为非线性大系统理论.