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Robust stability for singular systems with Frobenius norm-bounded uncertainties

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Abstract: This paper is to discuss the problems of stability and robust stabilization for singular systems with Frobenius norm-bounded uncertainties. The necessary and sufficient conditions of generalized quadratical stability, which guarantees that the uncertain singular system is stable, regular and impulse-free for all admissible uncertainties, can be obtained by solving an algebraic Riccati inequality or an algebraic Riccati equation. Furthermore, the design method of robustly stabilizing state feedback controller can be constructed in terms of the solution of a certain matrix equation. A numerical example is given illustrating the effectiveness of the proposed approach.

Key words: robustness; singular systems; Frobenius norm-bounded; uncertainty; Riccati equation

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具 Frobenius 有界不确定性广义系统的鲁棒稳定性

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摘要: 研究了具 Frobenius 有界不确定性广义系统的稳定与鲁棒镇定问题.通过对代数 Riccati 不等式或代数 Riccati 方程的求解,获得了不确定广义系统广义二次稳定的充要条件,使得对所有容许的不确定参数,系统是稳定,正则和无脉冲.而且,根据一类矩阵方程,构造了使不确定广义系统鲁棒镇定的状态反馈控制器的设计方法.实例说明了上述方法的有效性.

关键词: 鲁棒性; 广义系统; Frobenius 界; 不确定性; Riccati 方程

1 Introduction

Singular systems (sometimes referred to as descriptor systems, differential-algebraic-equation or semi-state systems) describe a broad class of systems, which are not only of theoretical interest but also of great practical significance. In recent years, much work has been focused on analysis and design techniques for singular systems (see [2,3], and references therein). Many of the standard design techniques for non-singular systems have been extended to singular systems. But it is well known that a common feature of robust stability for singular systems is that robust stability, regularity and impulse-free should be considered at the same time, while the latter two do not occur in the state-space systems. So a singular system has a more complicated structure. Meanwhile, even if the nominal singular systems are regular and impulse-free, a slight change of system matrices may result in impulse modes and destroy regularity of a singular system[4,5]. Recently some work dealing with the problems of robust stability analysis and robust control for uncertain singular systems has appeared in the literature (see, [1,6 ~ 10]).

On the other hand, the robust problems (stability, stabilization and control) have been extensively investigated for systems with uncertainties bounded by 2-norm or the maximum singular value (see [11], and reference therein). But there are few results on the uncertain singular systems with Frobenius norm-bounded uncertainties. The Frobenius norm is better than 2-norm, however, as a measure of uncertainties in some cases. Let us

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consider the following uncertainties[12]:

$$\Delta_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \Delta_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Obviously, $\| \Delta_2 \|_F (= \sqrt{2}) > \| \Delta_1 \|_F (= 1)$. However the maximum singular values of the uncertainties Δ_1 and Δ_2 are the same (= 1).

Motivated by notions of quadratic stability, Xu and Yang proposed the notions of generalized quadratic stability for uncertain singular systems with time-varying uncertainties and obtained some results of analyzing uncertain singular systems[1]. In this paper, we study the problems of stability and robust stabilization for singular systems with Fobenius norm-bounded uncertainties. By using the notions in [1], we propose necessary and sufficient conditions for the generalized quadratic stability which guarantees that the uncertain singular system is stable, regular and impulse-free for all admissible uncertainties by solving an algebraic Riccati inequality or an algebraic Riccati equation, and obtain a sufficient condition for the existence of the state feedback controller in terms of an algebraic Riccati equation. The main results in this paper can be viewed as extensions of what has been derived in [12].

2 Problem statement and definitions

Consider the following linear singular systems with Frobenius norm-bounded uncertainties

$$\begin{aligned} E\dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) = \\ &(A + H\Delta(t)E_a)x(t) + (B + H\Delta(t)E_b)u(t) = \\ &(A + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i \Delta_{ij} E_{ja})x(t) + \\ &(B + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i \Delta_{ij} E_{jb})u(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathbb{R}^m$ is the control input, $E \in \mathbb{R}^{n\times n}$, and rank $E = r \leqslant n$. A, B are known real constant matrices of appropriate dimensions. $H\Delta(t)E_a, H\Delta(t)E_b$ (e.g. the admissible uncertainties $\Delta A, \Delta B$) are real time-varying matrices representing parameter uncertainties with

$$H = [H_1 \quad H_2 \quad \cdots \quad H_r],$$
$$E_a = [E_{1a}^{\mathrm{T}} \quad E_{2a}^{\mathrm{T}} \quad \cdots \quad E_{sb}^{\mathrm{T}}]^{\mathrm{T}},$$
$$E_b = [E_{1b}^{\mathrm{T}} \quad E_{2b}^{\mathrm{T}} \quad \cdots \quad E_{sb}^{\mathrm{T}}]^{\mathrm{T}},$$

being known real constant matrices of appropriate dimensions, and

$$\Delta(t) = \begin{bmatrix} \Delta_{11}(t) & \Delta_{12}(t) & \cdots & \Delta_{1s}(t) \\ \Delta_{21}(t) & \Delta_{22}(t) & \cdots & \Delta_{2s}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \Delta_{r1}(t) & \Delta_{r2}(t) & \cdots & \Delta_{rs}(t) \end{bmatrix} \tag{2}$$

is an unknown real time-varying matrix satisfying the following inequality:

$$\sum_{i=1}^{r}\sum_{j=1}^{s} \| \Delta_{ij}(t) \|^2 \leqslant 1. \tag{3}$$

Let us denote $\Delta(t)$ and $\Delta_{ij}(t)$ by Δ and Δ_{ij}, respectively. Using the Frobenius norm, Ineq.(3) can be denoted by $\| \Delta_N \|_F \leqslant 1$, where Δ_N is given as

$$\Delta_N := \begin{bmatrix} \| \Delta_{11} \| & \| \Delta_{12} \| & \cdots & \| \Delta_{1s} \| \\ \| \Delta_{21} \| & \| \Delta_{22} \| & \cdots & \| \Delta_{2s} \| \\ \vdots & \vdots & \ddots & \vdots \\ \| \Delta_{r1} \| & \| \Delta_{r2} \| & \cdots & \| \Delta_{rs} \| \end{bmatrix}. \tag{4}$$

Remark 1 When each block uncertainty Δ_{ij} is a scalar δ_{ij}, (3) is equal to the following Frobenius norm-bound condition:

$$\sum_{i=1}^{r}\sum_{j=1}^{s} \| \delta_{ij}(t) \|^2 \leqslant 1.$$

Hence (3) is a generalization of the Frobenius norm-bound condition.

Remark 2 In many papers, the uncertain matrix $\Delta(t)$ satisfies $\Delta^{\mathrm{T}}\Delta \leqslant I$. In this paper, the Frobenius norm of the uncertainty $\Delta(t)$ is defined as

$$\| \Delta \|_F = (\mathrm{Trace}\ \Delta^{\mathrm{T}}\Delta)^{1/2}.$$

Without loss of generality, we assume that the matrix E in system (1) has the following special form as in [1]

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},$$

where I_r is an $r \times r$ identity matrix. In fact, E can always be brought to this form by a coordinate transformation

$$(Q_1 E Q_2, Q_1(A + \Delta A(t))Q_2, Q_1(B_1 + \Delta B_1(t))),$$

where Q_1 and Q_2 are non-singular matrices with $n \times n$ dimensions.

Before presenting the main results, we introduce the following definitions[1].

Definition 1 The uncertain singular system is said to be robustly stable if the unforced singular system [setting $u(t) \equiv 0$] is stable, regular, and impulse free for all admissible uncertainties.

Definition 2 The uncertain singular system is said to be generalized quadratically stable if there exist constant matrices $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$ with $Q^T = Q$, $Q > 0$ so that the unforced singular system [setting $u(t) \equiv 0$] satisfies

$$A_0^T(t)P + P^T A_0 \leqslant -Q, \qquad (5)$$

for all admissible uncertainty $\Delta A(t)$, where $A_0(t) = A + \Delta A(t)$, and P takes the form

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \qquad (6)$$

with $P_1 \in \mathbb{R}^{r\times r}$, $P_2 \in \mathbb{R}^{(n-r)\times r}$, $P_3 \in \mathbb{R}^{(n-r)\times(n-r)}$ and $P_1^T = P_1, P_1 > 0, P_3$ is invertible.

3 Main results

At first, we will show that generalized quadratic stability of uncertain singular system implies the robust stability of system (1).

Theorem 1 If the uncertain singular system (1) is generalized quadratically stable, it must be robustly stable.

Similar to the proof of Proposition 1 in [11], we can easily gain the result above.

Lemma 1[12] Given matrices

$$H = [H_1 \quad H_2 \quad \cdots \quad H_r],$$
$$E_a = [E_{1a} \quad E_{2a} \quad \cdots \quad E_{sa}]$$

being known real constant matrices of appropriate dimensions, and $\Delta_{ij}(i = 1,2,\cdots,r, j = 1,2,\cdots,s)$ satisfying (3), then for all vectors x and positive-definite matrices P, the equation

$$\max_{\|\Delta_N\|_F\leqslant 1}\left(\sum_{i=1}^{r}\sum_{j=1}^{s} x^T PH_i\Delta_{ij}E_{ja}x\right) = \sqrt{x^T\left(\sum_{i=1}^{r} PH_iH_i^TP\right)x x^T\left(\sum_{j=1}^{s} E_{ja}^TE_{ja}\right)x} \qquad (7)$$

holds.

It is noted that (7) can be denoted by

$$\max_{\|\Delta_N\|_F\leqslant 1}(x^TPH\Delta Ex) = \sqrt{x^TPHH^TPxx^TE^TEx}.$$

If each Δ_{ij} is a 1×1 matrix for all i, j, we have the following corollary.

Corollary 1 For all $x \in \mathbb{R}^n$

$$\max_{\|\Delta_N\|_F\leqslant 1}(x^TPH\Delta Ex) = \sqrt{x^TPHH^TPxx^TE^TEx} \qquad (8)$$

holds.

Using the above Lemma, we can prove the following theorem.

Theorem 2 The uncertain singular system (setting $u(t) \equiv 0$) with the uncertainties of (3) is generalized quadratically stable if and only if there exists a matrix P satisfying the inequality

$$A^TP + P^TA + P^T\left(\sum_{i=1}^{r} H_iH_i^T\right)P + \sum_{j=1}^{s} E_{ja}^TE_{ja} < 0, \qquad (9)$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \qquad (10)$$

with $P_1 \in \mathbb{R}^{r\times r}$, $P_2 \in \mathbb{R}^{(n-r)\times r}$, $P_3 \in \mathbb{R}^{(n-r)\times(n-r)}$ and $P_1^T = P_1, P_1 > 0, P_3$ is invertible.

Proof Sufficiency: Assume that there exists a matrix P with the form of (6) satisfying (9). Let

$$A_0(t) = A + \Delta A(t) = A + H\Delta E_a = A + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}E_{ja},$$

and by using Lemma 1, there is

$$\begin{aligned}
&x^T\{(A + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}E_{ja})^TP + P^T(A + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}E_{ja})\}x = \\
&x^T\{A^TP + P^TA + \sum_{i=1}^{r}\sum_{j=1}^{s}(P^TH_i\Delta_{ij}E_{ja} + E_{ja}^T\Delta_{ij}^TH_i^TP)\}x \leqslant \\
&x^T\{A^TP + P^TA + 2\max_{\|\Delta_N\|_F\leqslant 1}(\sum_{i=1}^{r}\sum_{j=1}^{s} P^T H_i\Delta_{ij}E_{ja})\}x = \\
&x^T(A^TP + P^TA)x + 2\sqrt{x^T(\sum_{i=1}^{r} P^TH_iH_i^TP)xx^T(\sum_{j=1}^{s} E_{ja}^TE_{ja})x}.
\end{aligned}$$

On the other hand,

$$2\sqrt{x^T(\sum_{i=1}^{r} P^TH_iH_i^TP)xx^T(\sum_{j=1}^{s} E_{ja}^TE_{ja})x} \leqslant x^T[P^T(\sum_{i=1}^{r} H_iH_i^T)P + \sum_{i=1}^{s} E_{ja}^TE_{ja}]x.$$

From the condition Ineq. (9), the following inequality can be obtained

$$\begin{aligned}
&x^T[A_0^T(t)P + P^TA_0(t)]x \leqslant \\
&x^T[A^TP + P^TA + P^T(\sum_{i=1}^{r} H_iH_i^T)P + \sum_{j=1}^{s} E_{ja}^TE_{ja}]x < 0.
\end{aligned}$$

Let

$$Q = -[A^TP + P^TA + P^T(\sum_{i=1}^{r} H_iH_i^T)P + \sum_{j=1}^{s} E_{ja}^TE_{ja}],$$

it is easy to show that $Q > 0$ and Ineq.(5) holds, i.e., the uncertain singular system is generalized quadratically stable.

Necessity: Assume that the uncertain singular system is generalized quadratically stable. Then there exists a matrix P_1 with the form (6) and a positive definite symmetric matrix Q such that the following inequality

$$A_0^T(t)P_1 + P_1^TA_0(t) \leqslant -Q$$

holds.

Let $U = A^TP_1 + P_1^TA$, there is

$$x^T\{(A + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}E_{ja})^TP_1 + P_1^T(A + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}E_{ja})\}x = x^TUx + 2(\sum_{i=1}^{r}\sum_{j=1}^{s} x^TP_1^TH_i\Delta_{ij}E_{ja}x) < 0$$

for all non-zero vectors $x \in \mathbb{R}^n$ and all admissible uncertainty $\Delta A(t)$ with Frobenius norm-bounded, and furthermore we have

$$x^TUx + 2\max_{\|\Delta_N\|_F\leqslant 1}(\sum_{i=1}^{r}\sum_{j=1}^{s} x^TP_1^TH_i\Delta_{ij}E_{ja}x) < 0.$$

That is

$$x^TUx < -2\max_{\|\Delta_N\|_F\leqslant 1}(\sum_{i=1}^{r}\sum_{j=1}^{s} x^TP_1^TH_i\Delta_{ij}E_{ja}x),$$

hence it follows that

$$(x^TUx)^2 > 4[\max_{\|\Delta_N\|_F\leqslant 1}(\sum_{i=1}^{r}\sum_{j=1}^{s} x^TP_1^TH_i\Delta_{ij}E_{ja}x]^2.$$

By using Lemma 1, there is

$$(x^TUx)^2 > 4x^T(\sum_{i=1}^{r} P_1^TH_iH_i^TP_1)xx^T(\sum_{j=1}^{s} E_{ja}^TE_{ja})x.$$

From Lemma 4 in [13], it follows that there exists a positive constant λ such that

$$\lambda(A^TP_1 + P_1^TA) + \lambda^2[P_1^T(\sum_{i=1}^{r} H_iH_i^T)P_1 + \sum_{j=1}^{s} E_{ja}^TE_{ja}] < 0.$$

Definite $P = \lambda P_1$, then we obtain that

$$A^TP + P^TA + P^T(\sum_{i=1}^{r} H_iH_i^T)P] + \sum_{j=1}^{s} E_{ja}^TE_{ja} < 0.$$

This ends the proof.

Remark 3 Under the theorem condition, the uncertain singular system (1) is robustly stable.

In the case when $E = 1$, system (1) becomes a normal system, i.e.,

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) = (A + H\Delta(t)E_a)x(t) + (B + H\Delta(t)E_b)u(t) = (A + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}E_{ja})x(t) + (B + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}E_{jb})u(t). \tag{11}$$

We have the following result.

Corollary 2 The uncertain system (11)(setting $u(t) = 0$) with the uncertainties of (3) is quadratically stable if and only if there exists a positive definite symmetric matrix P satisfying the inequality

$$A^TP + P^TA + P^T(\sum_{i=1}^{r} H_iH_i^T)P + \sum_{j=1}^{s} E_{ja}^TE_{ja} < 0. \tag{12}$$

Remark 4 Note that Corollary 2 is the same as Theorem 1 in [12], thus our result can be viewed as generalizations of some results for normal systems with Frobenius norm-bounded uncertainties.

Similar to the proof of Theorem 2, it is easy to obtain the following result.

Theorem 3 The uncertain singular system is generalized quadratically stable if and only if there exists a matrix P with the form (6) and a positive definite symmetric matrix Q satisfying the equality

$$A^TP + P^TA + P^T(\sum_{i=1}^{r} H_iH_i^T)P + \sum_{j=1}^{s} E_{ja}^TE_{ja} + Q = 0. \tag{13}$$

In the rest of this section we consider the problem of robust stabilization for singular system with Frobenius norm-bounded uncertainties. Let the state feedback controller be

$$u(t) = Kx(t), \tag{14}$$

then we can obtain the closed-loop system from(1)and (14) as

$$E\dot{x}(t) = A_cx(t), \tag{15}$$

where

$$A_c(t) = A + \Delta A(t) + (B + \Delta B)K = A + BK + H\Delta(t)(E_a + E_bK) = A + BK + \sum_{i=1}^{r}\sum_{j=1}^{s} H_i\Delta_{ij}(E_{ja} + E_{jb}K). \tag{16}$$

Definition 3 The uncertain singular system is said to be robustly stabilizable if there exists a linear state feedback controller $u(t) = Kx(t), K \in \mathbb{R}^{m\times n}$, such that the closed-loop system is robustly stable. In this case, $u(t) = Kx(t)$ is called a robust state feedback controller for system (1).

Theorem 4 If E_b is row full rank and there exists a matrix P with the form (6) and a positive definite symmetric matrix Q satisfying the equality

$$\begin{aligned}&[A - B(E_b^{\mathrm T}E_b)^{-1}E_b^{\mathrm T}E_a]^{\mathrm T}P + \\ &P^{\mathrm T}[A - B(E_b^{\mathrm T}E_b)^{-1}E_b^{\mathrm T}E_a] + \\ &\delta P^{\mathrm T}[HH^{\mathrm T} - B(E_b^{\mathrm T}E_b)^{-1}B^{\mathrm T}]P + \\ &\frac{1}{\delta}E_a^{\mathrm T}[I - E_b(E_b^{\mathrm T}E_b)^{-1}E_b^{\mathrm T}]E_a + Q = 0. \quad (17)\end{aligned}$$

Then the closed-loop system is robustly stable and a robust state feedback controller $u(t) = Kx(t)$ can be given by

$$u(t) = -\delta(E_b^{\mathrm T}E_b)^{-1}(B^{\mathrm T}P + \frac{1}{\delta}E_b^{\mathrm T}E_a)x(t). \quad (18)$$

Proof Suppose that there exists scalar $\delta > 0$ and a positive definite matrix Q such that (17) holds and the state feedback controller satisfies (18). On the other hand, noting that

$$\| \Delta \|_F = (\operatorname{tr} \Delta^{\mathrm T}\Delta)^{1/2} = \sum_i \lambda_i(\Delta^{\mathrm T}\Delta),$$

where $\lambda(A)$ stands for the eigenvalue of the matrix A and from $\| \Delta \|_F^2 \leqslant 1$, we can claim that $\Delta^{\mathrm T}\Delta \leqslant I$ (see [14]). By using Fact(A. l) in [15],

$$\begin{aligned}&A_c^{\mathrm T}(t)P + P^{\mathrm T}A_c(t) = \\ &[A + BK + H\Delta(t)(E_a + E_bK)]^{\mathrm T}P + \\ &P^{\mathrm T}[A + BK + H\Delta(t)(E_a + E_bK)] = \\ &(A + BK)^{\mathrm T}P + P^{\mathrm T}(A + BK) + \\ &[H\Delta(t)(E_a + E_bK)]^{\mathrm T}P + \\ &P^{\mathrm T}[H\Delta(t)(E_a + E_bK)] \leqslant \\ &(A + BK)^{\mathrm T}P + P^{\mathrm T}(A + BK) + \delta P^{\mathrm T}HH^{\mathrm T}P + \\ &\frac{1}{\delta}(E_a + E_bK)^{\mathrm T}(E_a + E_bK) = \\ &A^{\mathrm T}P + P^{\mathrm T}A + \delta P^{\mathrm T}HH^{\mathrm T}P + \frac{1}{\delta}E_a^{\mathrm T}E_a + K^{\mathrm T}(B^{\mathrm T}P + \\ &\frac{1}{\delta}E_b^{\mathrm T}E_a) + K^{\mathrm T}\frac{1}{\delta}E_b^{\mathrm T}Eb\,K + (P^{\mathrm T}B + \frac{1}{\delta}E_a^{\mathrm T}E_b)K = \\ &A^{\mathrm T}P + P^{\mathrm T}A + \delta P^{\mathrm T}HH^{\mathrm T}P + \frac{1}{\delta}E_a^{\mathrm T}E_a + (P^{\mathrm T}B + \\ &\frac{1}{\delta}E_a^{\mathrm T}E_b)[-\delta(E_b^{\mathrm T}E_b)^{-1}(B^{\mathrm T}P + \frac{1}{\delta}E_b^{\mathrm T}E_a)] = \\ &[A - B(E_b^{\mathrm T}E_b)^{-1}E_b^{\mathrm T}E_a]^{\mathrm T}P + \\ &P^{\mathrm T}[A - B(E_b^{\mathrm T}E_b)^{-1}E_b^{\mathrm T}E_a] + \\ &\delta P^{\mathrm T}[HH^{\mathrm T} - B(E_b^{\mathrm T}E_b)^{-1}B^{\mathrm T}]P + \\ &\frac{1}{\delta}E_a^{\mathrm T}[I - E_b(E_b^{\mathrm T}E_b)^{-1}E_b^{\mathrm T}]E_a = \\ &-Q < 0.\end{aligned}$$

Therefore, the closed-loop system is generalized quadratically stable, e. g. robustly stable. This completes the proof of the theorem.

4 A numerical example

Consider the system(1). Let

$$E = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix},\ A = \begin{bmatrix}1.5 & 2.2 & 1.4\\ 2.5 & 1.5 & 2.7\\ 1.2 & 1.4 & 1.8\end{bmatrix},$$

$$B = \begin{bmatrix}1.5 & 1\\ 1.2 & 0.8\\ 0.7 & 0.9\end{bmatrix},\ E_a = \begin{bmatrix}0.8 & 1 & 0.6\\ 0 & 0.3 & 0.5\\ 0.4 & 0.7 & 0.6\end{bmatrix},$$

$$E_b = \begin{bmatrix}0.5 & 0.2\\ 0.2 & 0.3\\ -0.3 & 0.3\end{bmatrix},\ H = \begin{bmatrix}1 & 0.5 & 0.9\\ 0.7 & 0.3 & 0.6\\ 0.6 & 0.8 & 0.7\end{bmatrix}.$$

Choosing $\delta = 1$ and using Theorem 4, we obtained that there exists a matrix P with the form (6) and a positive definite symmetric matrix Q

$$P = \begin{bmatrix}1.0155 & 0.6923 & 0\\ 0.6923 & 1.2945 & 0\\ 0.0997 & -0.4717 & 1.9473\end{bmatrix},$$

$$Q = \begin{bmatrix}17.1333 & 16.2951 & 12.6660\\ 16.2951 & 16.7614 & 11.7491\\ 12.6660 & 11.7491 & 9.9529\end{bmatrix},$$

such that Eq. (17) holds. The corresponding robust state feedback controller is

$$\begin{aligned}u(t) &= -\delta(E_b^{\mathrm T}E_b)^{-1}(B^{\mathrm T}P + \frac{1}{\delta}E_b^{\mathrm T}E_a)x(t) = \\ &\begin{bmatrix}-5.8335 & -5.6967 & -0.4664\\ -6.9579 & -6.3846 & -9.2271\end{bmatrix}x(t).\end{aligned}$$

5 Conclusions

In this paper, we deal with the problems of stability and robust stabilization for singular systems with Frobenius norm-bounded uncertainties. Our main contribution consists of necessary and sufficient conditions of generalized quadratically stability for uncertain singular systems. The proposed robust stabilization controller can be obtained by solving algebraic Riccati equation. The main results can be seen as the extensions of some existing results on uncertain normal systems to uncertain singular systems.

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