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Performance improvement in model reference adaptive control with the unknown high frequency gain

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Abstract: Under the assumption of the unknown high frequency gain k_p, a modified model reference adaptive controller is proposed. It is proved that the modified MRAC scheme has the stability and robustness properties, meanwhile the transient performance can be indeed improved by choosing the control law.

Key words: MRAC; performance improvement; stability
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具有未知高频增益的模型参考自适应控制的性能提高

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摘要: 在高频增益 k_p 未知的假设下,提出了一种修改的模型参考自适应控制器. 证明了这种修改的模型参考自适应控制方案具有稳定性和鲁棒性;同时通过选取的控制律, 其瞬态性能也明显提高.

关键词: 模型参考自适应控制; 性能提高; 稳定性

1 Introduction

Recently, more and more researchers are concentrating their attention on the performance issue of robust model reference adaptive control. Consider the following plant

$$y_p = G_p(s)(u_p + d) = k_p \frac{Z_p(s)}{R_p(s)}(u_p + d), \tag{1.1}$$

where $R_p(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_0$, $Z_p(s) = s^m + b_{m-1}s^{m-1} + \cdots + b_0$, a_i, b_j are unknown constants. d is bounded disturbance, i.e., there exists a constant $d_0 > 0$ such that $|d| \leqslant d_0$.

MRAC is to choose u_p such that all signals in the closed-loop system are all bounded, and y_p tracks the following reference model output y_m as close as possible for any given reference input $r(t)$.

$$y_m = W_m(s)r = k_m \frac{Z_m(s)}{R_m(s)}r. \tag{1.2}$$

Up to now, most of the results on MRAC with normalized adaptive laws can only guarantee that the tracking error $e \triangleq y_p - y_m$ satisfies

$$\int_t^{t+T} |e(\tau)|^2 \mathrm{d}\tau \leqslant c(f_0 + d_0^2)T + c, \ \forall t, T \geqslant 0, \tag{1.3}$$

where f_0 is a design parameter, and depending on the choices of the robust parameter estimation algorithms[1], c is some constant.

To further improve the performance of MRAC, in [1, 3], Sun considered the following control law

$$u_p = \theta_0^{\mathrm{T}} w_0 + c_0^* r - C(s)e,$$
$$C(s) = \frac{Q(S)}{1 - \frac{1}{c_0^*}W_m(s)Q(s)}, \tag{1.4}$$

where $Q(s) = \dfrac{c_0^* W_m^{-1}(s)}{(\tau s + 1)^{n^*}}$, $c_0^* = \dfrac{k_m}{k_p}$. From (1.1) and (1.4),

$$e = \frac{(\tau s + 1)^{n^*} - 1}{(\tau s + 1)^{n^*}} \frac{1}{c_0^*} W_m(s)[\tilde{\theta}_0^{\mathrm{T}} w_0 + d_1], \tag{1.5}$$

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where $\tilde{\theta}_0 = [\tilde{\theta}_1^{\mathrm{T}}, \tilde{\theta}_2^{\mathrm{T}}, \tilde{\theta}_3]^{\mathrm{T}}, \tilde{\theta}_i = \theta_i - \theta_i^*$, θ_i is an estimate of θ_i^*, $i = 1,2,3$, $d_1 = \dfrac{\Lambda(s) - \theta_1^{*\mathrm{T}} a(s)}{\Lambda(s)} d$. By (1.5), we can prove that

$$| e | \leqslant c\tau. \tag{1.6}$$

Obviously the result is better than (1.3).

In his work, we require the high-frequency gain k_p to be known. Based on the assumption, we are supposed to design $Q(s) = \dfrac{c_0^* W_m^{-1}(s)}{(\tau s + 1)^{n^*}}$ which is relevant to k_p ($c_0^* = \dfrac{k_m}{k_p}$) such that

$$1 - \frac{1}{c_0^*} W_m(s) Q(s) = \frac{(\tau s + 1)^{n^*} - 1}{(\tau s + 1)^{n^*}}, \tag{1.7}$$

while for the case of unknown k_p, since $c_0^* = \dfrac{k_m}{k_p}$, c_0^* is unknown; therefore we can not design $Q(s) = \dfrac{c_0^* W_m^{-1}(s)}{(\tau s + 1)^{n^*}}$ such that (1.8) holds. For this reason how to choose $Q(s)$ such that the tracking error has the form (1.6) constitutes the first part of our paper. It is worth emphasizing that the method used in this paper is completely different from those in [1,3]. Then, we prove that the modified MRAC has the stability and robustness properties, and meanwhile the transient performance can be indeed improved.

2 A modified model reference adaptive controller

We need the following assumptions for $G_p(s)$ and $W_m(s)$:

1) $Z_p(s)$ is monic Hurwitz polynomial.

2) The relative degree $n^* = n - m \geqslant 1$ is known.

3) The sign of k_p is known.

4) $Z_m(s)$ and $R_m(s)$ are monic Hurwitz polynomials of degree q_m, p_m, respectively, with $p_m \leqslant n, n^* = p_m - q_m$.

5) There exists the known constants $k_{\max}, k_{\min} > 0$ such that

$$k_p - \frac{k_p}{2^{n^*} - 1} < k_{\min} \leqslant k_p \leqslant$$

$$k_{\max} < k_p + \frac{k_p}{2^{n^*} - 1}.$$

We express (1.1) as

$$R_p(s) y_p = k_p Z_p(s)(u_p + d). \tag{2.1}$$

Adding and subtracting $\theta^{*\mathrm{T}} w$, we obtain

$$\begin{aligned} & R_p(s) y_p = \\ & k_p Z_p(s)(u_p - \theta^{*\mathrm{T}} w) + k_p Z_p(s) \theta^{*\mathrm{T}} w + k_p Z_p(s) d = \\ & k_p Z_p(s)(u_p - \theta^{*\mathrm{T}} w) + k_p Z_p(s)(\theta_1^{*\mathrm{T}} w_1 + \\ & \theta_2^{*\mathrm{T}} w_2 + \theta_3^* y_p + c_0^* r) + k_p Z_p(s) d, \end{aligned} \tag{2.2}$$

where $\theta^* = [\theta_1^{*\mathrm{T}}, \theta_2^{*\mathrm{T}}, \theta_3^*, c_0^*]^{\mathrm{T}}$ is the parameter of the controller, $w = [w_1^{\mathrm{T}}, w_2^{\mathrm{T}}, y_p, r]^{\mathrm{T}}$, $w_1 = \dfrac{a(s)}{\Lambda(s)} u_p$, $w_2 = \dfrac{a(s)}{\Lambda(s)} y_p$, $a(s) = [s^{n-2}, s^{n-3}, \cdots, s, 1]^{\mathrm{T}}$, $\Lambda(s)$ is known monic Hurwitz polynomial of degree $n - 1$, $\Lambda(s) = \Lambda_0(s) Z_m(s)$. The design parameter $\delta_0 > 0$ is chosen so that $\Lambda(s)$, $Z_m(s)$, and $R_m(s)$ have all their roots in Re $[s] < -\dfrac{\delta_0}{2}$. Consider the matching equations[1]

$$\begin{aligned} & (\Lambda(s) - \theta_1^{*\mathrm{T}} a(s)) R_p(s) - \\ & k_p(\theta_2^{*\mathrm{T}} a(s) + \theta_3^* \Lambda(s)) Z_p(s) = \\ & Z_p(s) \Lambda_0(s) R_m(s), \end{aligned} \tag{2.3}$$

$$c_0^* = \frac{k_m}{k_p}, \tag{2.4}$$

where $\Lambda(s) = \Lambda_0(s) Z_m(s)$. From (2.1) ~ (2.4) it is easy to see that

$$\begin{aligned} & y_p - y_m = \\ & \frac{1}{c_0^*} W_m(s)(u_p - \theta^{*\mathrm{T}} w) + \frac{(\Lambda(s) - \theta_1^{*\mathrm{T}} a(s))}{\Lambda(s) c_0^*} W_m(s) d. \end{aligned} \tag{2.5}$$

By (1.2), the definitions of θ^* and w, (2.5) can be rewritten as the following parametric model

$$z \triangleq W_m(s) u_p = \theta^{*\mathrm{T}} \varphi_p - \eta_0, \tag{2.6}$$

where

$$\varphi_p = [W_m(s) w_1^{\mathrm{T}}, W_m(s) w_2^{\mathrm{T}}, W_m(s) y_p, y_p]^{\mathrm{T}},$$

$$\eta_0 = \frac{(\Lambda(s) - \theta_1^{*\mathrm{T}} a(s))}{\Lambda(s)} W_m(s) d. \tag{2.7}$$

Since the controller parameter θ^* is unknown, we adopt the following algorithm to estimate θ^*.

$$\dot{\theta} = -\Gamma \varepsilon \bar{\varphi}_p + f, \ \varepsilon = \frac{z - \theta^{\mathrm{T}} \varphi_p}{m^2}, \tag{2.8}$$

$$\begin{cases} m^2 = 1 + n_s^2, \ n_s^2 = m_s, \\ \dot{m}_s = -\delta_0 m_s + u_p^2 + y_p^2, \\ m_s(0) = 0, \\ \dot{c}_0 = \sigma(-\gamma \varepsilon y_p), \end{cases} \tag{2.9}$$

$$\sigma=\begin{cases}\max\left\{0,\min\left[1,1+\frac{c_0-c_{\min}-\varepsilon_*}{\varepsilon_*}\right]\right\}, & \text{if } \varepsilon y_p>0,\\ \max\left\{0,\min\left[1,1+\frac{c_{\max}-\varepsilon_*-c_0}{\varepsilon_*}\right]\right\}, & \text{if } \varepsilon y_p<0,\\ 1, & \text{otherwise,}\end{cases} \tag{2.10}$$

where f can be chosen as dead-zone, or projection, or switching-σ modification, etc., Γ is adaptive gain,

$$\bar{\varphi}_p=[W_m(s)w_1^{\mathrm T},W_m(s)w_2^{\mathrm T},W_m(s)y_p]^{\mathrm T},$$

$$\varphi_p=[\bar{\varphi}_p^{\mathrm T},y_p]^{\mathrm T},\ \bar{\theta}=[\theta_1^{\mathrm T},\theta_2^{\mathrm T},\theta_3]^{\mathrm T},$$

$$\theta=[\bar{\theta}^{\mathrm T},c_0]^{\mathrm T},\ c_{\min}=\frac{k_m}{k_{\max}},\ c_{\max}=\frac{k_m}{k_{\min}},$$

$\varepsilon_*>0$ is an arbitrary small constant. The algorithm has the following properties[1]:

1) $\varepsilon,\varepsilon m,\theta,\dot{\theta}\in L_\infty$;

2) $\varepsilon,\varepsilon m,\dot{\theta}\in S\left(\frac{d_0^2}{m^2}\right)$,

where d_0 is a constant such that $|d|\leqslant d_0$,

$$S(w)\triangleq\{x,w\mid\int_t^{t+T}|x(\tau)|^2\mathrm{d}\tau\leqslant c\int_t^{t+T}|w(\tau)|\,\mathrm{d}\tau+c,c\geqslant 0 \text{ is some constant},\ \forall\, t,T\geqslant 0\}.$$

(2.5) can be further expressed as

$$y_p=-W_m(s)\alpha^{*\tau}\phi+\frac{\eta_0}{c_0^*}, \tag{2.11}$$

where

$$\alpha^*=[\frac{1}{c_0^*},\frac{\bar{\theta}^{*\tau}}{c_0^*}]^{\mathrm T},\ \phi=[-u_p,\bar{w}^{\mathrm T}]^{\mathrm T},$$

$$\bar{\theta}^*=[\theta_1^{*\tau},\theta_2^{*\tau},\theta_3^*]^{\mathrm T},$$

$$\bar{w}=[w_1^{\mathrm T},w_2^{\mathrm T},y_p]^{\mathrm T}.$$

Let us consider the control law

$$u_p=\theta^{\mathrm T}w+u_a, \tag{2.12}$$

which leads to

$$r=\frac{1}{c_0}[u_p-\bar{\theta}^{\mathrm T}\bar{w}-u_a], \tag{2.13}$$

where $\bar{\theta}=[\theta_1^{\mathrm T},\theta_1^{\mathrm T},\theta_3]^{\mathrm T}$ is the estimate of $\bar{\theta}^*$. Instituting (2.13) into (1.2), we have

$$y_m=W_m(s)\left[\frac{u_p}{c_0}-\frac{\bar{\theta}^\tau\bar{w}}{c_0}-\frac{u_a}{c_0}\right]=-W_m(s)\alpha^{\mathrm T}\phi-W_m(s)\frac{u_a}{c_0}, \tag{2.14}$$

where $\alpha=\left[\frac{1}{c_0},\frac{\bar{\theta}^{\mathrm T}}{c_0}\right]^{\mathrm T}$ is the estimate of α^*. Combining (2.11) and (2.14), we obtain

$$e=y_p-y_m=W_m(s)\left[\tilde{\alpha}^{\mathrm T}\phi+\frac{u_a}{c_0}+\frac{\Lambda(s)-\theta_1^{*\mathrm T}a(s)}{c_0^*\Lambda(s)}d\right], \tag{2.15}$$

where $\tilde{\alpha}=\alpha-\alpha^*$. Let us choose u_a as

$$u_a=-\frac{c_0Q(s)}{1-W_m(s)Q(s)}e,\ Q(s)=\frac{W_m^{-1}(s)}{(\tau s+1)^{n^*}}. \tag{2.16}$$

Introducing (2.16) into (2.15), we obtain the tracking error

$$e=\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}}W_m(s)\left(\tilde{\alpha}^{\mathrm T}\phi+\frac{\Lambda(s)-\theta_1^{*\mathrm T}a(s)}{c_0^*\Lambda(s)}d\right). \tag{2.17}$$

(2.17) is the desired form. Since c_0 can be obtained by (2.10), then (2.12) and (2.16) can be achieved.

3 Stability and performance analysis

The main results are as follows.

Theorem 1 Let $\tau\in(0,\tau_{\max}]$, where $\tau_{\max}>0$ is any finite number. If assumptions 1) ~ 5) hold, then the closed-loop plant consisting of (1.1), (1.2), (2.8) ~ (2.10), (2.12) and (2.16) has the following properties:

1) All signals in the closed-loop plant are all uniformly bounded;

2) $\sup_{t\geqslant 0}|e(t)|\leqslant c\tau(1+d_0),\ \forall\,\tau\in(0,\tau_{\max}]$,

where c is a constant independent of τ.

Before giving the proof of Theorem 1, we need a lemma.

Lemma 1 For the modified MRAC scheme, define

$$m_f(t)=1+\|(u_p)_t\|_{2\delta}+\|(y_p)_t\|_{2\delta},$$

$$\forall\,\delta\in\left(0,\min(\delta_0,\delta_1,\frac{1}{\tau_{\max}})\right), \tag{3.1}$$

where $\|x_t\|_{2\delta}\triangleq(\int_0^t e^{-\delta(t-\tau)}|x(\tau)|^2\mathrm{d}\tau)^{\frac{1}{2}}$, δ_1 is defined later, then

i) $\frac{|w_i|}{m_f},\frac{\|w\|_{2\delta}}{m_f},\ i=1,2$ and $\frac{n_s}{m_f}\in L_\infty$.

ii) If $\theta\in L_\infty$, then $\frac{u_p}{m_f},\frac{y_p}{m_f},\frac{w}{m_f},\frac{W(s)w}{m_f},\frac{\|u_p\|}{m_f},\frac{\|\dot{y}_p\|}{m_f}\in L_\infty$, where $W(s)$ is any proper transfer function that is analytic in Re $[s] \geqslant -\dfrac{\delta_0}{2}$.

iii) If θ, $r \in L_\infty$, then $\dfrac{\|\dot{w}\|}{m_f} \in L_\infty$.

iv) $\dfrac{m}{m_f}, \dfrac{\varphi_p}{m_f}, \dfrac{\phi}{m_f}, \dfrac{\|\varphi_p\|}{m_f}, \dfrac{W(s)\varphi_p}{m_f}, \dfrac{\|\phi\|}{m_f} \in L_\infty$.

where φ_p is defined by (2.7), $W(s)$ is any proper transfer function that is analytic in Re $[s] \geqslant -\dfrac{\delta_0}{2}$, $\|\cdot\|$ denotes the norm $\|(\cdot)_t\|_{2\delta}$.

Proof of the Theorem 1: From (2.17), (1.3) and (1.1), we can obtain that the output and input are

$$y_p = \frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}} W_m(s)\Big(\tilde{\alpha}^{\mathrm T}\phi + \frac{\Lambda(s)-\theta_1^{*\mathrm T}a(s)}{c_0^*\Lambda(s)} d\Big) + W_m(s) r, \tag{3.2}$$

$$u_p = \frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}} W_m(s) G_p^{-1}(s)\Big(\tilde{\alpha}^{\mathrm T}\phi + \frac{\Lambda(s)-\theta_1^{*\mathrm T}a(s)}{c_0^*\Lambda(s)} d\Big) + G_p^{-1}(s) W_m(s) r - d. \tag{3.3}$$

It is easy to see that

$$\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}} = \frac{\tau s(\tau s+1)^{n^*-1} + (\tau s+1)^{n^*-1}-1}{(\tau s+1)^{n^*}} = \cdots = \tau s W_1(s), \tag{3.4}$$

where $W_1(s) = \sum_{i=1}^{n^*} \dfrac{1}{(\tau s+1)^i}$. For any $\lambda < \dfrac{1}{\tau_{\max}}$, $\tau < \tau_{\max}$, since $1-\dfrac{\tau\lambda}{2} > \dfrac{\tau\lambda}{2}$, and $1-\dfrac{\tau\lambda}{2} > 1-\dfrac{\tau}{2}\dfrac{1}{\tau_{\max}} \geqslant \dfrac{1}{2}$, from the definition of $\|H(s)\|_{\infty\lambda} \triangleq \sup_\nu |H(j\nu - \dfrac{\lambda}{2})|$, we have

$$\Big\|\frac{\tau s}{\tau s+1}\Big\|_{\infty\lambda} = 1,\ \Big\|\frac{1}{\tau s+1}\Big\|_{\infty\lambda} < 2.$$

Let $\delta_1 > 0$ be such that $Z_p(s)$ has all roots in Re $[s] < -\dfrac{\delta_1}{2}$, then for $\forall \delta \in (0, \min(\delta_0, \delta_1, \dfrac{1}{\tau_{\max}}))$, by (3.2), (3.4), Assumptions 2) and 4), Lemma 1 and Lemma 3.3.2 in [1], we have

$$\|(y_p)_t\|_{2\delta} \leqslant c\tau\|\tilde{\alpha}\|_\infty \|\phi_t\|_{2\delta} + c d_0 + c \leqslant c_1 \tau m_f(t) + c, \tag{3.5}$$

where $\alpha = \Big[\dfrac{1}{c_0}, \dfrac{\bar{\theta}^{\mathrm T}}{c_0}\Big]^{\mathrm T} \in L_\infty$ is deduced from the property (1) of the estimation algorithm, c_1 is a constant independent of τ. By (1.2), Assumptions 1), 2) and 4), $W_m(s)G_p^{-1}(s) = c_0^* + D(s)$ and $D(s)$ is strictly proper stable transfer function, thus (3.3) can be expressed as

$$\begin{aligned} u_p = &-\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}} c_0^*\Big(\frac{1}{c_0}-\frac{1}{c_0^*}\Big) u_p + \\ &\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}} c_0^*\Big(\frac{\bar{\theta}}{c_0}-\frac{\bar{\theta}^*}{c_0^*}\Big)^{\mathrm T} \bar{w} + \\ &\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}} D(s)\tilde{\alpha}^{\mathrm T}\phi + \\ &\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}} W_m(s)G_p^{-1}(s)\frac{\Lambda(s)-\theta_1^{*\mathrm T}a(s)}{c_0^*\Lambda(s)} d + \\ &G_p^{-1}(s)W_m(s) r - d. \end{aligned} \tag{3.6}$$

For any $\lambda < \dfrac{1}{\tau_{\max}}$, by (3.4), we can obtain that

$$\Big\|\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}}\Big\|_{\infty\lambda} \leqslant 2^{n^*}-1.$$

For

$$\delta \in \Big(0, \min(\delta_0, \delta_1, \frac{1}{\tau_{\max}})\Big),$$

it is easily concluded that

$$\Big\|\frac{(\tau s+1)^{n^*}-1}{(\tau s+1)^{n^*}}\Big\|_{\infty\delta} \leqslant 2^{n^*}-1.$$

From (3.1), Lemma 1 and Lemma 3.3.2 in [1], $\alpha \in L_\infty$, it follows that

$$\tau\|sW_1(s)\|_{\infty\delta} \Big\|\Big(c_0^*\big(\frac{\theta_1}{c_0}-\frac{\theta_1^*}{c_0^*}\big)^{\mathrm T} w_1\Big)_t\Big\|_{2\delta} \leqslant c\tau\|(u_p)_t\|_{2\delta} \leqslant c\tau m_f(t), \tag{3.7}$$

$$\tau\|sW_1(s)\|_{\infty\delta} \Big\|\Big(c_0^*\big(\frac{\theta_2}{c_0}-\frac{\theta_2^*}{c_0^*}\big)^{\mathrm T} w_2\Big)_t\Big\|_{2\delta} \leqslant c\tau\|(y_p)_t\|_{2\delta} \leqslant c\tau m_f(t), \tag{3.8}$$

where

$$w_1 = \frac{a(s)}{\Lambda(s)} u_p,\ w_1 = \frac{a(s)}{\Lambda(s)} y_p,$$

c is some constant independent of τ. Therefore, by assumptions 1) and 5), Remark 2, (3.6) ~ (3.8), Lemma 1 and 3.3.2 in [1], we have

$$\|(u_p)_t\|_{2\delta} \leqslant c_2\|(u_p)_t\|_{2\delta} + c_3\tau m_f(t) + c, \tag{3.9}$$

where $0 < c_2 < 1$ and c_3 are some constants independent of τ, which leads to

$$\|(u_p)_t\|_{2\delta} \leqslant c_4 \tau m_f(t) + c, \tag{3.10}$$

where c_4 is a constant independent of τ. For any $\delta \in \left(0, \min\left(\delta_0, \delta_1, \frac{1}{\tau_{\max}}\right)\right)$, combining (3.1), (3.5), and (3.10), we get

$$m_f(t) \leqslant c_5 \tau m_f(t) + c, \tag{3.11}$$

where c_5 is a constant independent of τ. Thus for any $0 < \tau < \frac{1}{c_5}$, from (3.11) we can conclude that $m_f(t) < \infty, t \geqslant 0$. For $\tau \in [\frac{1}{c_5}, \tau_{\max}]$, since $\tau_{\max}$ is finite, thus

$$m_f(t) < c_5 \tau_1 m_f(t) + c_5(\tau_{\max} - \tau_1) m_f(t) < c,$$

where $\tau_1 \in (0, \frac{1}{c_5})$ is a fixed number, c is a constant dependent upon $\tau_{\max}$.

Since the roots of $(\tau s + 1)^i$ equal to $s = -\frac{1}{\tau}$, noticing

$$-\frac{1}{\tau} \leqslant -\frac{1}{\tau_{\max}} \leqslant -\frac{1}{2\tau_{\max}} < -\frac{\delta}{2},$$

we conclude that $W_1(s)$ is analytic in $\mathrm{Re}[s] \geqslant -\frac{\delta}{2}$ for all $\delta \in \left(0, \min\left(\delta_0, \delta_1, \frac{1}{\tau_{\max}}\right)\right)$. From (2.17), (3.4), $m_f(t) < \infty$, Lemma 1 and Lemma 3.3.2 in [1], it follows that

$$|e(t)| \leqslant \tau \| sW_1(s) W_m(s) \|_{2\delta} ((\|(\tilde{\alpha}^{\mathrm{T}}\phi)t\|_{2\delta} + \|(\frac{\Lambda(s) - \theta_1^{*\mathrm{T}} a(s)}{c_0 \Lambda(s)} d)_t\|_{2\delta}) \leqslant$$

$$c\tau(m_f + d_0) \leqslant c\tau(1 + d_0),$$

where c is some constant independent of τ.

4 Conclusion

In this paper, under the assumption that k_p is unknown, we propose a modified model reference adaptive controller, which can greatly improve transient performance of adaptive systems.

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