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一类非线性系统基于Backstepping的自适应鲁棒神经网络控制

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摘要: 针对一类未知非线性系统提出了一种基于Backstepping的自适应神经网络控制方法,放松了满足匹配条件,要求神经网络逼近误差的边界已知等一些限制性的假设.扩展了自适应backstepping和自适应神经控制的适用范围,整个闭环系统表明是最终一致有界的,跟踪误差收敛于原点的一个大小可调的邻域.

关键词: 非线性自适应控制; backstepping; 神经网络; 自适应界化

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Adaptive robust neural network control for a class of nonlinear systems using backstepping

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Abstract: The adaptive neural control scheme was formulated for a class of unknown nonlinear systems based on backstepping technique. The proposed scheme relaxed the requirements of matching condition and of a known bound on the network reconstruction. The method expands the applicable scope of the class of nonlinear systems, which can successfully utilize the backestepping algorithm and adaptive NN control . The resulting closed-loop system is proven to be ultimately uniform bounded, and the output tracking error converges to an adjustable neighborhood of zero.

Key words: nonlinear adaptive control; backstepping; neural network; adaptive bounding

1 引言(Introduction)

在不确定非线性系统中,最受关注的两类不确定性是:参数性不确定和来自建模误差、外部干扰的不确定性.自适应backstepping和鲁棒控制是不确定非线性系统的重要设计工具[1,2],自适应神经网络(NN)控制对具有未知非线性项的系统取得了较好的效果.文[3]对高阶的不满足匹配条件且非线性项未知的一类系统,将径向基(RBF)网络与backstepping相结合,在网络逼近误差的边界已知的假设下提出一种神经网络自适应控制.文[4]中使用多层神经网络作为函数逼近器,结合backstepping,对一类未知非线性系统提出一种稳定的NN自适应设计,但要求网络逼近误差的边界已知.文[5,6]中提出了一种自适应界化理论,对未知的NN逼近误差的边界做自适应估计.本文将NN与backstepping进一步结合,对一类具有外部扰动的高阶未知系统,使用线性参数化NN,基于Lyapunov稳定性理论,对NN权值和逼近误差的上界做自适应估计,提出一种稳定的自适应鲁棒神经网络backstepping设计.

2 问题描述(Problem statement)

对高阶严格反馈系统:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + \Delta_{d_1}(x_1,u,t), \\ \dot{x}_2 = x_3 + f_2(x_1,x_2) + \Delta_{d_2}(x_1,x_2,u,t), \\ \quad\vdots \\ \dot{x}_{n-1} = x_n + f_{n-1}(\bar{x}_{n-1}) + \Delta_{d_{n-1}}(\bar{x}_{n-1},u,t), \\ \dot{x}_n = \beta(x)u + f_n(x) + \Delta_{d_n}(x,u,t), \end{cases} \tag{1}$$

其中 $x=[x_1,x_2,\cdots,x_n]^{\mathrm{T}}\in\mathbb{R}^n$ 为系统状态, $y=x_1\in\mathbb{R}^1$ 为系统输出, $u\in\mathbb{R}^1$ 为系统输入.定义

$$\bar{x}_i=[x_1,\cdots,x_i],\ \bar{y}_r^{(i)}=[y_r,\dot{y}_r,\cdots,y_r^{(i)}],$$

$f_i(\bar{x}_i)$ 是未知的光滑函数, $\Delta_{d_i}(\bar{x}_i,u,t)$ 为来自建模误差,外部扰动的不确定性,设其对所有的 x,u,t 是一致有界的, $i=1,\cdots,n$. $\beta(x)\neq 0$ 为已知函数.控制目标是使输出 $y=x_1$ 跟踪期望轨迹 $y_r(t)$. 由于RBF网络的逼近能力,因此

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$$f_i(\bar{x}_i)=\zeta_i^{\mathrm{T}}(\bar{x}_i)\theta_i^*+\varepsilon_i(\bar{x}_i),$$

其中 $\zeta_i(\bar{x}_i):\mathbb{R}^i\to\mathbb{R}^{m_i}$ 为已知的基函数向量，$\theta_i^*\in\mathbb{R}^{m_i}$ 为网络连接权，m_i 为该网络的隐层神经元个数. $\varepsilon_i(\bar{x}_i)$ 为逼近误差.

$$\delta_i(\bar{x}_i,u,t)=\varepsilon_i(\bar{x}_i)+\Delta_{d_i}(\bar{x}_i,u,t),$$

定义为网络重构误差，它包括了有界的模型不确定性. 定义最优权向量

$$\theta_i^*:=\arg\min_{\theta_i\in\mathbb{R}^{m_i}}\{\sup_{\bar{x}_i\in\Omega_i}|f_i(\bar{x}_i)-\theta_i^{\mathrm{T}}\zeta_i(\bar{x}_i)|\},$$

其中紧集 $\Omega_i\in\mathbb{R}^i$.

假设 2.1 在紧集 $\Omega_i\subset\mathbb{R}^i$ 上，$|\delta_i(\bar{x}_i,u,t)|\leqslant\Psi_i^*$，其中 $\Psi_i^*\geqslant 0$ 为未知上界.

定义

$$\begin{cases}\theta^{*\mathrm{T}}=[\theta_1^{*\mathrm{T}},\cdots,\theta_n^{*\mathrm{T}}],\\ \Psi^{*\mathrm{T}}=[\Psi_1^*,\cdots,\Psi_n^*],\\ \delta^{\mathrm{T}}=[\delta_1,\cdots,\delta_n],\\ \varphi_i^{\mathrm{T}}=\left[0^{\mathrm{T}}_{(1\times\sum_{j=1}^{i-1}m_j)}\,\middle|\,\zeta^{\mathrm{T}}_{i(1\times m_i)}\,\middle|\,0^{\mathrm{T}}_{(1\times\sum_{j=i+1}^{n}m_j)}\right],\end{cases}\tag{2}$$

e_i 为 n 维向量空间单位向量，则系统(1)能够写为：

$$\begin{cases}\dot{x}_1=x_2+\varphi_1^{\mathrm{T}}\theta^*+e_1^{\mathrm{T}}\delta(t),\\ \dot{x}_2=x_3+\varphi_2^{\mathrm{T}}\theta^*+e_2^{\mathrm{T}}\delta(t),\\ \quad\vdots\\ \dot{x}_{n-1}=x_n+\varphi_{n-1}^{\mathrm{T}}\theta^*+e_{n-1}^{\mathrm{T}}\delta(t),\\ \dot{x}_n=\beta(x)u+\varphi_n^{\mathrm{T}}\theta^*+e_n^{\mathrm{T}}\delta(t).\end{cases}\tag{3}$$

系统(3)具有两种类型的不确定性：未知 θ^* 引起的参数不确定性和未知上界 Ψ^* 引起的边界不确定性. 设 $\hat{\theta}$ 和 $\hat{\Psi}$ 分别为 θ^* 和 Ψ^* 的估计值，$\tilde{\theta}=\theta^*-\hat{\theta}$，$\tilde{\Psi}=\Psi^*-\hat{\Psi}$. 标记：

$$|x|_1=\sum_{i=1}^n|x_i|,$$

$$|x|=\left(\sum_{i=1}^n|x_i|^2\right)^{1/2},$$

$$|x|_\infty=\max_i|x_i|.$$

3 自适应神经网络 backsepping 设计(Adaptive neural network backstepping design)

由于存在 NN 的逼近误差和未知扰动，重构误差 $\delta(t)\neq 0$，其边界 Ψ^* 未知，本文对未知参数 θ^* 和 Ψ^* 在线做自适应估计，提出如下设计算法：

$$x_0=0,\ z_0=0,\ \tau_0=0,\ \iota_0=0.$$

坐标变换：

$$z_i=x_i-y_r^{(i-1)}-\alpha_{i-1}.\tag{4}$$

镇定函数：

$$\begin{aligned}&\alpha_i(\bar{x}_i,\hat{\theta},\hat{\Psi},\bar{y}_r^{(i-1)})=\\&-z_{i-1}-c_iz_i-\omega_i^{\mathrm{T}}\hat{\theta}+\\&\sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}x_{k+1}+\frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right)+\\&\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma(\tau_i-\sigma\hat{\theta})+\sum_{k=2}^{i-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma\omega_iz_k-\bar{\omega}_i^{\mathrm{T}}\hat{\Psi}+\\&\sum_{k=2}^{i-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\Psi}}\gamma\bar{\omega}_iz_k+\frac{\partial\alpha_{i-1}}{\partial\hat{\Psi}}\gamma(\iota_i-\sigma\hat{\Psi}).\end{aligned}\tag{5}$$

对 $\hat{\theta}$ 的调节函数：

$$\tau_i(\bar{x}_i,\hat{\theta},\hat{\Psi},\bar{y}_r^{(i-1)})=\tau_{i-1}+\omega_iz_i.\tag{6}$$

对 $\hat{\Psi}$ 的调节函数：

$$\iota_i(\bar{x}_i,\hat{\theta},\hat{\Psi},\bar{y}_r^{(i-1)})=\iota_{i-1}+\bar{\omega}_iz_i.\tag{7}$$

回归向量：

$$\omega_i(\bar{x}_i,\hat{\theta},\hat{\Psi},\bar{y}_r^{(i-2)})=\varphi_i-\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k,\tag{8}$$

$$\begin{aligned}&v_i(\bar{x}_i,\hat{\theta},\hat{\Psi},\bar{y}_r^{(i-2)})=\\&[v_{i1},\cdots,v_{in}]^{\mathrm{T}}=e_i-\sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}e_k,\end{aligned}\tag{9}$$

$$\begin{aligned}&\bar{\omega}_i(\bar{x}_i,\hat{\theta},\hat{\Psi},y_r^{(i-2)})=\\&[v_{i1}\tanh(\frac{v_{i1}z_i}{\varepsilon}),v_{i2}\tanh(\frac{v_{i2}z_i}{\varepsilon}),\cdots,\\&v_{in}\tanh(\frac{v_{in}z_i}{\varepsilon})]^{\mathrm{T}},\end{aligned}\tag{10}$$

$i=1,\cdots,n$，其中 $\varepsilon>0$ 为设计常数.

回归矩阵：

$$\begin{cases}W=[\omega_1,\cdots,\omega_n],\\ V=[v_1,\cdots,v_n],\\ U=[\bar{\omega}_1,\cdots,\bar{\omega}_n].\end{cases}\tag{11}$$

控制律：

$$u=\frac{1}{\beta(x)}[\alpha_n(x,\hat{\theta},\hat{\Psi},\bar{y}_r^{(n-1)})+y_r^{(n)}].\tag{12}$$

参数自适应律：

$$\dot{\hat{\theta}}=\Gamma(\tau_n-\sigma\hat{\theta})=\Gamma(Wz-\sigma\hat{\theta}),\tag{13}$$

$$\dot{\hat{\Psi}}=\gamma(\iota_n-\sigma\hat{\Psi})=\gamma(Uz-\sigma\hat{\Psi}),\tag{14}$$

其中 $c_i>0$，$\Gamma=\Gamma'>0$，$\gamma=\gamma'>0$.

定理 3.1 对系统(3)，控制律(12)，参数自适应律(13)，(14)构成的闭环系统具有如下性质：

1) 闭环系统的所有信号是一致有界的；

2) 跟踪误差均方值满足

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t z_1^2(\tau)\mathrm{d}\tau\leqslant\lambda/c_0,$$

$\lambda>0$，$c_0>0$ 为常数.

证　记 $x_{n+1}=\beta(x)u$，误差变量 z_i 的导数为($i=1,\cdots,n$)

$$
\begin{aligned}
\dot z_i &= \dot x_i - y_r^{(i)} - \dot\alpha_{i-1} = \\
&x_{i+1}+\varphi_i^{\mathrm T}\theta + e_i^{\mathrm T}\delta - y_r^{(i)} - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}(x_{k+1}+\varphi_k^{\mathrm T}\theta + \\
&e_k^{\mathrm T}\delta) - \sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)} - \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta} - \frac{\partial\alpha_{i-1}}{\partial\hat\Psi}\dot{\hat\Psi} = \\
&z_{i+1}-z_{i-1}-c_iz_i+\omega_i^{\mathrm T}\tilde\theta + v_i^{\mathrm T}\delta + \\
&\frac{\partial\alpha_{i-1}}{\partial\hat\theta}\Gamma(\tau_i-\sigma\hat\theta)+\sum_{k=2}^{i-1}\frac{\partial\alpha_{k-1}}{\partial\hat\theta}\Gamma\omega_i z_k - \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta}+ \\
&\frac{\partial\alpha_{i-1}}{\partial\hat\Psi}\gamma(\tau_i-\sigma\hat\Psi)+\sum_{k=2}^{i-1}\frac{\partial\alpha_{k-1}}{\partial\hat\Psi}\gamma\bar\omega_i z_k - \\
&\bar\omega_i^{\mathrm T}\hat\Psi - \frac{\partial\alpha_{i-1}}{\partial\hat\Psi}\dot{\hat\Psi}.
\end{aligned}
$$

记

$$
\sigma_{k,i} = -\frac{\partial\alpha_{k-1}}{\partial\hat\theta}\Gamma\omega_i,\ \delta_{k,i} = -\frac{\partial\alpha_{k-1}}{\partial\hat\Psi}\gamma\bar\omega_i,
$$

闭环误差系统方程为

$$
\begin{cases}
\dot z = A_z(z,\hat\theta,\hat\Psi,t)z + W^{\mathrm T}\tilde\theta + V^{\mathrm T}\delta - U^{\mathrm T}\hat\Psi, \\
\dot{\hat\theta} = \Gamma(\tau_n - \sigma\hat\theta), \\
\dot{\hat\Psi} = \gamma(\iota_n - \sigma\hat\Psi),
\end{cases}
\tag{15}
$$

其中

$$
A_z = \begin{bmatrix}
-c_1 & 1 & 0 & \cdots & 0 \\
-1 & -c_2 & 1+\sigma_{2,3}+\delta_{2,3} & \cdots & \sigma_{2,n}+\delta_{2,n} \\
0 & -1-\sigma_{2,3}-\delta_{2,3} & -c_3 & \ddots & \vdots \\
\vdots & \vdots & \ddots & \ddots & 1+\sigma_{n-1,n}+\delta_{n-1,n} \\
0 & -\sigma_{2,n}-\delta_{2,n} & \cdots & -1-\sigma_{n-1,n}-\delta_{n-1,n} & -c_n
\end{bmatrix}
\tag{16}
$$

选择 Lyapunov 函数

$$
V = 1/2[z^{\mathrm T}z + \tilde\theta^{\mathrm T}\Gamma^{-1}\tilde\theta + \tilde\Psi^{\mathrm T}\gamma^{-1}\tilde\Psi].
$$

$$
\begin{aligned}
\dot V &= z^{\mathrm T}\dot z - \tilde\theta^{\mathrm T}\Gamma^{-1}\dot{\hat\theta} - \tilde\Psi^{\mathrm T}\gamma^{-1}\dot{\hat\Psi} = \\
&\sum_{i=1}^{n}[-c_iz_i^2 + z_i\omega_i^{\mathrm T}\tilde\theta + z_iv_i^{\mathrm T}\delta - z_i\bar\omega_i^{\mathrm T}\hat\Psi] - \\
&\tilde\theta^{\mathrm T}(Wz-\sigma\hat\theta) - \tilde\Psi^{\mathrm T}(Uz-\sigma\hat\Psi) \leqslant \\
&-c_0|z|^2 + \sigma\tilde\theta^{\mathrm T}\hat\theta + \sum_{i=1}^{n}(z_iv_i^{\mathrm T}\delta - z_i\bar\omega_i^{\mathrm T}\hat\Psi) - \\
&\tilde\Psi^{\mathrm T}(Uz-\sigma\hat\Psi),\ c_0 = \min_{1\leqslant i\leqslant n} c_i.
\end{aligned}
\tag{17}
$$

令

$$
\begin{aligned}
\Lambda &= \sum_{i=1}^{n}(z_iv_i^{\mathrm T}\delta - z_i\bar\omega_i^{\mathrm T}\hat\Psi) - \tilde\Psi^{\mathrm T}(Uz-\sigma\hat\Psi) \leqslant \\
&\sum_{i=1}^{n}[-z_i\bar\omega_i^{\mathrm T}(\Psi^* - \tilde\Psi) + |z_i|p^{\mathrm T}\Psi^*] - \\
&\tilde\Psi^{\mathrm T}(Uz-\sigma\hat\Psi) = \\
&\sum_{i=1}^{n}[(|z_i|p - z_i\bar\omega_i)^{\mathrm T}\Psi^* + z_i\bar\omega_i^{\mathrm T}\tilde\Psi] - \\
&\tilde\Psi^{\mathrm T}(Uz-\sigma\hat\Psi) = \\
&\sum_{i=1}^{n}(|z_i|p - z_i\bar\omega_i)^{\mathrm T}\Psi^* + \sigma\tilde\Psi^{\mathrm T}\hat\Psi,
\end{aligned}
\tag{18}
$$

其中 $p=[|v_{i,1}|,\cdots,|v_{i,n}|]$.

由文[5]中引理得向量 $|z_i|p - z_i\bar\omega_i$ 的第j个分量($j=1,\cdots,n$)满足

$$
|z_iv_{i,j}| - z_iv_{i,j}\tanh\left(\frac{z_iv_{i,j}}{\varepsilon}\right) \leqslant \kappa\varepsilon,
$$

所以

$$
\Lambda \leqslant n\kappa\varepsilon|\Psi^*|_1 + \sigma\tilde\Psi^{\mathrm T}\hat\Psi = \bar\varepsilon|\Psi^*|_1 + \sigma\tilde\Psi^{\mathrm T}\hat\Psi,
$$

其中 $\bar\varepsilon = n\kappa\varepsilon$. 代入式(17)得

$$
\begin{aligned}
\dot V \leqslant &-c_0|z|^2 - \frac{\sigma}{2}|\tilde\theta|^2 - \frac{\sigma}{2}|\tilde\Psi|^2 + \\
&\bar\varepsilon|\Psi^*|_1 + \frac{\sigma}{2}|\theta^*|^2 + \frac{\sigma}{2}|\Psi^*|^2,
\end{aligned}
\tag{19}
$$

则

$$
\dot V \leqslant -cV + \lambda,
$$

$$
c := \min\left\{2c_0, \frac{\sigma}{\lambda_{\min}(\Gamma^{-1})}, \frac{\sigma}{\lambda_{\min}(\gamma^{-1})}\right\},
$$

$$
\lambda := \bar\varepsilon|\Psi^*|_1 + \frac{\sigma}{2}|\theta^*|^2 + \frac{\sigma}{2}|\Psi^*|^2,
$$

所以 $0\leqslant V(t)\leqslant V(0)\mathrm e^{-ct}+\lambda/c$，$z(t)$，$\hat\theta(t)$，$\hat\Psi(t)$ 都是一致有界的，从而 $x(t)$ 也是一致有界的. 由式(19)得

$$
\dot V \leqslant -c_0|z|^2 + \lambda = -c_0\sum_{i=1}^{n}z_i^2 + \lambda,
$$

对其两边在 $[0,t]$ 上积分得

$$
\int_0^t z_1^2(\tau)\mathrm d\tau \leqslant \frac{1}{c_0}[v(0)+t\lambda],
$$

$$
\lim_{t\to\infty}\frac{1}{t}\int_0^t z_1^2(\tau)\mathrm d\tau \leqslant \lambda/c_0.
$$

4　仿真研究(Simulation studies)

考虑下面的二阶系统:

$$
\begin{cases}
\dot x_1 = x_2 + \varphi_1(x_1) + \Delta_{d_1}, \\
\dot x_2 = u, \\
y = x_1,
\end{cases}
$$

其中 $\varphi_1(x_1) = x_1 + x_2^2$ 假定为未知函数,$\Delta_{d_1} = 0.01\sin(x_1)$,代表系统不确定性.期望轨迹 $y_r = \sin(0.5t)$. RBF 网络选取高斯型函数,

$$\zeta(x) = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{(x-c_i)^2}{2\sigma^2}\right),$$

$\sigma = 0.25$, c_i 在[0,1]中选择,神经元个数为30,参数初值为0,状态初值为$[1,0]^T$. 仿真结果见图1.

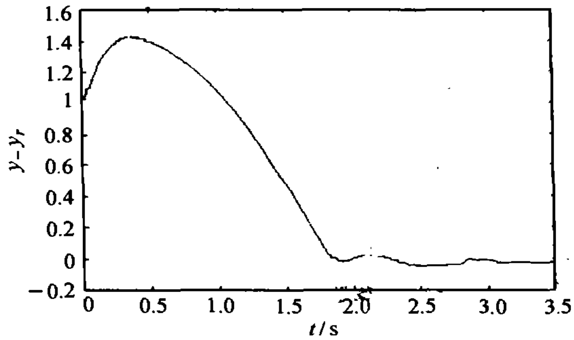

图 1 跟踪误差曲线

Fig. 1 The curve of tracking error

5 结论(Conclusion)

本文对一类未知非线性高阶系统结合 backstepping 和 NN 技术提出了一种稳定的自适应设计,保证了闭环系统的稳定性,对有界的外部扰动具有鲁棒性,跟踪误差收敛于原点的邻域,其大小可通过设计参数来调节.仿真结果表明了算法的可行性.

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