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随机非线性时滞大系统的输出反馈分散镇定

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摘要：针对具有严格反馈形式的随机非线性时滞大系统，设计了含有时滞项的随机控制 Lyapunov 函数，运用 Backstepping 技术，构造出一类输出反馈无记忆控制器．在此控制器作用下，所考虑的闭环系统实现概率意义上的时滞无关全局渐近稳定．并在无限时区优化指标函数的约束下，对控制器进行逆优再设计，以满足一定的性能要求．

关键词：随机非线性时滞大系统；状态观测器；输出反馈分散镇定；反向递推；逆优
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Decentralized output feedback stabilization for large scale stochastic nonlinear system with time delays

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Abstract: Recursive design scheme of decentralized output feedback global stabilization in probability was presented for a class of large-scale parameter strict-feedback nonlinear stochastic systems with time delay. A new form of stochastic control Lyapunov function with time delay was proposed to construct decentralized output feedback memoryless controllers which achieved global asymptotic stability in probability for the considered systems. The inverse optimal controllers are redesigned with cost functional includes penalty on control effort, and when it has an infinite gain margin.

Key words: large-scale stochastic nonlinear system; state observer; decentralized output-feedback stabilization; backstepping; inverse optimality

1 引言(Introduction)

工业系统和社会经济等领域存在的许多问题，以及自然界中的各类现象，其动态规律都可以用确定性模型和随机模型来描述．在理想化或对系统精度要求不高的情况下，系统常表述为确定性模型．当对于系统研究有较高的精度要求时，就必须充分考虑随机因素的影响[1]．

非线性系统理论在近20年中取得了许多重要的成果，获得了许多有意义的结果．例如 Sontag 引入了控制 Lyapunov 函数[2]和输入状态稳定(ISS)的概念[3]，Teel 进一步发展了 ISS 小增益定理[4]．在此基础上，非线性系统的构造性控制得到了很大的发展[5]．

近5年来，随着随机稳定性理论中许多概念的提出[6]，相当一部分确定型非线性系统的研究结果被拓展到随机框架下[7]．在研究随机镇定问题的同时，人们又将注意力集中到基于性能指标的控制上，并引入了逆优设计的思想[8]．所谓逆优设计，是指依据某个具有一定逻辑意义的性能指标来设计控制器，该性能指标的一些组成部分预先没有给定，将随控制器设计的需要而定．

由于传输、测量、反应等因素的影响，时滞在现实系统中普遍存在着，它们常常是引起系统不稳定的一个重要因素．时滞系统鲁棒控制是近年来的研究热点，由于随机时滞系统控制有广泛的应用前途，逐渐受到了人们的重视[9]．目前，有关随机非线性时滞系统鲁棒控制的研究报道还不多见，尤其是利用反向递推技术实现非线性时滞大系统输出反馈的文献更少．这是因为，利用反向递推技术构造控制器的过程中，控制 Lyapunov 函数的选取是一个难题，选取不当将会造成时滞项在递推过程中发散．本文将对此进行较为详细的分析．

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本文主要研究了随机非线性时滞系统的输出反馈分散镇定问题.首先,依据提出的状态观测器,构造了便于反推设计的增广系统;接着通过提出一种含有时滞的Lyapunov函数,构造出输出反馈无记忆分散控制器;最后,在满足一定性能指标的前提下,对控制器进行了逆优再设计.

2 系统描述(System description)

考虑一类由 N 个子系统组成的随机非线性时滞大系统:

$$\begin{aligned}&\mathrm{d}x_{ij}(t) = \\&x_{i,j+1}(t)\mathrm{d}t + \phi_{ij}(y(t))\mathrm{d}t + \\&f_{ij}(y_i(t), y_i(t-\tau_{i1}), \cdots, y_i(t-\tau_{i,M_i}))\mathrm{d}t + \\&g_{ij}^{\mathrm{T}}(y_i(t))\mathrm{d}w_i,\ 1 \leqslant i \leqslant N,\ 1 \leqslant j \leqslant n_i-1,\\&\mathrm{d}x_{i,n_i}(t) = \\&u_i(t)\mathrm{d}t + \phi_{i,n_i}(y(t))\mathrm{d}t + \\&f_{i,n_i}(y_i(t), y_i(t-\tau_{i1}), \cdots, y_i(t-\tau_{i,M_i}))\mathrm{d}t + \\&g_{i,n_i}^{\mathrm{T}}(y_i(t))\mathrm{d}w_i,\\&y_i(t) = x_{i1}(t).\end{aligned} \tag{1}$$

其中

$$x_i(t) = [x_{i1}(t), \cdots, x_{i,n_i}(t)]^{\mathrm{T}} \in \mathbb{R}^{n_i}$$

为第 i 个子系统的状态($i = 1,2,\cdots,N$),$y_i(t) \in \mathbb{R}$ 为第 i 个子系统的输出,且 $y(t) = [y_1(t), \cdots, y_N(t)]^{\mathrm{T}} \in \mathbb{R}^N$,$u_i(t) \in \mathbb{R}$ 为第 i 个子系统的控制输入,$\tau_{i1}, \cdots, \tau_{i,M_i} > 0$ 表示系统多重状态滞后,$f_{ij}(y_i(t), y_i(t-\tau_{i1}), \cdots, y_i(t-\tau_{i,M_i}))$ 为非线性函数,表示第 i 个子系统中的非线性成分,$\phi_{ij}(y(t))$ 表示第 i 个子系统和其它子系统之间的交互作用,它们均为局部Lipschitz连续函数.ω_i 是定义在具有自然流 $\{F_t\}_{t\geqslant 0}$($F_t = \sigma\{\omega_i(s): 0 \leqslant s \leqslant t\}$)的完备概率空间($\Omega, F, P$)上的相互独立的布朗运动.

本文为了便于推导,引入下列假设:

假设 1 系统中 $f_{ij}(\cdot)$ 和 $g_{ij}(\cdot)$ 为光滑函数,且满足条件 $f_{ij}(0) = 0$,满足下列约束条件:

1) $$f_{ij}(y_i(t), y_i(t-\tau_{i1}), \cdots, y_i(t-\tau_{i,M_i})) \leqslant \sum_{m=0}^{M_i} \pi_{ijm} \hat{f}_{ijm}(y_i(t-\tau_{i,m})). \tag{2}$$

其中,多项式系数 $\pi_{ijm} > 0$,$\tau_0 = 0$,$\hat{f}_{ijm}(\cdot)$ 为光滑函数,满足 $\hat{f}_{ijm}(0) = 0$.

2) 由于 $\hat{f}_{ijm}(\cdot)$ 和 $g_{ij}(\cdot)$ 均为光滑函数,满足 $\hat{f}_{ijm}(0) = 0$ 和 $g_{ij}(\cdot) = 0$,因而存在光滑函数 $\bar{f}_{ijm}(\cdot)$ 和 $\bar{g}_{ij}(\cdot)$,使得下面等式成立:

$$\begin{aligned}&\hat{f}_{ijm}(y_i(t-\tau_{i,m})) = \\&y_i(t-\tau_{i,m})\bar{f}_{ijm}(y_i(t-\tau_{i,m})),\ m = 0, \cdots, M_i,\end{aligned} \tag{3}$$

$$\begin{aligned}&g_{ij}^{\mathrm{T}}(y_i(t)) = y_i(t)\bar{g}_{ij}^{\mathrm{T}}(y_i(t)),\\&1 \leqslant i \leqslant N,\ 1 \leqslant j \leqslant n_i.\end{aligned} \tag{4}$$

假设 2 系统中的关联项 $\phi_{ij}(\cdot)$ 满足下列条件:

$$|\phi_{ij}(y(t))| \leqslant \sum_{l=1}^{N} \eta_{ijl}\zeta_{ijl}(|y_l(t)|). \tag{5}$$

其中,η_{ijl} 为非负常数,$\zeta_{ijl}(\cdot)$ 为非负光滑函数,且 $\zeta_{ijl}(0) = 0$,故存在光滑函数 $\bar{\zeta}_{ijl}(\cdot)$,使得

$$\zeta_{ijl}(|y_l(t)|) = |y_l(t)|\bar{\zeta}_{ijl}(|y_i(t)|). \tag{6}$$

注 1 在假设1中,系统非线性时滞函数

$$f_{ij}(y_i(t), y_i(t-\tau_{i1}), \cdots, y_i(t-\tau_{i,M}))$$

满足组合非线性函数不等式约束.由于函数的光滑性和原点的零值性,运用中值定理可以得到函数的分解表示式(3)和(4)[11].

注 2 在假设2中,第 i 个子系统和其他子系统之间关联项 $\phi_{ij}(\cdot)$ 的范数小于各个子系统输出变量范数的组合非线性函数不等式限制,其非线性函数可以表示为第 i 个子系统的输出变量范数和其他子系统输出变量非线性函数的乘积.假设2物理意义可以解释如下:其他子系统通过输出变量对第 i 个子系统产生作用,作用量大小满足线性增长约束,其增长系数为第 i 个子系统输出变量的非线性函数.

引理 1[7,11] 若存在 $\varepsilon > 0$,常数 $p > 1$ 和 $q > 1$,且满足 $(p-1)(q-1) = 1$,则对于 $(x, y) \in \mathbb{R}^2$ 有下列不等式成立:

$$xy \leqslant \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q, \tag{7}$$

此不等式称为Young不等式.

3 主要结果(Main results)

3.1 状态观测器设计(State observer design)

系统(1)中的状态变量不可直接得到,故首先设计如下状态观测器进行状态估计:

$$\begin{cases}\dot{\hat{x}}_{ij}(t) = \hat{x}_{i,j+1}(t) + k_{ij}(y_i(t) - \hat{x}_{i1}(t)) + \\ \qquad f_{ij}(y_i(t), y_i(t-\tau_{i1}), \cdots, y_i(t-\tau_{i,M_i})),\\ \hat{x}_{i,n_i+1}(t) = u_i(t).\\ 1 \leqslant i \leqslant N,\ 1 \leqslant j \leqslant n_i,\end{cases} \tag{8}$$

令观测器误差为 $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$,故有

$$\mathrm{d}\tilde{x}_i(t) = A_i\tilde{x}_i(t)\mathrm{d}t + \phi_i(y(t))\mathrm{d}t + g_i^{\mathrm{T}}(y_i(t))\mathrm{d}w_i, \tag{9}$$

其中

$$x_i(t) = [x_{i1}(t), \cdots, x_{i,n_i}(t)]^{\mathrm{T}},$$

$$\hat{x}_i(t) = [\hat{x}_{i1}(t), \cdots, \hat{x}_{i,n_i}(t)]^{\mathrm{T}},$$

$$\tilde{x}_i(t) = [\tilde{x}_{i1}(t),\cdots,\tilde{x}_{i,n_i}(t)]^{\mathrm T},$$

$$A_i = \begin{bmatrix} -k_{i1} & & \\ \vdots & & I \\ -k_{i,n_i} & 0 & \cdots & 0 \end{bmatrix},$$

$$\phi_i(y(t)) = \begin{bmatrix} \phi_{i1}(y(t)) \\ \vdots \\ \phi_{i,n_i}(y(t)) \end{bmatrix},$$

$$g_i^{\mathrm T}(y_i(t)) = \begin{bmatrix} g_{i1}^{\mathrm T}(y_i(t)) \\ \vdots \\ g_{i,n_i}^{\mathrm T}(y_i(t)) \end{bmatrix}.$$

选择合适的观测器增益 k_{ij},使得 A_i 为渐近稳定矩阵.此时,随机非线性时滞大系统可以表示为

$$\begin{cases} \mathrm d\tilde{x}_i(t) = A_i\tilde{x}_i(t)\mathrm dt + \phi_i(y(t))\mathrm dt + g_i^{\mathrm T}(y_i(t))\mathrm dw_i, \\ \mathrm dy_i(t) = [\hat{x}_{i2}(t) + \tilde{x}_{i2}(t) + \phi_{i1}(y(t))]\mathrm dt + \\ \qquad f_{i1}(y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M}))\mathrm dt + g_{i1}^{\mathrm T}(y_i(t))\mathrm dw_i, \\ \mathrm d\hat{x}_{i2}(t) = [\hat{x}_{i3}(t) + k_{i2}(y_i(t) - \hat{x}_{i1}(t)) + \\ \qquad f_{i2}(t,y_i(t),y_i(t-\tau_{i1}),\cdots, y_i(t-\tau_{i,M}))]\mathrm dt, \\ \qquad \vdots \\ \mathrm d\hat{x}_{i,n_i}(t) = [u_i(t) + k_{i,n_i}(y_i(t) - \hat{x}_{i1}(t)) + \\ \qquad f_{i,n_i}(t,y_i(t),y_i(t-\tau_{i1}),\cdots, y_i(t-\tau_{i,M}))]\mathrm dt. \end{cases} \tag{10}$$

明显可以看出,系统(10)具有三角结构,本文将运用反向递推(backstepping)技术对此系统构造输出反馈控制器.

3.2 控制器设计(Controller design)

定义误差变量 $z_{ij}(t)(1 \leqslant i \leqslant N,2 \leqslant j \leqslant n_i)$ 为

$$\begin{cases} z_{i1}(t) = y_i(t), \\ z_{ij}(t) = \hat{x}_{ij}(t) - \alpha_{i,j-1}(\bar{\hat{x}}_{i,j-1}(t),y_i(t)). \end{cases} \tag{11}$$

其中 $\bar{\hat{x}}_{ij} = [\hat{x}_{i2},\cdots,\hat{x}_{ij}]^{\mathrm T}$, 函数 $\alpha_{i,j-1}(\cdot)$ 为虚拟控制,满足平衡条件,即

$$\alpha_{i,j-1}(0,0) = 0. \tag{12}$$

虚拟控制的具体形式本文将在反推过程中得到.

根据 Itô 微分法则[1],有

$$\mathrm dz_{i1}(t) = [\hat{x}_{i2}(t) + \tilde{x}_{i2}(t) + \phi_{i1}(y(t))]\mathrm dt + f_{i1}(y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i}))\mathrm dt + g_{i1}^{\mathrm T}(y_i(t))\mathrm dw_i,$$

$$\begin{aligned} \mathrm dz_{ij}(t) = {} & [\hat{x}_{i,j+1}(t) + k_{ij}\tilde{x}_{i1}(t) + f_{ij}(t,y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i})) - \\ & \sum_{l=2}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}(t)}(\hat{x}_{i,l+1}(t) + k_{il}\tilde{x}_{i1}(t) + f_{il}(t,y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i}))) - \\ & \frac{\partial\alpha_{i,j-1}}{\partial y_i(t)}(\hat{x}_{i2}(t) + \tilde{x}_{i2}(t) + \phi_{i1}(y(t)) + f_{i1}(t,y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i}))) - \\ & \frac{1}{2}\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2(t)}g_{i1}^{\mathrm T}(y_i(t))g_{i1}(y_i(t))]\mathrm dt - \frac{\partial\alpha_{i,j-1}}{\partial y_i(t)}g_{i1}^{\mathrm T}(y_i(t))\mathrm dw_i, \\ \hat{x}_{i,n_i+1}(t) = {} & u_i(t). \end{aligned} \tag{13}$$

定义 Lyapunov 函数为

$$V(\tilde{x},y,z):= \sum_{i=1}^{N}[\frac{b_i}{2}(\tilde{x}_i^{\mathrm T}P_i\tilde{x}_i)^2 + \frac{1}{4}y_i^4 + \frac{1}{4}\sum_{j=2}^{n_i}z_{ij}^4 + \sum_{m=1}^{M_i}\int_{t-\tau_{i,m}}^{t}\Phi_{im}(y_i(s))\mathrm ds]. \tag{14}$$

其中 P_i 是正定矩阵,且满足 $A_i^{\mathrm T}P_i + P_iA_i = -I$, $\Phi_{i,m}(\cdot)$ 为非负的待定函数.

由 Itô 微分法则, $V(\tilde{x},y,z)$ 沿系统(10)的微分生成算子为

$$\begin{aligned} LV = {} & \sum_{i=1}^{N}[-b_i\tilde{x}_i^{\mathrm T}P_i\tilde{x}_i\|\tilde{x}_i\|^2 + 2b_i\tilde{x}_i^{\mathrm T}P_i\tilde{x}_i\phi_i(y)P_i\tilde{x}_i + \\ & 2b_i\mathrm{tr}\{g_i(y_i)(2P_i\tilde{x}_i\tilde{x}_i^{\mathrm T}P_i + \tilde{x}_i^{\mathrm T}P_i\tilde{x}_iP_i)g_i^{\mathrm T}(y_i)\}] + \\ & \sum_{i=1}^{N}[y_i^3(\alpha_{i1} + z_{i2} + \tilde{x}_{i2} + \phi_{i1}(y_i) + f_{i1}(y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i}))) + \\ & \frac{3}{2}y_i^2g_{i1}^{\mathrm T}(y_i)g_{i1}(y_i)] + \\ & \sum_{i=1}^{N}\sum_{j=2}^{n_i}z_{ij}^3[\alpha_{ij} + z_{i,j+1} + k_{ij}\tilde{x}_{i1} + f_{ij}(y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i})) - \\ & \sum_{l=2}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}}(\hat{x}_{i,l+1} + k_{il}\tilde{x}_{i1} + f_{il}(y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i}))) - \\ & \frac{\partial\alpha_{i,j-1}}{\partial y_i}(\hat{x}_{i2}(t) + \tilde{x}_{i2}(t) + \phi_{i1}(y(t)) + f_{i1}(y_i(t),y_i(t-\tau_{i1}),\cdots,y_i(t-\tau_{i,M_i}))) - \end{aligned}$$

$$\frac{1}{2}\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2(t)}g_{i1}^{\mathrm{T}}(y_i(t))g_{i1}(y_i(t))] +$$

$$\frac{3}{2}\sum_{i=1}^{N}\sum_{j=2}^{n_i}z_{ij}^2(\frac{\partial\alpha_{i,j-1}}{\partial y_i})^2g_{i1}^{\mathrm{T}}(y_i)g_{i1}(y_i) +$$

$$\sum_{i=1}^{N}\sum_{m=1}^{M_i}[\Phi_{im}(y_i(t)) - \Phi_{im}(y_i(t-\tau_{i,m}))]. \tag{15}$$

运用引理1,有下列不等式:

$$LV \leqslant \sum_{i=1}^{N}[-M_{i0}\|\tilde{x}_i\|^4 + y_i^3(\alpha_{i1}+M_{i1}) +$$

$$\sum_{j=2}^{n_i-1}z_{ij}^3(\alpha_{ij}+M_{ij}) + z_{i,n_i}^3(u_i + M_{i,n_i}) +$$

$$\sum_{m=1}^{M}\pi_{ijm}^4(\sum_{j=2}^{n_i}\frac{1}{4\gamma_{ij}^4} + \sum_{j=2}^{n_i}\frac{1}{4\gamma_{ij}^4} + \sum_{j=2}^{n_i}\sum_{l=2}^{j-1}\frac{1}{4\mu_{ijl}^4} +$$

$$\frac{1}{4\xi_{i1}^4})y_i^4(t-\tau_{i,m})\|\bar{f}_{ijm}(y_i(t-\tau_{i,m}))\|^4 +$$

$$\sum_{m=1}^{M_i}(\Phi_{im}(y_i(t)) - \Phi_{im}(y_i(t-\tau_i)))]. \tag{16}$$

其中

$$M_{i0} =$$

$$b_i\lambda_{\min}(P_i) - 3b_i(n_i)^{\frac{3}{2}}\kappa_i^2\|P_i\|^4 -$$

$$\frac{1}{4\epsilon_{i1}^4} - b_i\lambda_{\max}(P_i)(\frac{\|P_i\|^2}{\rho_i} +$$

$$\beta\rho_i^2b_i\lambda_{\max}(P_i)\sum_{j=1}^{n_i}\bar{\omega}_{ij}^2) - \sum_{j=2}^{n_i}\frac{1}{4\epsilon_{ij}^4}, \tag{17}$$

$$\beta = \sum_{l=1}^{N}\beta_l,\ \bar{\omega}_{ij} = \sum_{l=1}^{N}\eta_{ijl}^2. \tag{18}$$

$$M_{i1} =$$

$$\frac{3}{4}\delta_{i1}^{\frac{4}{3}}y_i + \frac{3}{4}\epsilon_{i1}^{\frac{4}{3}}y_i + \pi_{i10}y_i\bar{f}_{i10}(y_i) +$$

$$\frac{3}{4}\xi_{i1}^{\frac{4}{3}}y_i + \frac{3}{4}\sum_{j=2}^{n_i}\xi_{ij}^2y_i(\bar{g}_{i1}^{\mathrm{T}}(y_i)\bar{g}_{i1}(y_i))^2 +$$

$$\frac{3b_i(n_i)^{\frac{3}{2}}}{\kappa_i^2}y_i\|\bar{g}_i(y_i)\|^4 + \frac{1}{4}\sum_{l=1}^{N}(\eta_{i1l}^2 + \frac{1}{\sigma_{i1l}})y_i +$$

$$\frac{3}{2}y_i\bar{g}_{i1}^{\mathrm{T}}(y_i)\bar{g}_{i1}(y_i) + \sum_{l=1}^{N}\sigma_{i1l}\bar{\zeta}_{i1l}^4(|y_i|)y_i +$$

$$\sum_{j=2}^{n_i}\sum_{l=1}^{N}\sigma_{i1l}\bar{\zeta}_{i1l}^4(|y_i|)y_i +$$

$$\frac{1}{4}\sum_{j=1}^{n_i}\sum_{l=1}^{N}\frac{1}{\beta_i}\bar{\zeta}_{i1l}^4(|y_i|)y_i, \tag{19}$$

$$M_{ij} =$$

$$k_{ij}\tilde{x}_{i1} + \pi_{ij0}y_i\bar{f}_{ij0}(y_i) + \frac{3}{4}\delta_{ij}^{\frac{4}{3}}z_{ij} -$$

$$\sum_{l=2}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}}(\hat{x}_{i,l+1} + k_{il}\tilde{x}_{i1} + \pi_{il0}y_i\bar{f}_{il0}(y_i)) -$$

$$\frac{\partial\alpha_{i,j-1}}{\partial y_i}\hat{x}_{i2} + \frac{1}{4\delta_{i,j-1}^4}z_{ij} + \frac{3}{4}\lambda_{ij}^{\frac{4}{3}}z_{ij} -$$

$$\frac{1}{2}\frac{\partial^2\alpha_{i,j-1}}{\partial y_i^2}g_{i1}^{\mathrm{T}}(y_i)g_{i1}(y_i) +$$

$$\frac{3}{4}\epsilon_{ij}^{\frac{4}{3}}(\frac{\partial\alpha_{i,j-1}}{\partial y_i})^{\frac{4}{3}}z_{ij} + \frac{3}{4}\xi_{ij}^{-2}(\frac{\partial\alpha_{i,j-1}}{\partial y_i})^4z_{ij} +$$

$$\frac{3}{4}\gamma_{ij}^{\frac{4}{3}}(\frac{\partial\alpha_{i,j-1}}{\partial y_i})^{\frac{4}{3}}z_{ij} + \frac{3}{4}\sum_{l=2}^{j-1}\mu_{ijl}^{\frac{4}{3}}(\frac{\partial\alpha_{i,j-1}}{\partial\hat{x}_{il}})^{\frac{4}{3}}z_{ij} +$$

$$\frac{1}{4}\sum_{l=1}^{N}[\eta_{i1l}^2(\frac{\partial\alpha_{i,j-1}}{\partial y_i})^2 + \frac{1}{\sigma_{i1l}}]z_{ij},\ 2\leqslant j\leqslant n_i-1, \tag{20}$$

$$M_{i,n_i} =$$

$$k_{i,n_i}\tilde{x}_{i1} + \pi_{i,n_i,0}y_i\bar{f}_{i,n_i,0}(y_i) -$$

$$\sum_{l=2}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{il}}(\hat{x}_{i,l+1} + k_{il}\tilde{x}_{i1} + \pi_{il0}y_i\bar{f}_{il0}(y_i)) -$$

$$\frac{1}{2}\frac{\partial^2\alpha_{i,n_i-1}}{\partial y_i^2}g_{i1}^{\mathrm{T}}(y_i)g_{i1}(y_i) + \frac{3}{4}\lambda_{i,n_i}^{\frac{4}{3}}z_{i,n_i} +$$

$$\frac{3}{4}\epsilon_{i,n_i}^{\frac{4}{3}}(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i})^{\frac{4}{3}}z_{i,n_i} + \frac{1}{4\delta_{i,n_i-1}^4}z_{i,n_i} +$$

$$\frac{3}{4}\xi_{i,n_i}^{-2}(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i})^4z_{i,n_i} + \frac{3}{4}\gamma_{i,n_i}^{\frac{4}{3}}(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i})^{\frac{4}{3}}z_{i,n_i} +$$

$$\frac{3}{4}\sum_{l=2}^{n_i-1}\mu_{i,n_i,l}^{\frac{4}{3}}(\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{il}})^{\frac{4}{3}}z_{i,n_i} +$$

$$\frac{1}{4}\sum_{l=1}^{N}[\eta_{i1l}^2(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i})^2 + \frac{1}{\sigma_{i1l}}]z_{i,n_i}. \tag{21}$$

其中, $\xi_{ij},\gamma_{ij},\delta_{ij},\lambda_{ij},\mu_{ijl},\sigma_{i1l},\epsilon_{ij},\kappa_i,\rho_i,\beta_i$ 和 b_i 为给定参数.

通过适当选择参数 $b_i,\epsilon_{ij},\kappa_i,\rho_i$ 和β_i, 满足

$$M_{i0} > 0, \tag{22}$$

且令 $\Phi_i(\cdot),\alpha_{ij}(\cdot),u_i(\cdot)$ 具有如下形式:

$$\Phi_{im}(y_i(t)) = \pi_{ijm}^4(\frac{1}{4\xi_{i1}^4} + \sum_{j=2}^{n_i}\frac{1}{4\gamma_{ij}^4} + \sum_{j=2}^{n_i}\frac{1}{4\lambda_{ij}^4} + \sum_{j=2}^{n_i}\sum_{l=2}^{j-1}\frac{1}{4\mu_{ijl}^4})y_i^4(t)\|\bar{f}_{ijm}(y_i(t))\|^4, \tag{23}$$

$$\alpha_{i1} = -c_{i1}y_i(t) - M_{i1} - \frac{1}{y_i^3(t)}\sum_{m=1}^{M}\Phi_{im}(y_i(t)), \tag{24}$$

$$\alpha_{ij} = -c_{ij}z_{ij}(t) - M_{ij},\ 2 \leqslant j \leqslant n_i - 1, \tag{25}$$

$$u_i = -c_{i,n_i}z_{i,n_i}(t) - M_{i,n_i}, \tag{26}$$

其中 $c_{ij} > 0$，则有

$$LV \leqslant -\sum_{i=1}^{N}\Big(\sum_{j=1}^{n_i} c_{ij}z_{ij}^4 + M_{i0}\|\tilde{x}_i\|^4\Big), \tag{27}$$

于是，得到下列定理：

定理 1　随机非线性时滞大系统在输出反馈分散控制律(26)作用下，其闭环系统的平衡点在概率意义下为时滞无关全局稳定的.

3.3 逆优镇定(Inverse optimal stabilization)

通常，人们在实现控制目标的同时还需要满足一定的性能指标. 下面本文将在定理 1 的基础上进行输出反馈逆优控制器的再设计.

利用引理 1，不等式(16)进一步转化为

$$\begin{aligned}LV \leqslant \sum_{i=1}^{N}\Big[&-\Big(M_{i0} - \frac{1}{4\zeta_{i4}^4} - \frac{1}{4\zeta_{i3}^4}K_{i,n_i}^4\Big)\|\tilde{x}_i\|^4 + \\ &y_i^3\Big(\alpha_{i1} + M_{i1} + \frac{y_i}{4\zeta_{i1}^4} + \frac{y_i}{4\zeta_{i2}^4} + \frac{y_i}{4\zeta_{i7}^4} + \\ &\frac{y_i}{8\zeta_{i5}^4}\Big) + \sum_{j=2}^{n_i-1} z_{ij}^3\Big(\alpha_{ij} + M_{ij} + \frac{z_{ij}}{4\zeta_{i6}^4} + \frac{z_{ij}}{4\zeta_{i7}^4}\Big) + \\ &z_{i,n_i}^3(u_i + M_i(y_i,\hat{x}_i)) + \sum_{m=1}^{M_i}\Phi_{im}(y_i(t))\Big].\end{aligned} \tag{28}$$

选择合适的正参数 $\zeta_{i1},\zeta_{i2},\zeta_{i3},\zeta_{i4},\zeta_{i5},\zeta_{i6}$ 和 ζ_{i7}，使得下列条件成立：

$$M_{i0} - \frac{1}{4\zeta_{i4}^4} - \frac{1}{4\zeta_{i3}^4}k_{i,n_i}^4 = \bar{M}_{i0} > 0, \tag{29}$$

$$\frac{1}{4\zeta_{i1}^4} + \frac{1}{4\zeta_{i2}^4} + \frac{1}{4\zeta_{i7}^4} + \frac{1}{8\zeta_{i5}^4} = c_{i1}, \tag{30}$$

$$\frac{1}{4\zeta_{i6}^4} + \frac{1}{4\zeta_{i7}^4} = c_{il}. \tag{31}$$

其中，c_{i1} 和 c_{il} 与式(24)和(25)相同.

此时，分散控制器由下式确定：

$$u_i = -M_i(y_i,\hat{x}_i)z_{1,n_i}, \tag{32}$$

$$\begin{aligned}&M_i(y_i,\hat{x}_i) = \\ &c_{i,n_i} + \frac{3}{4}\zeta_{i1}^{\frac{4}{3}}(\pi_{i,n_i,0}\bar{f}_{i,n_i,0}(y_i))^{\frac{4}{3}} + \\ &\frac{3}{4}\zeta_{i2}^{\frac{4}{3}}\Big(\sum_{l=2}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{il}}\pi_{i,n_i,0}\bar{f}_{i,n_i,0}(y_i)\Big)^{\frac{4}{3}} + \\ &\frac{3}{4}\sum_{l=2}^{n_i-1}\Big(\zeta_{i6}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{il}}\Big)^{\frac{4}{3}} + \frac{3}{4}\zeta_{i6}^{\frac{4}{3}}\Big(\frac{\partial\alpha_{i,n_i}}{\partial y_i}\Big)^{\frac{4}{3}} + \\ &\frac{1}{4}\zeta_{i6}^{-4} + \frac{3}{4}\sum_{k=1}^{n_i-1}\Big(\zeta_{i7}\sum_{l=k}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{il}}\alpha_{ilk}\Big)^{\frac{4}{3}} + \\ &\frac{3}{8}\zeta_{i5}^{\frac{4}{3}}\Big(\frac{\partial^2\alpha_{i,n_i-1}}{\partial y_i^2}\bar{g}_{i1}^{\mathrm{T}}(y_i)\bar{g}_{i1}(y_i)\Big)^{\frac{4}{3}} + \\ &\frac{3}{4}\sum_{l=2}^{n_i-1}\Big(\zeta_{i4}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{il}}k_{il}\Big)^{\frac{4}{3}} + \frac{3}{4}\zeta_{i3}^{\frac{4}{3}} + \\ &\frac{3}{4}\varepsilon_{i,n_i}^{\frac{4}{3}}\Big(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i}\Big)^{\frac{4}{3}}z_{i,n_i} + \frac{3}{4}\lambda_{i,n_i}^{\frac{4}{3}}z_{i,n_i} + \\ &\frac{3}{4}\xi_{i,n_i}^{-2}\Big(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i}\Big)^4 z_{i,n_i} + \frac{1}{4\delta_{i,n_i-1}^4}z_{i,n_i} + \\ &\frac{3}{4}\gamma_{i,n_i}^{\frac{4}{3}}\Big(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i}\Big)^{\frac{4}{3}}z_{i,n_i} + \\ &\frac{1}{4}\sum_{l=1}^{N}\Big[\eta_{i1l}^2\Big(\frac{\partial\alpha_{i,n_i-1}}{\partial y_i}\Big)^2 + \frac{1}{\sigma_{i1l}}\Big]z_{i,n_i} + \\ &\frac{3}{4}\sum_{l=2}^{n_i-1}\mu_{i,n_i,l}^{\frac{4}{3}}\Big(\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{x}_{il}}\Big)^{\frac{4}{3}}z_{i,n_i},\end{aligned} \tag{33}$$

当同时满足式(23)～(26)和式(29)、(33)时，有

$$LV \leqslant -\sum_{i=1}^{N}\Big(\sum_{j=1}^{n_i} c_{ij}z_{ij}^4 + \bar{M}_{i0}\|\tilde{x}_i\|^4\Big), \tag{34}$$

此时，本文不但实现了概率意义下的时滞无关全局渐近镇定，而且达到逆优.

定理 2　在分散控制律

$$u_i = -B_iM_i(y_i,\hat{x}_i),\ B_i > \frac{4}{3} \tag{35}$$

作用下，闭环系统原点为概率意义下时滞无关全局渐近稳定平衡点，且使下列指标函数

$$J(u_i) = E\Big\{\sum_{i=1}^{N}\int_0^{\infty}[L_i(x_i,\tilde{x}_i) + R_i(u_i)]\Big\}\mathrm{d}\tau, \tag{36}$$

$$R_i(u_i) = \frac{27}{16B_i^2}M_i^{-3}(y_i,\hat{x}_i)u_i^4 \tag{37}$$

的代价最小.

注 3　定理 2 中的指标函数为一个标准的积分形式，在风险灵敏度随机控制意义下，可看成是风险灵敏度参数趋近零情况下的零风险控制问题. 此指标函数是文献[11]中指标函数的一个拓展，具体含义请参阅该文及其参考文献.

注 4　本文所研究的系统具有一定的代表性，文献[10, 11]所研究的系统可以视为本文所考虑系统的一个特例. 当不考虑时滞和各子系统内部的非线性项，系统将简化为如文献[10]所研究的系统. 若在无时滞情况下考虑集中控制，则简化为文献[11]所研究的系统.

4 结论(Conclusion)

本文针对随机非线性时滞系统，基于反向递推技术，构造了输出反馈无记忆分散控制器. 接着在一定性能指标约束下，对其进行了逆优再设计. 在此控

制律作用下,闭环系统的原点为概率意义下时滞无关全局渐近稳定平衡点,并满足一定的性能指标.

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