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Robust H-infinity adaptive control for uncertain cascaded nonlinear system

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Abstract: A robust adaptive controller with L_2-gain is derived for a class of cascaded non-minimum phase nonlinear systems with unknown parameters and disturbances. A recursive Lyapunov-based design approach was developed to construct the controller explicitly so as to avoid solving Hamilton-Jacobi-Isaacs inequality. The state feedback controller guaranteed that the closed system was input-to-state stable and the L_2-gain from the disturbance input to the controlled output was not larger than a prescribed value for all admissible parameter uncertainties. In the end, a simulation example was given to demonstrate the controller's feasibility.

Key words: nonlinear system; robust control; adaptive control; backstepping design; L_2-gain
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不确定串联非线性系统 H_∞ 鲁棒自适应控制

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摘要: 针对一类含有未知参数和干扰的非最小相位串联非线性系统,结合 H_∞ 控制和自适应控制方法并利用李雅普诺夫函数递推设计方法设计了状态反馈 H_∞ 自适应控制器,避免了求解 Hamilton-Jacobi-Isaacs 不等式设计控制器的困难.该控制器不仅保证闭环系统 ISS(input-to-state)稳定,而且使得系统对于所有允许的参数不确定从干扰输入到可控输出的 L_2 增益不大于给定的值.最后,给出了一个仿真例子,仿真结果充分表明了所设计的控制器的可行性和有效性.

关键词: 非线性系统; 鲁棒控制; 自适应控制; 递推设计; L_2 增益

1 Introduction

In the past decade, robust and adaptive control has been an active research area and many remarkable results have been obtained for a class of uncertain nonlinear systems (see [1 ~ 5]). The problem of H_∞ control of general nonlinear systems has been addressed by [5] using the notion of dissipativity, but the results in [5] involve solving Hamilton-Jacobi-Isaacs inequality, which imposes a formidable difficulty. The problem of H_∞ control has been solved without the need of solving HJI inequality for a class of uncertain cascaded nonlinear system in the literature [1], the problem of parameter estimate was not discussed yet. Although robust adaptive control problem has been investigated for a class of uncertain nonlinear systems in the literature [2], L_2-gain from the disturbance input to the controlled output was not considered. Robust adaptive control with L_2 gain has been addressed for a class of special systems by [4]. Nevertheless, very few researches have been reported on designing nonlinear H_∞ adaptive controller without the need of sovling Hamilton-Jacobi-Isaacs (HJI) for a class of non-minimum phase nonlinear systems.

In the present paper, motivated by [4], based on the literatures [1,2], a state feedback H_∞ adaptive controller is derived for a class of non-minimum phase nonlinear systems by integrating H_∞ control with adaptive control approach and using recursive Lyapunov-based approach, which avoids the difficulty of solving HJI inequality. The controller guarantees that the closed system is input-to-state stable and the L_2-gain from the disturbance input to the controlled output is not larger than a prescribed value for all admissible parameter uncertainties.

2 Problem presentation

Consider the uncertain cascaded nonlinear system

$$\begin{cases} \dot{\zeta} = \mu(\zeta, x_1), \\ \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + \theta^{\mathrm{T}}\phi_i(\bar{x}_i) + p_i(\zeta, \bar{x}_i)w, \\ \qquad 1 \leqslant i \leqslant n-1, \\ \dot{x}_n = u + f_n(\bar{x}_n) + \theta^{\mathrm{T}}\phi_n(\bar{x}_n) + p_n(\zeta, \bar{x}_n)w, \\ z = h_0(\zeta, \bar{x}_n) + d_0(\zeta, \bar{x}_n)w, \end{cases} \tag{1}$$

where, $\bar{x}_i = (x_1, \cdots, x_i)^{\mathrm{T}} \in \mathbb{R}^i (i = 1,2,\cdots,n)$ and $\zeta \in \mathbb{R}^{n_0}$ is the state. $u \in \mathbb{R}$ is the control input, $\theta \in \mathbb{R}^l$

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is an unknown constant parameter vector, $w \in \mathbb{R}^r$ is the disturbance input and $w \in L_\infty[0,\infty) \cup L_2[0,\infty)$, $z \in \mathbb{R}^m$ is the controllable output, μ, ϕ_i, p_i, h_0, d_0 are known smooth real function vectors, f_i is a known smooth real function, $i = 1,2,\cdots,n$, with $f_i(0) = 0$, $\mu(0,0) = 0, \phi_i(0) = 0, h_0(0,0) = 0$.

Assume that the system (1) satisfies the following assumptions.

Assumption 1 For the ζ-subsystem, there exist a smooth real-valued function $\alpha_0(\zeta)$ with $\alpha_0(0) = 0$ and a real-valued function $V_0(\zeta)$, which is smooth and positive definite, such that

$$(\partial V_0/\partial \zeta)\mu(\zeta,\alpha_0(\zeta)) \leqslant -\beta_1 V_0(\zeta),\ \beta_1 > 0, \quad (2)$$

$$\beta_2 \|\zeta\|^2 \leqslant V_0(\zeta),\ \beta_2 > 0, \quad (3)$$

for some positive real numbers β_1 and β_2.

Assumption 2 There exists a positive real number γ_{d_0} such that $d_0(\zeta,\bar{x}_n)$ in equation (1) satisfies

$$\| d_0(\zeta,\bar{x}_n)\| \leqslant \gamma_{d_0},\ \forall [\zeta^{\mathrm{T}},\bar{x}_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n+n_0}.$$

Lemma 1 The nonlinear system $\dot{x} = f(x,u)$ is input-to-state stable if and only if there exist K_∞ function $k_i(\cdot), i = 1,\cdots,4$, and a smooth function $V(x)$ such that the following conditions hold for any $x \in \mathbb{R}^n$ and $u \in L_\infty[0,\infty)$:

$$k_1(\|x\|) \leqslant V(x) \leqslant k_2(\|x\|),$$

$$(\partial V/\partial x)f(x,u) \leqslant -k_3(\|x\|) + k_4(\|u\|),$$

where $f:\mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$ is continuously differentiable and $f(0,0) = 0$.

This paper addresses the following control problem: Given any $\gamma > \gamma_{d_0}$, design a state feedback controller

$$\dot{\vartheta} = \tau_n(\vartheta,\zeta,\bar{x}_n),\ u = u(\vartheta,\zeta,\bar{x}_n),\ \vartheta \in \mathbb{R}^l, \quad (4)$$

where ϑ is the estimate of θ, for the closed-loop system composed of equation (1) and (4) such that the following design specifications hold for any admissible uncertainty θ:

(S_1) Stability: The resulting closed-loop system is input-to state stable;

(S_2) L_2-gain performance: For some real-valued function $\varepsilon_0:\mathbb{R}^{n_0} \times \mathbb{R}^n \times \mathbb{R}^l \to \mathbb{R}$ with $\varepsilon_0(0,0,0) = 0$, the following integrator inequality holds,

$$\int_0^\infty z^{\mathrm{T}}(\tau)z(\tau)\mathrm{d}\tau \leqslant \gamma^2\int_0^\infty w^{\mathrm{T}}(\tau)w(\tau)\mathrm{d}\tau + \varepsilon_0(\zeta_0,\bar{x}_{n0},\vartheta_0),$$

for all $w \in L_2[0,\infty)$ and any initial condition $[\zeta_0^{\mathrm{T}},\bar{x}_{n0}^{\mathrm{T}},\vartheta_0^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n+n_0+l}$, where $\tilde{\theta} = \vartheta - \theta$. γ is a positive constant specifying attenuation level.

Notation $\mathbb{R}_+$ is the set of nonnegative real numbers, $\mathbb{R}^n$ is an n-dimension real vector space, $\|\cdot\|$ is the Euclidean vector norm, and $L_2[0,\infty)$ and $L_\infty[0,\infty)$ denote the spaces of square integrable and uniformly bounded functions on $[0,\infty)$, respectively. $\alpha_i = \alpha_i(\zeta,\bar{x}_i,\vartheta)$, is the virtual control law, ϕ_i, p_i and μ denote $\phi_i(\bar{x}_i), p_i(\zeta,\bar{x}_i)$, and $\mu(\zeta,x_1)$ respectively, $i = 0,\cdots,n$.

3 Design of robust adaptive controller

The design steps of robust adaptive controller are as follows:

Step 1 Letting $e_1 = x_1 - \alpha_0, e_2 = x_2 - \alpha_1$, then we have $\mu(\zeta,x_1) = \mu(\zeta,\alpha_0) + \bar{\mu}(\zeta,e_1)e_1$ (the existence of $\bar{\mu}$ is followed by the smooth property of μ). For convenience, letting $\mu_0 = \mu(\zeta,\alpha_0), \mu = \bar{\mu}(\zeta,e_1)$, Eq.(1) subsystem can be transformed into

$$\begin{aligned}\dot{\zeta} &= \mu_0 + \bar{\mu}e_1,\\ \dot{e}_1 &= e_2 + \alpha_1 + f_1 + \theta^{\mathrm{T}}\phi_1 + p_1 w - \dot{\alpha}_0.\end{aligned} \quad (5)$$

According to Assumption 1, we have

$$(\partial V_0/\partial \zeta)\mu_0 \leqslant -\beta_1 V_0(\zeta). \quad (6)$$

In addition, note that by using equation (1) and completing the squares, it can be easily obtained that

$$\begin{aligned}&\|z\|^2 - \gamma_1^2\|w\|^2 \leqslant\\ &h_0^{\mathrm{T}}[I + (1/\gamma_0^2)d_0d_0^{\mathrm{T}}]h_0 -\\ &w^{\mathrm{T}}[\gamma_1^2 - \gamma_0^2)I - d_0^{\mathrm{T}}d_0]w \leqslant\\ &[1 + (\gamma_{d_0}^2/\gamma_0^2)]\|h_0\|^2 - \bar{\gamma}_1^2\|w\|^2,\end{aligned} \quad (7)$$

where $\gamma_1 > \gamma_{d_0}$, γ_{d_0} is as defined in Assumption 2, and γ_0 and $\bar{\gamma}_1$ are positive real numbers such that $\bar{\gamma}_1^2 = \gamma_1^2 - \gamma_0^2 - \gamma_{d_0}^2$. In the case where $d_0 \equiv 0$, we have $\gamma_{d_0} = 0$, $\bar{\gamma}_1 = \gamma_1$ and γ_0 becomes redundant.

Since the transformation $e_1 = x_1 - \alpha_0$ is a diffeomorphism, there exists a positive-definite function $H(\zeta,e_1)$ such that

$$[1 + (\gamma_{d_0}^2/\gamma_0^2)]\|h_0(\zeta,x_1)\|^2 \leqslant H(\zeta,e_1). \quad (8)$$

Also, since $H(0,0) = 0$, then $H(\zeta,e_1)$ can be decomposed as

$$H(\zeta,e_1) = H_0(\zeta) + H_1(\zeta,e_1)e_1, \quad (9)$$

where $H_0(\zeta) = H_0(\zeta,0)$, and $H_0(0) = 0, H_1(0,0) = 0$.

Next, considering $V_0(\zeta)$ is radially unbounded and positive definite, there exists a class of K_∞ function $k_0(\cdot)$ such that

$$H_0(\zeta) + (1/2)\|\zeta\|^2 \leqslant k_0(V_0(\zeta)). \quad (10)$$

Define a storage function candidate V_1 for the system of equation (6)

$$\begin{aligned}V_1 = &(1/\beta_1)S_0(V_0(\zeta)) + (1/2)e_1^2 +\\ &(1/2)(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}(\vartheta - \theta),\end{aligned} \quad (11)$$

where $\Gamma > 0$ is the design parameter matrix, ϑ is the estimate of θ, β_1, is the constant in Assumption 1, $S_0(V_0)$ is the following class K_∞ function:

$$S_0(V_0) = V_0 \sup_{0\leqslant t\leqslant 1}\frac{\mathrm{d}k_0(t)}{\mathrm{d}t} + \int_{V_0}^{2V_0}k_0(t)\mathrm{d}t. \quad (12)$$

Observe that the function $S_0(V_0)$ satisfies[1]:

$$S_0(V_0) \geqslant k_0(V_0), V_0(\mathrm{d}S_0(V_0)/\mathrm{d}V_0) \geqslant S_0(V_0). \quad (13)$$

Considering equation (5), (6), (11) and (13), the time derivative of V_1 satisfies

$$\begin{aligned}\dot{V}_1 = &\frac{1}{\beta_1}\frac{\mathrm{d}S_0(V_0)}{\mathrm{d}V_0}\frac{\partial V_0}{\partial \zeta}(\mu_0 + \bar{\mu} e_1) + \\ &e_1\dot{e}_1 + (\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}\dot{\vartheta} \leqslant \\ &- S_0(V_0) + \varphi(\zeta, e_1)e_1 + e_1(e_2 + \alpha_1 + f_1 + \\ &\vartheta^{\mathrm{T}}\phi_1 + p_1 w - (\partial\alpha_0/\partial\zeta)\mu) \\ &(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}(\dot{\vartheta} - \tau_1) - \sigma_\theta(\vartheta - \theta)^{\mathrm{T}}\vartheta, \quad (14)\end{aligned}$$

where σ_θ is a design parameter,

$$\tau_1 = \Gamma(e_1\phi_1 - \sigma_\theta\vartheta),$$

$$\varphi(\zeta, e_1) = \frac{1}{\beta_1}\frac{\mathrm{d}S_0(V_0)}{\mathrm{d}V_0}\frac{\partial V_0}{\partial \zeta}\bar{\mu}.$$

By using equations (7), (8), (9), (10), (13) and (14), we have

$$\begin{aligned}&\dot{V}_1 + \|z\|^2 - \gamma_1^2\|w\|^2 \leqslant \\ &- (1/2)\|\zeta\|^2 - \bar{\gamma}_1^2\|w\|^2 + e_1[\alpha_1 + e_2 + \\ &H_1 + \varphi + f_1 + \vartheta^{\mathrm{T}}\phi_1 - (\partial\alpha_0/\partial\zeta)\mu] + \\ &e_1 p_1 w + (\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}(\dot{\vartheta} - \tau_1) - \sigma_\theta(\vartheta - \theta)^{\mathrm{T}}\vartheta. \quad (15)\end{aligned}$$

Also, there exists a bounding $\delta_{p_1}(\zeta, x_1)$ such that $\|p_1\| \leqslant \delta_{p_1}$. By completing the square for inequality (15), inequality (15) yields

$$\begin{aligned}&\dot{V}_1 + \|z\|^2 - \gamma_1^2\|w\|^2 \leqslant \\ &- (1/2)\|\zeta\|^2 + e_1(\alpha_1 + e_2 + H_1 + \varphi + \\ &f_1 + \vartheta^{\mathrm{T}}\phi_1 - (\partial\alpha_0/\partial\zeta)\mu + (1/4\bar{\gamma}_1^2)e_1\delta_{p_1}^2) + \\ &(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}(\dot{\vartheta} - \tau_1) - \sigma_\theta(\vartheta - \theta)^{\mathrm{T}}\vartheta. \quad (16)\end{aligned}$$

Now choose a virtual control law α_1 as

$$\begin{aligned}\alpha_1 = - [&\frac{1}{2}e_1 + H_1 + \varphi + f_1 + \vartheta^{\mathrm{T}}\phi_1 - \\ &(\partial\alpha_0/\partial\zeta)\mu + (1/4\bar{\gamma}_1^2)e_1\delta_{p_1}^2]. \quad (17)\end{aligned}$$

Substituting equation (17) into (16), we have

$$\begin{aligned}&\dot{V}_1 + \|z\|^2 - \gamma_1^2\|w\|^2 \leqslant \\ &- \bar{V}_1 + e_1 e_2 + (\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}(\dot{\vartheta} - \tau_1) + (1/2)\sigma_\theta\theta^{\mathrm{T}}\theta. \quad (18)\end{aligned}$$

where

$$\begin{aligned}\bar{V} = &\frac{1}{2}\|\zeta\|^2 + \frac{1}{2}e_1^2 + (\sigma_\theta/2\lambda_{\max}(\Gamma^{-1}))(\vartheta - \\ &\theta)^{\mathrm{T}}\Gamma^{-1}(\vartheta - \theta),\end{aligned}$$

$\lambda_{\max}(\Gamma^{-1})$ is maximum eigenvalue of Γ^{-1}.

Step 2 By letting $e_3 = x_3 - \alpha_2$, we have

$$\begin{aligned}\dot{e}_2 = &\dot{x}_2 - \alpha_1 = \\ &e_3 + \alpha_2 + f_2 + \theta^{\mathrm{T}}\phi_2 + p_2 w - \\ &(\partial\alpha_1/\partial x_1)(x_2 + f_1 + \theta^{\mathrm{T}}\phi_1 + p_1 w) - \\ &(\partial\alpha_1/\partial\zeta)\mu - (\partial\alpha_1/\partial\vartheta)\dot{\vartheta} = \\ &e_3 + \alpha_2 + \bar{f}_2 - (\partial\alpha_1/\partial\vartheta)\dot{\vartheta} + \\ &\theta^{\mathrm{T}}(\phi_2 - (\partial\alpha_1/\partial x_1)\phi_1) + \bar{p}_2 w, \quad (19)\end{aligned}$$

where

$$\bar{f}_2 = f_2 - (\partial\alpha_1/\partial x_1)(x_2 + f_1) - (\partial\alpha_1/\partial\zeta)\mu,$$

$$\bar{p}_2 = (p_2 - (\partial\alpha_1/\partial x_1)p_1).$$

Define a storage function candidate as $V_2 = V_1 + (1/2)e_2^2$. By using (18) and (19), we have

$$\begin{aligned}&\dot{V}_2 + \|z\|^2 - \gamma_2^2\|w\|^2 \leqslant \\ &- \bar{V}_1 + e_1 e_2 - \bar{\gamma}_2^2\|w\|^2 + (\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}(\dot{\vartheta} - \tau_1) + \\ &(1/2)\sigma_\theta\theta^{\mathrm{T}}\theta + e_2 e_3 + e_2[\alpha_2 + \bar{f}_2 - (\partial\alpha_1/\partial\vartheta)\dot{\vartheta} + \\ &\theta^{\mathrm{T}}(\phi_2 - (\partial\alpha_1/\partial x_1)\phi_1) + \bar{p}_2 w], \quad (20)\end{aligned}$$

where $\bar{\gamma}_2^2 = \gamma_2^2 - \gamma_1^2$, γ_2 is a positive constant large than γ_1.

Also since there exists a smooth upper bound function $\delta_{p_2}(\zeta, \bar{x}_2, \vartheta)$ such that $\|\bar{p}_2\| \leqslant \delta_{p_2}$, by completing the squares for inequality (20), inequality (20) yields

$$\begin{aligned}&\dot{V}_2 + \|z\|^2 - \gamma_2^2\|w\|^2 \leqslant \\ &- \bar{V}_1 + e_2 e_3 + [(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1} - e_2(\partial\alpha_1/\partial\vartheta)](\dot{\vartheta} - \tau_2) + \\ &e_2[\alpha_2 + \bar{f}_2 + e_1 + \vartheta^{\mathrm{T}}(\phi_1 - (\partial\alpha_1/\partial x_1)\phi_1) - \\ &(\partial\alpha_1/\partial\vartheta)\tau_2 + (1/4\bar{\gamma}_2^2)e_2\delta_{p_2}^2] + (1/2)\sigma_\theta\theta^{\mathrm{T}}\theta, \quad (21)\end{aligned}$$

where

$$\tau_2 = \tau_1 + \Gamma(\phi_2 - (\partial\alpha_1/\partial x_1)\phi_1)e_2.$$

Choose virtual control law α_2 as

$$\begin{aligned}\alpha_2 = - [&(1/2)e_1 + \bar{f}_2 + e_1 + \vartheta^{\mathrm{T}}(\phi_2 - (\partial\alpha_1/\partial x_1)\phi_1) - \\ &(\partial\alpha_1/\partial\vartheta)\tau_2 + (1/4\bar{\gamma}_2^2)e_2\delta_{p_2}^2]. \quad (22)\end{aligned}$$

Substituting equation (22) into (21), we have

$$\begin{aligned}&\dot{V}_2 + \|z\|^2 - \gamma_2^2\|w\|^2 \leqslant \\ &- \bar{V}_2 + e_2 e_3 + (1/2)\sigma_\theta\theta^{\mathrm{T}}\theta + [(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1} - \\ &e_2(\partial\alpha_1/\partial\vartheta)](\dot{\vartheta} - \tau_2), \quad (23)\end{aligned}$$

where

$$\begin{aligned}\bar{V}_2 = &(1/2)\|\zeta\|^2 + (1/2)e_1^2 + (1/2)e_2^2 + \\ &(\sigma_\theta/2\lambda_{\max}(\Gamma^{-1}))(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1}(\vartheta - \theta).\end{aligned}$$

Step i Let $e_{j+1} = x_{j+1} - \alpha_j (1 \leqslant j \leqslant i - 1)$.

Define a storage function candidate as

$$\begin{aligned}V_{i-1} = &(1/\beta_1)S_0(V_0) + \sum_{j=1}^{i-1}(1/2)e_j^2 + \\ &(1/2)(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1} - (\vartheta - \theta). \quad (24)\end{aligned}$$

Assume that according to the above recursive design approach we have designed virtual control laws α_j and estimate functions $\tau_j (1 \leqslant j \leqslant i - 1)$, by choosing $\gamma_1 < \gamma_2 < \cdots < \gamma_{i-1}$, such that the time derivative of V_{i-1} along equation (1) satisfies

$$\begin{aligned}&\dot{V}_{i-1} + \|z\|^2 - \gamma_{i-1}^2\|w\|^2 \leqslant \\ &- \bar{V}_{i-1} + e_{i-1}e_i + (1/2)\sigma_\theta\theta^{\mathrm{T}}\theta + [(\vartheta - \theta)^{\mathrm{T}}\Gamma^{-1} - \\ &\sum_{j=1}^{i-2} e_{j+1}(\partial\alpha_j/\partial\vartheta)](\dot{\vartheta} - \tau_{i-1}), \quad (25)\end{aligned}$$

where

$$\bar{V}_{i-1} = (1/2)\|\zeta\|^2 + \sum_{j=1}^{i-1}(1/2)e_j^2 +$$

$(\sigma_\theta/2\lambda_{\max}(\Gamma^{-1}))(\vartheta-\theta)^{\mathrm T}\Gamma^{-1}(\vartheta-\theta)$.

Define

$$V_i = V_{i-1} + (1/2)e_i^2. \tag{26}$$

Note that the variable e_i satisfies the following equation:

$$\dot e_i = e_{i+1} + \alpha_i + \bar f_i + \theta^{\mathrm T}(\phi_i - \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)\phi_j) - (\partial\alpha_{i-1}/\partial\vartheta)\dot\vartheta + \bar p_i w, \tag{27}$$

where

$$\bar f_i = f_i - [\sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)(x_{j+1}+f_j) + (\partial\alpha_{i-1}/\partial\zeta)\mu],$$

$$\bar p_i = p_i - \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)p_j.$$

By choosing $\gamma_i > \gamma_{i-1}$, letting $\bar\gamma_i^2 = \gamma_i^2 - \gamma_{i-1}^2$ and using Eqs. (25), (26) and (27), we have

$$\begin{aligned}&\dot V_i + \|z\|^2 - \gamma_i^2\|w\|^2 \leqslant \\ &-\bar V_{i-1} + e_ie_{i+1} + (1/2)\sigma_\theta\theta^{\mathrm T}\theta - \bar\gamma_i^2\|w\|^2 + \\ &[(\vartheta-\theta)^{\mathrm T}\Gamma^{-1} - \sum_{j=1}^{i-2}e_{j+1}(\partial\alpha_j/\partial\vartheta)](\vartheta - \tau_{i-1}) + \\ &e_i[\alpha_i + \bar f_i + e_{i-1} + \theta^{\mathrm T}(\phi_i - \\ &\sum_{j=1}^{i-1}(\partial\alpha_i/\partial x_j)\phi_j) - (\partial\alpha_{i-1}/\partial\vartheta)\dot\vartheta + \bar p_i w].\end{aligned} \tag{28}$$

Also, since there exists a bounding function $\delta_{p_i}(\zeta,\bar x_i,\vartheta)$ such that $\|\bar p_i\| \leqslant \delta_{p_i}$, by completing squares for the right side of inequality (28), inequality (28) yields

$$\begin{aligned}&\dot V_i + \|z\|^2 - \gamma_i^2\|w\|^2 \leqslant \\ &-\bar V_{i-1} + e_ie_{i+1} + (1/2)\sigma_\theta\theta^{\mathrm T}\theta + [(\vartheta-\theta)^{\mathrm T}\Gamma^{-1} - \\ &\sum_{j=1}^{i-1}e_{j+1}(\partial\alpha_j/\partial\vartheta)](\vartheta - \tau_i) + e_i[\alpha_i + \bar f_i + e_{i-1} - \\ &(\partial\alpha_{i-1}/\partial\vartheta)\tau_i + (\vartheta^{\mathrm T} - \sum_{j=1}^{i-2}e_{j+1}(\partial\alpha_j/\partial x_j)\Gamma)(\phi_i - \\ &\sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)\phi_j) + (1/4\bar\gamma_i^2)e_i\delta_{p_i}^2],\end{aligned} \tag{29}$$

where

$$\tau_i = \tau_{i-1} + \Gamma(\phi_i - \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)\phi_j)e_i.$$

Choose the virtual control law α_i as

$$\begin{aligned}\alpha_i = &-[(1/2)e_i + e_{i-1} + \bar f_i - (\partial\alpha_{i-1}/\partial\vartheta)\tau_i - \\ &(\vartheta^{\mathrm T} - \sum_{j=1}^{i-2}e_{j+1}(\partial\alpha_i/\partial\theta)\Gamma) + (\phi_i - \\ &\sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)\varphi_j)(1/4\bar\gamma_i^2)e_i\delta_{p_i}^2].\end{aligned} \tag{30}$$

Substituting Eq. (30) into (29), we have

$$\begin{aligned}&\dot V_i + \|z\|^2 - \gamma_i^2\|w\|^2 \leqslant \\ &-\bar V_i + e_ie_{i-1} + (1/2)\sigma_\theta\theta^{\mathrm T}\theta + [(\vartheta-\theta)^{\mathrm T}\Gamma^{-1} - \\ &\sum_{j=1}^{i-1}e_{j+1}(\partial\alpha_j/\partial\vartheta)](\dot\vartheta - \tau_i),\end{aligned} \tag{31}$$

where

$$\bar V_i = (1/2)\|\zeta\|^2 + \sum_{j=1}^{i}(1/2)e_j^2 + (\sigma_\theta/2\lambda_{\max}(\Gamma^{-1}))(\vartheta-\theta)^{\mathrm T}\Gamma^{-1}(\vartheta-\theta).$$

Step *n* Define $V_n = V_{n-1} + (1/2)e_n^2$ and choose the control input u as

$$\begin{aligned}u = &-[(1/2)e_n + e_{n-1} + \bar f_n - (\partial\alpha_{n-1}/\partial\vartheta)\tau_n - \\ &(\vartheta^{\mathrm T} - \sum_{j=1}^{n-2}e_{j+1}(\partial\alpha_j/\partial\vartheta)\Gamma) \\ &(\phi_n - \sum_{j=1}^{n-1}(\partial\alpha_{n-1}/\partial x_j)\phi_j) + (1/4\bar\gamma_n^2)e_n\delta_{p_n}^2].\end{aligned} \tag{32}$$

By letting $i = n$ in equation (29) and substituting Eq. (32) into Eq. (29), we have

$$\begin{aligned}&\dot V_n + \|z\|^2 - \gamma_n^2\|w\|^2 \leqslant \\ &-\bar V_n + (1/2)\sigma_\theta\theta^{\mathrm T}\theta + [(\vartheta-\theta)^{\mathrm T}\Gamma^{-1} - \\ &\sum_{j=1}^{n-1}e_{j+1}(\partial\alpha_j/\partial\vartheta)](\dot\vartheta - \tau_n),\end{aligned} \tag{33}$$

where

$$\bar V_n = (1/2)\|\zeta\|^2 + \sum_{j=1}^{n}(1/2)e_j^2 + (\sigma_\theta/2\lambda_{\max}(\Gamma^{-1}))(\vartheta-\theta)^{\mathrm T}\Gamma^{-1}(\vartheta-\theta).$$

Choose the adaptive control law as

$$\dot\vartheta = \tau_n = \tau_{n-1} + \Gamma(\phi_n - \sum_{j=1}^{n-1}(\partial\alpha_{n-1}/\partial x_j)\phi_j)e_n. \tag{34}$$

Substituting Eq. (34) into Eq. (33), we have

$$\dot V_n + \|z\|^2 - \gamma_n^2\|w\|^2 \leqslant -\bar V_n + (1/2)\sigma_\theta\theta^{\mathrm T}\theta. \tag{35}$$

In view of the definition of V_n, V_n is a radially unbounded and positive definite C^1 function. Evidently, there exist class K_∞ functions $k_1(\cdot)$ and $k_2(\cdot)$ such that V_n satisfies the first inequality of Lemma 1. In addition, letting $(1/2)\sigma_\theta = \gamma_n^2$ and $\omega = [w^{\mathrm T},\theta^{\mathrm T}]^{\mathrm T}$, Eq. (35) yields

$$\dot V_n \leqslant -\|z\|^2 - \bar V_n + \gamma_n^2\|\omega\|^2. \tag{36}$$

According to the known condition, $\omega \in L_\infty[0,\infty)$. If regarding ω as the general input for the closed-loop system, then Eq. (36) satisfies the second inequality of Lemma 1. Hence, in view of Lemma 1, this implies that the closed-loop system is input-to-state stable for all admissible uncertainties.

Choosing an appropriate positive constant σ_θ satisfying $\frac{1}{2}\sigma_\theta\theta^{\mathrm T}\theta \leqslant \bar V_n$, Eq. (35) yields

$$\dot V_n + \|z\|^2 - \gamma_n^2\|w\|^2 \leqslant 0. \tag{37}$$

Integrating both sides of Eq. (37) and noting that $V_n(\zeta, e, \tilde\theta \geqslant 0\ (\forall[\zeta^{\mathrm T}, e^{\mathrm T}, \tilde\theta^{\mathrm T}]^{\mathrm T} \in \mathbb{R}^{n_0+n+1})$, it follows that

$$\int_0^\infty z^{\mathrm T}z\mathrm d\tau \leqslant \gamma^2\int_0^\infty w^{\mathrm T}w\mathrm d\tau + \varepsilon_0(\zeta(0), e(0), \tilde\theta(0)), \tag{38}$$

where

$\gamma = \gamma_n$, $e(t) = [e_1, \cdots, e_n]^{\mathrm{T}}$,

$\varepsilon_0(\zeta(0), e(0), \tilde{\theta}(0)) = V_n(\zeta(0), e(0), \tilde{\theta}(0))$.

In view of Eqs. (36) and (38), the closed-loop system composed of Eqs. (1), (32) and (34) satisfies design specifications (S_1) and (S_2). According to the above proof, we have the following theorem:

Theorem 1 Under Assumption 1 and 2, there exists a state feedback adaptive controller such that the closed-loop system composed of Eqs. (1), (32) and (34) satisfies design specifications (S_1) and (S_2) for all admissible uncertainties.

Remark 1 In fact, according to the definition of $\bar{V}_n$, there exits a σ_θ such that $\frac{1}{2}\sigma_\theta\theta^{\mathrm{T}}\theta \leq \bar{V}_n$ if $(1/2)\parallel \zeta \parallel^2 + \sum_{j=1}^{n}(1/2)e_j^2 > 0$, then inequality (37) holds. Hence, In simulation, we can choose an appropriate positive constant σ_θ such that all error states of the closed-loop system converge to zero or the neighborhood of zero.

4 Simulation example

Consider the three-dimensional nonlinear system

$$\begin{cases} \dot{\zeta} = -\zeta + x_1^2, \\ \dot{x}_1 = x_2 + x_1^2 + \theta \sin x_1, \\ \dot{x}_2 = u + w, \\ z = \begin{bmatrix} \zeta \\ x_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w. \end{cases} \tag{39}$$

Choose ζ-subsystem storage function $V_0(\zeta) = (1/2)\zeta^2$. Assumption 1 is satisfied with $\alpha_0(\zeta) = 0$, $\beta_1 = 2$ and $\beta_2 = 0.5$. Moreover, Assumption 2 is satisfied with $\gamma_{d_0} = \sqrt{2}$. Therefore Assumption 1, 2 hold and thus, we will apply the above approach to design the state feedback controller. Choosing $\gamma_0 = 1$, $\gamma_1 = 2$ and $\gamma_2 = 3$, we can obtain $\bar{\gamma}_1 = 1$, $\bar{\gamma}_2 = \sqrt{5}$, $\delta_{p_1} = 0$ and $\delta_{p_2} = 1$. In view of the above design approach, we can obtain the virtual control law α_1, control input u and adaptive law $\dot{\vartheta} = \tau_2$ as follows

$\alpha_1 = -(3.5x_1 + x_1^2 + 3.5\zeta x_1 + 5.25\zeta^3 x_1 + \vartheta \sin x_1)$,

$u = -[0.55e_2 + x_1 + f_2 + \tau_2 \sin x_1 - \vartheta \sin x_1 (\partial\alpha_1/\partial x_1)$,

$\dot{\vartheta} = \tau_2 = \Gamma x_1 \sin x_1 - \Gamma \sin x_1 e_2 (\partial\alpha_1/\partial x_1) - \Gamma\sigma_\theta\vartheta$,

where

$$f_2 = (3.5x_1 + 15.75\zeta^2 x_1)(-\zeta + x_1^2) - (\partial\alpha_1/\partial x_1)(x_2 + x_1^2),$$

$$\sigma_\theta = 2\gamma_2^2 = 18.$$

In simulation, parameter $\Gamma = 1$, $\theta = 0.1$, disturbance input $w = \sin(4t)\exp(-0.1t)$, initial states was set to 1, i.e. $\zeta(0) = x_1(0) = x_2(0) = x_2(0) = \vartheta(0) = 1$, performance index

$$J(t) = \left[\int_0^t z^{\mathrm{T}}(\tau)z(\tau)\mathrm{d}\tau \Big/ \int_0^t w(\tau)w(\tau)\mathrm{d}\tau\right]^{0.5}$$

was calculated. Note that $J(t) < 3$. In view of simulation Figs. 1, 2 and 3, it implies that the closed-loop system satisfies design specifications (S_1) and (S_2) for all admissible uncertainties, Fig. 4 is the control input. Hence, the controller designed is feasible and effective.

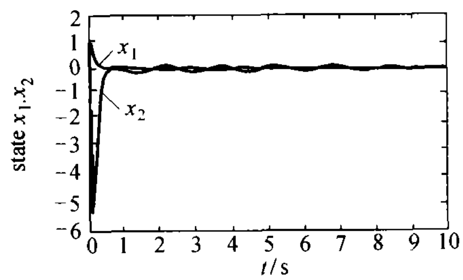

Fig. 1 State x_1 and x_2.

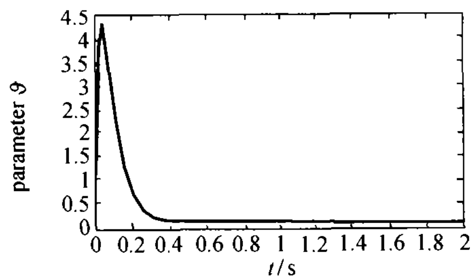

Fig. 2 Parameter estimate ϑ .

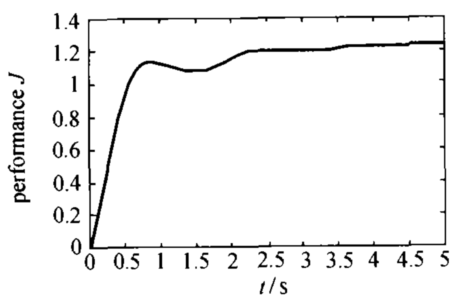

Fig 3. Performance index $J(t)$.

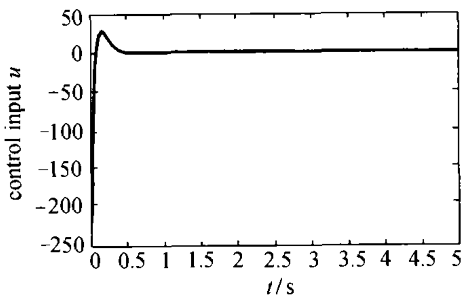

Fig. 4 Control input u.

5 Conclusion

In the present paper, a state feedback H_∞ adaptive controller is derived for a class of cascaded nonlinear systems with unknown parameter and disturbance by integrating H_∞ control with adaptive control approach. The controller guarantees that the closed system is input-to-state stable and the L_2-gain from the disturbance input to the controlled output is not larger than a prescribed value for all admissible parameter uncertainties. Simulation results demonstrate that the controller designed by the above approach is feasible and effective.

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(上接第530页)

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